This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Educ T
120
01.934

Educ T 129.01.934

3 2044 097 012 801

Purchased 9/1/1994

Desk!

EXERCISES IN ALGEBRA.

BY

G. A. WENTWORTH

AND

G. A. HILL.

PART I.: EXERCISE MANUAL.
PART II.: EXAMINATION MANUAL.

WESTFIELD SCHOOL PROPERTY,
Purchased 7/1/1904

BOSTON, U.S.A.:
PUBLISHED BY GINN & COMPANY.
1901.

Entered, according to Act of Congress, in the year 1883,
BY GEORGE A. WENTWORTH AND GEORGE A. HILL,
in the Office of the Librarian of Congress, at Washington.

ALL RIGHTS RESERVED.

44.5

TYPOGRAPHY BY J. S. CUSHING & CO., BOSTON, U.S.A.

PRESSWORK BY GINN & CO., BOSTON, U.S.A.

PREFACE.

THIS book has been prepared to meet a generally-expressed wish for more problems than are found in most text-books on Algebra. It contains about 4500 problems, classified by subjects, and including all the topics usually taught in schools. The exercises have been selected mainly from the rich French collection of Morf and Tzaut. The simplest and the most difficult of the 12,000 problems in that collection have been rejected, and it is believed that these exercises will be not only new to American teachers but well adapted to their wants.

The book may be used in connection with any Algebra for supplementary or review work; and, in order that it may be brought within the reach of scholars as well as teachers, the price has been made as low as possible.

G. A. WENTWORTH.
G. A. HILL.

MAY, 1884.

CONTENTS.

EXERCISE.		PAGE.
1.	NOTATION	1
2.	ADDITION	4
3.	SUBTRACTION	6
4.	MULTIPLICATION	9
5.	DIVISION	13
6.	PARENTHESES	17
7.	MULTIPLICATION AND DIVISION BY INSPECTION	20
8.	RESOLUTION INTO FACTORS	23
9.	HIGHEST COMMON FACTOR	31
10.	LOWEST COMMON MULTIPLE	34
11.	REDUCTION OF FRACTIONS TO LOWEST TERMS	35
12.	ADDITION AND SUBTRACTION OF FRACTIONS	38
13.	MULTIPLICATION AND DIVISION OF FRACTIONS	43
14.	COMPLEX FRACTIONS	47
15.	THE SYMBOLS $\frac{0}{m}, \frac{m}{0}, \frac{0}{0}$	51
16.	VARIOUS EXERCISES IN FRACTIONS	53
17.	SIMPLE NUMERICAL EQUATIONS WITH ONE UNKNOWN QUANTITY	56
18.	LITERAL EQUATIONS WITH ONE UNKNOWN QUANTITY	62
19.	PROBLEMS WITH ONE UNKNOWN QUANTITY	66

20. Simple Numerical Equations with two Unknown Quantities 71
21. Literal Equations with two Unknown Quantities . 76
22. Numerical Equations with More than two Unknown Quantities 80
23. Literal Equations with More than two Unknown Quantities 86
24. Problems with two or More Unknown Quantities . 88
25. Square and Cube Roots 93
26. Pure Quadratics 97
27. Affected Quadratics 98
28. Problems Involving Quadratics 107
29. Roots of Quadratic Equations 114
30. Maxima and Minima 116
31. Simultaneous Quadratics 119
32. Problems Involving Simultaneous Quadratics . . . 122
33. Indeterminate Equations 126
34. Inequalities 129
35. Exponents, Multiplication 132
36. Exponents, Division 144
37. Negative Exponents, Multiplication 138
38. Negative Exponents, Powers, Division 140
39. Fractional Exponents 143
40. Fractional and Negative Exponents 145
41. Powers of Binomials 147
42. Transformation of Radicals 148
43. Reduction of Radicals 150
44. Addition and Subtraction of Radicals 152
45. Multiplication of Radicals 153

46. Division of Radicals 156
47. Simplification of Radicals 159
48. Rationalization of the Denominators of Radicals . 160
49. Imaginary Expressions 161
50. Radical Equations 163
51. Solution of Problems by Logarithms 169
52. Ratio and Proportion 176
53. Arithmetical Progression 179
54. Arithmetical Progression of a Higher Order. . . 183
55. Geometrical Progression 185
56. Permutations and Combinations 188
57. Binomial Theorem 195
58. Probabilities 201
59. Continued Fractions 204
60. Indeterminate Coefficients 208
61. Differential Method 210
62. Equations of Higher Degrees 212
63. Exponential Equations 215

EXERCISE MANUAL.

ALGEBRA.

Ex. 1.

Notation.

1. Write the sum of a and b.
2. Increase a by 8 units.
3. What is the value of m, if $m = n + p$, and $n = 27$, and $p = 36$?
4. If z is a whole number, what is the whole number which immediately follows it?
5. John takes d steps forward, and then f steps backward; how many steps does he take in all?
6. To-day the thermometer indicates m degrees above 0; yesterday it indicated n degrees below 0; how many degrees warmer is it to-day than yesterday?
7. A man sells a horse for c dollars, and loses a dollars; what is the price paid for the horse?
8. If a is a whole number, what whole number immediately precedes it? and what precedes this last?
9. The sum of two numbers is m, and one of them is 18; what is the other?
10. What value does x have when (i.) $x+5=12$? (ii.) $x+37=63$: $x+n=18$: $x+m=p$?
11. A man bought a house for p dollars, and sold it for n dollars; how much did he gain?

12. A man sold a horse for c dollars, and gained 25 dollars; what did he pay for the horse?

13. A man has an income of m dollars; his expenses amount to n dollars; how much has he left (i.) if m is greater than n? (ii.) if m equals n? (iii.) if n is greater than m?

14. A man makes m steps forward, and then n steps backward; at what distance from the point of starting is he?

15. A body moves a feet in one direction, and then b feet in the opposite direction; at what distance from the point of starting is it if $a > b$? $a = b$? $a < b$?

16. Three towns, A, B, and C, are situated in a straight line. The distance from A to B is a miles, that from A to C is b miles; what is the distance from B to C?

17. If the length of a day is s hours, what is the length of the night?

18. Two steamboats move in the same direction, one behind the other. The first goes m yards per minute, and the second n yards. At the end of one minute, what distance separates them when $m > n$? when $n > m$?

19. A steamboat makes, without taking account of the current of the river and the force of the wind, 491 yards per minute. The current, by itself alone, will carry the boat 71 yards per minute, and the wind 100 yards. What distance will the boat go in one minute (i.) when it goes with the current and with the wind? (ii.) when it goes with the current but against the wind? (iii.) when it goes against the current but with the wind? (iv.) when it goes against the current and against the wind?

20. What will the results of the preceding problem become if d, s, and w are substituted for 491, 71, and 100, respectively?

21. What is the product if the multiplier is 7 and the multiplicand is a? if the multiplier is p and the multiplicand q?

22. A rectangular field is a rods long and b rods wide; what is its area?

23. A man pays m dollars for 1 yard of cloth; what must he pay for n yards?

24. A locomotive passes over a yards per second; how many yards will it pass over in t seconds?

25. A man has oats enough for n horses during t days; how long will the oats last 1 horse?

26. A product is p, the multiplicand is m; what is the multiplier?

27. What is the value of y when $9y = 63$? when $8y = 72$? when $43y = 2451$? when $53y = 1537$?

28. A body moves s yards in t seconds; how many yards does it move in 1 second?

29. How many times greater is the velocity of a bullet making m yards per second, than that of a stone making n yards per second?

30. What value has x when $x \div 7 = 9$? when $x \div 23 = 17$? when $35 \div x = 7$? when $703 \div x = 19$?

31. How many rolls of paper, g feet long by h feet wide, will be required to paper a wall a feet long by b feet high?

32. The cogs of 2 wheels fit into one another; the large wheel has n cogs, and the small wheel has m cogs. How many revolutions will the small wheel make while the large wheel is making 1 revolution?

33. Find the values of 3^2; 4^3; 2^{10}; 10^2; 2^4; 4^2.

34. Express the fifth power of 4, the xth power of y, the mth power of n, the pth power of q, the $x+y$th power of a?

35. If $x^y = z$, and $x = 5$, and $y = 7$, what is the value of z?

36. What is the shortest way of representing the product of 7 factors, each equal to 3, and what is its value?

37. What is the shortest way of representing $a \times a \times a \times a \times a \times a \times a \times a \times a \times a$?

38. What is the shortest way of representing the product of x factors, each equal to m?

39. Into how many parts is a line divided, when it is first divided into m parts, and afterwards each of these m parts is divided into m new parts, and so on x times?

40. A man mixes a drop of a certain liquid with 4 drops of water; then he takes a drop of this mixture and mixes it again with 4 drops of water, and so on 6 times. What part of the first drop will finally remain in a drop of this mixture?

Ex. 2.

Addition.

Add:

1. $a^3 + 3a^2b + 3ab^2 + b^3$; $a^3 - 3a^2b + 3ab^2 - b^3$;
$-a^3 + 3a^2b - 3ab^2 + b^3$; $-a^3 - 3a^2b - 3ab^2 - b^3$;
$a^3 + 6a^2b + 6ab^2 + b^3$.

2. $x^4 + 3ax^3 - bx^2 + 3cx - d$; $4x^4 - 6ax^3 + 5bx^2 - 3cx + 2d$;
$-5x^4 - 6ax^3 - 5bx^2 - 3cx - 2d$;
$6x^4 + 7ax^3 - 8bx^2 + 6cx + 7d$;
$-10x^4 - ax^3 + 10bx^2 - cx + d$;
$-x^4 - ax^3 - bx^2 - cx - d$.

3. $a^5+5a^4b+6a^3b^2c-7ab$; $6a^5-a^4b-6a^3b^3c+10ab$;
$-2a^5+4a^4b+12a^3b^3c-10ab$;
$5a^5-16a^4b-11a^3b^3c+13ab$;
$-10a^5+8a^4b+a^3b^3c-6ab$.

4. $-7ax-5a\sqrt{x}+12y$; $5ax+a\sqrt{x}-8y$; $-ax+9a\sqrt{x}-y$;
$4ax-3a\sqrt{x}+5y$; $-ax+a\sqrt{x}+y$.

5. $a^3+\sqrt{x}-10+2\sqrt{ax}$; $2a^3-3\sqrt{x}+20+\sqrt{ax}$;
$-5a^3-3\sqrt{x}+30-5\sqrt{ax}$;
$-4a^3+2\sqrt{x}+1+12\sqrt{ax}$;
$-25a^3-25\sqrt{x}-25-25\sqrt{ax}$.

6. $a^m-2b^n+3c^p-4d^q$; $23a^m+16b^n-13c^p+12d^q$;
$-14a^m+15b^n-17c^p+19d^q$; $18a^m-b^n+c^p-d^q$;
$-13a^m+5b^n-3c^p+11d^q$.

7. $5a^2b-7a^3bc-13b^2c^4+10$; $\frac{1}{2}a^2b+8a^3bc-10b^2c^4+\frac{2}{15}$;
$-\frac{7}{8}a^2b-15a^3bc-7b^2c^4-\frac{4}{21}$;
$-\frac{11}{12}a^2b+23a^3bc+\frac{1}{3}b^2c^4+\frac{11}{14}$;
$-\frac{8}{10}a^2b-\frac{13}{15}a^3bc+\frac{1}{5}b^2c^4+\frac{4}{9}$.

8. $7a-3b+5c-10d$; $2b-3c+d-4e$;
$5c-6a-4e+2d$; $-3b-8c+7a-e$;
$21e-16c+a-5d$.

9. $3ab^2-4a^2b+a^3$; $-4ac^2+5ab^2-c^3$;
$-7b^3+2a^2b-6ac^2$; $5a^3-11ab^2-12ac^2$.

10. $a^3+3a^2b+3ab^2+b^3$; $-5ab^2+3a^2b-b^3+3a^3$;
$+3ab^2-5a^2b+3b^3-3a^3$;
$-5b^3+2a^2b-4a^3+3ab^2$;
$7a^3+6b^3-5a^2b+5ab^2$; $4a^2b+3ab^2-3b^3+4a^3$.

11. $4x^4y^5z^6-3x^3y^4z^5+17x^2y^3z^4-8xy^2z^3$;
$14x^2y^3z^4+4xy^2z^3+5x^3y^4z^5-3x^4y^5z^6$;
$-x^4y^5z^6-2x^3y^4z^5+4xy^2z^3+19x^2y^3z^4$;
$2x^3y^4z^5+5xy^2z^3-7x^4y^5z^6+9x^2y^3z^4$;
$-12xy^2z^3+4x^4y^5z^6-15x^2y^3z^4-x^3y^4z^5$;
$3x^4y^5z^6+41x^2y^3z^4-x^3y^4z^5+7xy^2z^3$.

12. $a^m + 6a^{m-1}b + 10a^{m-2}b^2 + 6a^{m-3}b^3 + b^4$;
$-7b^4 - 3a^{m-2}b^2 + 7a^{m-3}b^3 + 8a^m + 4a^{m-1}b$;
$8a^{m-2}b^2 + 3a^{m-3}b^3 - 5a^m + 3b^4 + 6a^{m-1}b$;
$-3a^{m-1}b + 5a^{m-2}b^2 + 3a^m - 5b^4 - 3a^{m-3}b^3$.

13. $ax^3 + bx^2 - cx$; $5x^2 - bx^3 + 7x$; $-dx^3 + 4x - cx^2$;
$dx^2 + fx^3 - gx$; $mx^3 - nx^2 + px$; $-3x^3 + 7ax^2 - 3dx$.

14. $mx^4y^3 + nx^3y^2 - px^2y + 9x$; $-mx^3y^2 + 8x^4y^3 - px + px^2y$;
$-ax + px^2y - px^4y^3 + mx^3y^2$; $x^4y^3 - x^3y^2 + x - x^2y$;
$px + ax^2y - 2mx^3y^2 + 5mx^4y^3$.

15. $x^p + y^q + z^k - t^m$; $abx^p - mnz^k + amy^q + bt^m$;
$-5abx^p + 3aby^q + 8t^m - az^k$; $at^m - 3bz^k + mx^p + 14y^q$;
$3bcx^p - 4y^q + 3mz^k + nt^m$.

16. $abm^{p+2} + bcn^{q-1} - cdu^{y-3} + fgv^x$;
$-3a^2bm^{p+2} + 3bc^2n^{q-1} + 6c^2d^2u^{y-3} + 5fgv^x$;
$-cn^{q-1} + 3a^2bm^{p+2} - ku^{y-3} + av^x$;
$+5m^{p+2} - 3bn^{q-1} - 3c^2d^2u^{y-3} - 8fgv^x$;
$a^2m^{p+2} - bcn^{q-1} - fv^x + cdu^{y-3}$.

Ex. 3.

SUBTRACTION.

1. From $m^2 - n^2$ take n^2.
2. From $m^2 - n^2$ take m^2.
3. From n^2 take $m^2 - n^2$.
4. From m^2 take $m^2 - n^2$.
5. From $-m^2$ take $m^2 + n^2$.
6. From $-x^n$ take $x^n - y^p$.
7. From $-y^p$ take $x^n - y^p$.
8. From $x^n - y^p$ take $-x^n$.
9. From $x^n + y^p$ take $-y^p$.
10. From y^p take $-x^n - y^p$.
11. From $m - n$ take $2m$.
12. From $m + n$ take $2n$.
13. From $2m$ take $m + n$.
14. From $3n$ take $m + n$.
15. From $8n$ take $3n - 5m$.
16. From $a - 5b$ take $+3b$.
17. From $a - 4b$ take $-3a$.
18. From $2a - b$ take $-2b$.
19. From $a + 2b$ take $+3b$.
20. From $3a - 4b$ take $-b$.

21. From $\frac{2}{3}n - \frac{1}{4}$ take $\frac{2}{3}n + \frac{1}{4}$.

22. From $\frac{4}{5}a - \frac{3}{7}b$
take $\frac{3}{7}b - \frac{4}{5}a$.

23. From $\frac{2}{3}x + \frac{4}{5}y$
take $\frac{2}{3}x - \frac{4}{5}y$.

24. From $1\frac{7}{8}m^5 + \frac{5}{6}$
take $1\frac{7}{8}m^5 - \frac{2}{3}$.

25. From $\frac{3}{5}a - \frac{4}{5}b$
take $\frac{5}{6}a - \frac{7}{6}b$.

26. From $\frac{3}{4}a - \frac{5}{6}b$ take $-\frac{3}{5}a$.

27. From $\frac{5}{6}x - \frac{7}{8}y$ take $\frac{3}{8}y$.

28. From $\frac{4}{5}x^3 - \frac{2}{3}x^2$
take $-\frac{5}{7}x^2$.

29. From $1\frac{1}{2}a^2 - 1\frac{5}{4}b^3$
take $+ 2\frac{1}{2}b^3$.

30. From $a - b$ take $+\frac{5}{6}b$.

31. From $n + x$ take $x - 1$.

32. From $n - x$ take $1 - x$.

33. From $m - x$ take $m + 1$.

34. From $x + y$ take $y - a$.

35. From $a + b$ take $a + c$.

36. From $a + 2$ take $b - 3$.

37. From $a - 5$ take $5 - b$.

38. From $2a - 3b$ take $2b - 3a$.

39. From $3x - y$ take $y + 3x$.

40. From $7x - 5y$ take $2y - x$.

41. From $4x^2 + 2xy + 3y^2$ take $x^2 - xy + 2y^2$.

42. From $a^3 + 3a^2b + 3ab^2 + b^3$ take $a^3 - 3a^2b + 3ab^2 - b^3$.

43. From $a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$
take $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$.

44. From $2x + 11a + 10b - 5c - 23$
take $2c - 10 + 5a - 3b$.

45. From $x^4 + 3ax^3 - 2bx^3 + 3cx - 4d$
take $3x^4 + ax^3 - 4bx^3 - 3cx + d$.

46. From $72x^4 - 78x^3y - 10x^2y^2 + 17xy^3 + 3y^4$
take $-x^4 + 36x^3y - 17xy^3 - 34y^4 + 10x^2y^2$.

47. From $6x^2 + 7xy - 5y^2 - 12xyz - 8yz - 5z^2$
take $-8xy - 7yz + 3x^2 - 4y^2 + 5xyz$.

48. From $-a^2x^2 + 4axy - 3aby^2$
take $4aby^2 - 5axy + 2a^2x^2$.

49. From $10a^m - 15b^n - c^p + 5d^q$
take $-9a^m + 2b^n + c^p - 5d^q$.

50. From $\frac{5}{2}x^2 + 3ax - \frac{7}{8}a^2$ take $2x^2 - ax - \frac{1}{2}a^2$.

51. From $\frac{1}{2}x - 1\frac{3}{6}a + \frac{1}{3}b - \frac{1}{4}y$ take $-\frac{1}{2}y - \frac{3}{5}b + \frac{1}{6}a + 1\frac{1}{4}x$.

52. From $\frac{5}{2}a - 76 - 3bc + \frac{1}{2}c$ take $\frac{3}{4}a - 56\frac{4}{5} + \frac{1}{4}c - 3bc$.

53. From $\frac{1}{2}y - \frac{5}{2}a - \frac{3}{4}x + \frac{1}{3}b$ take $3y + 1\frac{1}{4}a - \frac{2}{3}x$.

54. From the sum of the first four following polynomials take the sum of the last four:
$m^3 + n^3 + p^2 + q^2$, $q^3 + p^2 + n^2$, $m^3 - p^2 + n^2 - q^2$,
$m^2 - n^2 + p^2 + q^2$, $n^2 + p^2 + q^2 - m^2$.

55. From $x^4 + 3ax^3 - 2bx^2 + 3cx - 4d$
take $3ax^4 + ax^3 - 4x^2 + 6cx + d$.

56. From $ax^3 + bx^2 + cx + d$ take $ex^3 - ax^2 + bx + 4$.

57. From $ax^3 + bc^2y - cxy^2 - dy^3$
take $ey^3 + fxy^2 - gx^2y - bx^3$.

58. From $5a^2xy^3 - 4bxy^3 + 9ax^2y + 15y^4$
take $3a^2xy^3 + 5b^2x^2y - 9cxy^3 - 5ay^4$.

59. From $mx^5 - 4mnx^4y + 3mx^3y^2 - 7mx^2y^3$
take $-x^5 + 7abx^3y^2 - 7px^4y + 7x^2y^3$.

60. From $abx^5 - bcx^4 + 3cdx^3 - 2dfx^2$
take $2fgx^2 - 6dex^3 - acx^4 + bcx^5$.

61. From $3mnyz + 7pqy^2z^2 + 4aby^3z^3$
take $-14qy^2z^2 + 9myz - 8ay^3z^3$.

62. From $48a^3b^2 + 56a^2b^3 - 34ab^4 + 59b^5$
take successively the following polynomials:
$13a^3b^2 - 14a^2b^3 + 26ab^4 - b^5$;
$15a^3b^2 + 70a^2b^3 - 60ab^4 + 30b^5$;
$20a^3b^2 + 30b^5$.

If $m = a^2 + b^2 + c^2$, $n = a^2 + b^2 - c^2$, $o = a^2 - b^2 + c^2$, $p = b^2 + c^2 - a^2$, find the value of:

63. $m + n + o + p$.

64. $m - n + o - p$.

65. $m - n - o + p$.

66. $m - n - o - p$.

If $x = 3a^3 - 2ab + 5b^2$, $y = 7a^2 - 8ab + 5b^2$,
$z = 9a^2 - 5ab + 3b^2$, $t = 11a^2 - 3ab - 4b^2$,
find the value of:

67. $x + y + z + t$.
70. $x + y - z - t$.
68. $x - y - z + t$.
71. $x - (y - z - t)$.
69. $y - (x + z - t)$.
72. $y - \{z - [x - (y + t)]\}$.

If $E = 5a^3 + 3a^2b - 7b^3$, $F = 8a^3 - 9a^2b - 3b^3$,
$G = 9a^2b - 3a^3 - 7b^3$, $H = 8a^2b - 7a^3 - 2b^3$,
find the value of:

73. $E - \{F - (G - H)\}$.
74. $G - \{F - [H - (E + G)]\}$.

Ex. 4.

Multiplication.

Multiply:

1. $2x - 4y + z$ by $3x$.
2. $a^3 + 2ab - b^2$ by a^2b^2.
3. $4a + 2ac^3 - 3cd^4 - 1$ by $-a^2d$.
4. $\frac{3}{4}b^2 - 4c^3 + \frac{2}{5}ad^2 - 3$ by $-\frac{2}{3}a^2c$.
5. $-5b^2 + 2bc^3 - 4c$ by $\frac{1}{2}b^2c^3$.
6. $x^3 - 3x^2y + 3xy^2 - 2$ by $-3xy^2$.
7. $3x^3 - 2xy + 4y^2$ by $2a^2x$.
8. $-2a^2b^2 + 5ab^3 - 7b^4$ by $-4ab$.
9. $a^2 - ax + x^2$ by $-ax$.
10. $x^3 - 3x^2y + 3xy^2 - y^3$ by xy.
11. $3x^{2m-1} - \frac{3}{7}y^{3n-5} + x^{2m}y^{3n} - y^3 - 3$ by $-x^{3-2m}y^{5-3n}$.
12. $-a^xb^{2y} + 3a^{4+x}b^{3+2y} - 5a^{3+x}b^{5+2y} + 2$
by $-\frac{5}{6}a^{1-x}b^{3-2y}$.
13. $-25c^p + 14c^{p+1}d + 40c^{2p+1}d^x$ by $-36c^{3-p}d^{3-2x}$.
14. $18m^{x-3}y^4 - 24m^{2x-4}y^5 + 1$ by $75m^{5-x}y^{p-4}$.

15. $34b^{m+n}d^{p-2q} - 27b^{2m-n-1}d^{p-q+2} + \frac{1}{2}b^{1-n}d^{q-1}$ by $-50b^{2n-2m+4}d^{2p+2q}$.

16. $39d^{x+y-1} - 54d^{x-2y+1} + 60d^{x+3y}$ by $30d^{2-x+2y}$.

17. $24x^{m+2n-1} - 42x^{2m-3n+2} + 25x^{2n+3m-2}$ by $25x^{2-m-2n}$.

18. $70y^{m-p-1} - 65y^{3-3m-2p} + 5y^{3p+m}$ by $\frac{3}{5}y^{3p-m+1}$.

19. $3a - 4b$ by $a - b$.

20. $4b - 5c$ by $3b + 4c$.

21. $4x + 9y$ by $x - 5y$.

22. $3a - 2b$ by $2a - b$.

23. $5a^2 - 3ab - 2b^2$ by $a^2 + 2ab$.

24. $8x^3 - 4x^2y + 2xy^2 - y^3$ by $2x - 3y$.

25. $3a^2 - 5ab + 2b^2$ by $a^2 - 7ab$.

26. $5a^3 - 2a^2x + ax^2$ by $2a^2 - ax + x^2$.

27. $3x^2 - 4x + 7$ by $5x^2 - x - 4$.

28. $x^2 + 7x - 5$ by $x^2 - 3x + 7$.

29. $3a^3b - 2a^2b^2 + ab^3$ by $2a^2 - ab - 5b^2$.

30. $x^3 - 6ax^2 + 12a^2x - 8a^3$ by $x^2 - 4ax + 4a^2$.

31. $x^3 - 2x + 1$ by $x^2 - 3x + 2$.

32. $x^4 - 7x^2y^2 + 6xy^3 - y^4$ by $x^2 - 2xy^2 + y^3$.

33. $x^2 - 5ax - 2a^2$ by $x^2 + 2ax + 3a^2$.

34. $2x^4 - 4x^2 - 4x - 1$ by $2x^4 - 4x^2 - 4x - 1$.

35. $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$ by $x^2 - 3x^2y + 3xy^2 - y^3$.

36. Expand $(a - 2b + 3c)^3$.

Multiply:

37. $3x^4 + 6x^3y - 9xy^3 + 12y^4$ by $2x^2 - 3xy + y^2$.

38. $3a^3x - 2a^2x^2 + 5ax^3 - 8x^4$ by $-2a^2 + 6ax - 4x^2$.

39. $2by^5 - 3b^2y^4 + 4b^3y^3 - 5b^4y^2$ by $y^3 - 4by^2 + 6b^2y$.

40. $c^3 - 8c^2z + 5cz^2 + 11z^3$ by $6c^2 - 2cz + 12z^2$.

41. $a^4 - 7a^3b + 6a^2b^2 + 8ab^3 - 2b^4$ by $a^2 - 3ab + 2b^2$.

42. $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$ by $x^2 + 2xy + y^2$.

43. $3a^4b^2 - 6a^3b^3 + 4a^2b^4 + ab^5$ by $2a^2b - 5ab^2 + 2b^3$.

44. $x^4 - 3x^3 + 2x^2 - x + 5$ by $3x^3 - 2x^2 - 5x + 3$.

45. $7a - 2a^2 + 8a^3 - 4a^4$ by $5a + 3a^2 - 7a^3$.

46. $2p^3 + 4p^2q - 3pq^2 - q^3$ by $5p^3 - 2pq - q^2$.

47. $a^5 - 2a^4b + 3a^3b^2 - 2a^2b^3 + ab^4 - b^5$ by $2a^2 - ab + b^2$.

48. $5c^3 - 3c^6 + 9c^4 - 10c^2 + 4$ by $9c^6 + 5c^4 - 13c^3$.

Expand:

49. $(a^2 - 5ab - b^2)^3$.

50. $(x^3 - 3x^2 - 2x)^4$.

Multiply:

51. $2d^5 - 7d^4e + 3d^3e^2 - 9d^2e^3 + 6de^4 + 8e^5$
by $7d^3 + 24d^2e + 70de^2 - 2e^3$.

52. $a^3 - 2a^2x + ax^2 - \frac{1}{3}x^3$ by $3a^2 - ax + 3x^2$.

53. $c^4 - 2c^3 - \frac{1}{2}c^2 + 120c - 8$ by $50c^2 + 18c - \frac{1}{10}c - \frac{1}{2}$.

54. $d^5 - 5cd^4 + \frac{1}{3}c^2d^3 - 9c^3d^2 + \frac{1}{4}c^4d$
by $8d^3 + 7cd^2 - \frac{1}{3}c^2d - c^3$.

55. $24p^5 - 36p^4q - 16p^3q^2 + 20p^2q^3 - 16pq^4 + \frac{1}{2}q^5$
by $-\frac{1}{4}p^3 + 8p^2q + 7pq^2 - 11q^3$.

56. $14c^4 - 18c^3d + 15c^2d^2 - 8cd^3 + \frac{1}{3}d^4$
by $12c^2 + \frac{1}{3}c^4d - 10c^3d^2 - \frac{1}{2}c^2d^3$.

Expand:

57. $\left(\frac{c^2}{3} - \frac{c}{4} - \frac{1}{2}\right)^3$.

58. $\left(\frac{x}{2} - \frac{x^2}{3} + \frac{x^3}{4}\right)^2$.

Multiply:

59. $0.5m^4 - 0.4m^3n + 1.2m^2n^2 + 0.8mn^3 - 1.4n^4$
by $0.4m^3 - 0.6mn - 0.8n^2$.

60. $7x^3 + y^3 - 3xy$ by $2x^3 + y - x$.

61. $2x - x^3 + 4x^2 - 10 + 7x^5$ by $2x^4 - 2x^3 + 3x^2 + 2 + 4x$.

62. $2x^4 - 3x^3y + 2y^4 - 2xy^3 + 4x^2y^2$ by $2xy^2 - y^3 + \frac{1}{2}x^3 - \frac{1}{3}x^2y$.

63. $2b^2d^2+3b^4-7bd^3-5b^3d+d^4$ by $3b^2+3d^2-5bd$.

64. Expand $(-7ab^2+2a^3-3a^2b)^3$.

Multiply:

65. $5p^2x^2+3p^4-9px^3-3p^3x$ by $2p^2x-8x^3+9p^3-7px^2$.

66. $4b^3-2b+1-3b^2$ by $4b^2+5b+2b^3$.

67. $a^3+3a-2a^2+1$ by $4a^2+3-2a$.

68. $bc+ac+ab$ by $a-b+c$.

69. $1-2x+x^2$ by $1-2y+y^2$.

70. $bxz-cxy+ayz$ by $bcx+acy-abz$.

71. x^2+bx-c by x^2-ax+b.

72. x^2-ax-b^2 by $px+x^2-q^2$.

73. $x^2-3ax-6a^2$ by $5bx+8b^2+x^2$.

74. $bx-ax^2+x^3-c$ by $n-mx+x^2$.

75. $bx-ax^2+x^3-c$ by $n+mx+x^2$.

76. $1-ax+bx^2-cx^3$ by $1-x^2+x$.

77. az^2+bz+c by dz^2+ez+f.

78. mz^3-nz^2+q-pz by sz^2+rz^3-u+pz.

79. a^3+3a^2-4a by $-4na+3ma^2+p$.

80. $5nx^2+4mx^3-2px-v$ by $2ax^2-3bx+4d$.

81. $a^m+2a^{m+1}-3a^{m+2}-1$ by $a+1$.

82. $a^n-4a^{n-1}+5a^{n-2}+a^{n-3}$ by $a-1$.

83. $a^n-a^{n-1}b+a^{n-2}b^n$ by $a-b$.

84. $x^{n-1}-x^{n-2}y+x^{n-3}y^2-x^{n-4}y^3+x^{n-5}y^4$ by $x+y$.

85. $a^p-3a^{p-1}+4a^{p-2}-6a^{p-3}+5a^{p-4}$ by $2a^3-a^2+a$.

86. $a^{2n+1}-a^{n+1}-a^n+a^{n-1}$ by $a^{n+2}-a^2-a+1$.

87. $3x^{4n+1}-4x^{3n}+2x^{2n-1}-x^{n-2}$
by $2x^{4n+1}-5x^{3n}-2x^{2n-1}+x^{n-2}$.

88. $m^p+3m^{p-1}n+m^{p-2}n^2-m^{p-3}n^3$ by $m^r-3m^{r-1}n-3m^{r-2}n^2$.

89. Expand $(e^m-e^{m-1}d+e^{m-2}d^2-e^{m-3}d^3)^2$.

Multiply:

90. $2x^{r+5}y^{q-1}+3x^{r+4}y^{2q-1}-x^{r+3}y^{3q-1}+4x^{r+2}y^{4q-1}$
by $2x^{r+2}y^{2-4q}-3x^{r+1}y^{2-3q}+x^{r}y^{2-2q}+4x^{r-1}y^{2-q}$.

91. $a^{p+4}b^{1-q}+a^{p+3}b^{2-q}-2a^{p+2}b^{3-q}-4a^{p+1}b^{4-q}+a^{p}b^{5-q}$
by $2a^{5-p}b^{q}-2a^{4-p}b^{q+1}+6a^{3-p}b^{q+2}-2a^{2-p}b^{q+3}$
$+4a^{1-p}b^{q+4}$.

92. $u^{p+1}-3u^{p+2}+u^{p+3}-2u^{p+4}$ by $2u^{q-p}+3u^{2q-p}-4u^{3q-p}$.

93. $5x^{p-3}y^{r+3}-2x^{p-1}y^{r+1}-x^{p-2}y^{r+2}$
by $3x^{p+4}y^{r-1}+4x^{p+5}y^{r-2}-x^{p+3}y^{r}$.

94. $a^{p}+3a^{p-2}-2a^{p-1}$ by $2a^{p+1}+a^{p+2}-3a^{p}$.

95. $3y^{m}-2y^{m+1}-5y^{m+2}+y^{m+3}$
by $3y^{p-3}+2y^{p-2}-5y^{p-1}-y^{p}$.

Ex. 5.

Division.

Divide:

1. $5a^{6}+15a^{5}+5a+15$ by $a+3$.
2. $35x^{3}+47x^{2}+13x+1$ by $5x+1$.
3. $z^{4}-bz^{3}-b^{2}z^{2}-b^{3}z-2b^{4}$ by $z-2b$.
4. $2a^{4}-6a^{3}+3a^{2}-3a+1$ by $a^{2}-3a+1$.
5. $42a^{4}+41a^{3}-9a^{2}-9a-1$ by $7a^{2}+8a+1$.
6. $x^{6}+4x^{5}-3x^{4}-16x^{3}+11x^{2}+12x-9$
by $x^{3}+x^{2}-5x+3$.
7. $6b^{5}+20b^{4}-12b^{3}-48b^{2}+22b+12$ by $6b^{2}+14b+4$.
8. $24m^{5}+16m^{4}n-36m^{3}n^{2}-36m^{2}n^{3}-8mn^{4}$
by $2m^{2}-2mn-n^{2}$.
9. $d^{6}-3d^{5}+6d^{4}-7d^{3}+6d^{2}-3d+1$ by $d^{2}-d+1$.
10. $2m^{4}-6m^{3}+3m^{2}-3m+1$ by $m^{2}-3m+1$.
11. $p^{6}-p^{5}+2p^{4}-p^{3}+2p^{2}-p+1$ by $p^{4}-2p^{3}+3p^{2}-2p+1$.
12. $6a^{3}x-17a^{2}x^{2}+14ax^{3}-3x^{4}$ by $2a-3x$.

12. $y^5 - 2y^4 - 6y^3 + 4y^2 + 13y - 6$ by $y^3 + 3y^2 + 3y + 1$.

13. $4y^4 - 18y^3 + 22y^2 - 7y + 5$ by $2y - 5$.

14. $4x^2 - 28xy + 49y^2$ by $2x - 7y$.

15. $4a^3 + 4a^2 - 29a + 21$ by $2a - 3$.

16. $6a^3 - 13a^2b + 4ab^2 + 3b^3$ by $2a - 3b$.

17. $6x^4 - 13ax^3 + 13a^2x^2 - 13a^3x - 5a^4$ by $2x^2 - 3ax - a^2$.

18. $a^5 + a^2x^3 + a^4x + a^3x^2 + ax^4 + x^5$ by $a + x$.

19. $45x^4 + 35x^2 - 4 + 18x^3 + 4x$ by $9x^2 + 7x - 2$.

20. $6x^4 + 13xy^3 - x^2y + 4y^4 + 2x^2y^2$ by $2x^2 - 3xy + 4y^2$.

21. $x^6 - a^4x^2 - a^2x^4 - 2ax^5 - 2a^5x + 4a^3x^3 + a^6$
by $x^4 + 2xa^3 - 2x^3a - a^4$.

22. $2x^3 + 7x^2y - 9y^2(x + y)$ by $2x - 3y$.

23. $2x^4 + x^3y - 3xy^3 - 13x^2y^3 + y^4$ by $x^2 - 2xy - y^2$.

24. $x^5 + 4x^3 - 92x - 3x^4 + 26x^2 + 55$ by $x^2 - 3x + 11$.

25. $x^7 - 3x^6 + x^5 - 4 + 12x - 4x^2$ by $x^5 - 4$.

26. $15x^5 - 3xy^4 + 4x^3y^2 + 10x^4y + 6x^2y^3$ by $5x^3 + 3xy^2$.

27. $x^5 - 125x^2 - 9x^3 + 5x^4 - 340x - 300$ by $x^3 - 15x - 50$.

28. $5x^7 - 4x^5y^4 - 22x^6y - 6x^4y^3 + 12x^5y^2 + 8x^2y^5$
by $x^3 - 4x^2y + 2y^3$.

29. $21a^4 - 16a^3b - 5ab^3 + 16a^2b^2 + 2b^4$ by $3a^2 - ab + b^2$.

30. $\frac{2}{3}b^3 + 1\frac{3}{8}bd^2 - \frac{44}{270}b^2d - \frac{5}{12}d^3$ by $\frac{3}{2}b - \frac{5}{8}d$.

31. $-\frac{1}{8}d^5 + d^2 - \frac{4}{24}d^3 + \frac{5}{6}d^4$ by $-\frac{3}{4}d^2 + 2d$.

32. $\frac{9}{16}a^4 - \frac{7}{8}a^3b + \frac{19}{36}a^2b^2 + \frac{1}{6}ab^3$ by $\frac{3}{8}a + \frac{1}{3}b$.

33. $\frac{1}{14}m^3 + \frac{1}{24}m^2n - \frac{25}{12}mn^2 + \frac{14}{9}n^3$ by $\frac{1}{2}m - \frac{7}{8}n$.

34. $\frac{2}{15}x^3 - \frac{1}{12}x^2y - \frac{19}{24}xy^2 + \frac{5}{12}y^3$ by $\frac{1}{3}x - \frac{5}{6}y$.

35. $\frac{1}{2}a^5 + \frac{23}{24}a^4b - \frac{69}{72}a^3b^2 + 1\frac{3}{4}a^2b^3 - 1\frac{3}{8}ab^4 + \frac{3}{8}b^5$
by $\frac{3}{4}a^2 + 2ab - \frac{3}{8}b^2$.

36. $x^5 - \frac{28}{80}x^4 + \frac{81}{10}x^3 - \frac{7}{8}x^2 - \frac{181}{18}x + \frac{5}{8}$ by $x^2 - \frac{x}{6} + 5$.

37. $\frac{1}{8}a^5 + \frac{11}{45}a^4b - 2\frac{18}{80}a^3b^2 - 2\frac{1}{15}a^2b^3 + \frac{7}{10}ab^4 - 3b^5$
by $\frac{3}{8}a^2 - \frac{2}{3}ab - 4b^2$.

39. $\frac{16}{15}y^7+\frac{2}{15}y^6z-3\frac{1}{80}y^5z^2+\frac{283}{180}y^4z^3+1\frac{2}{8}y^3z^4-\frac{7}{12}y^2z^5-\frac{5}{8}yz^6$ by $\frac{8}{5}y^3+\frac{2}{3}y^2z-\frac{5}{8}yz^2$.

40. $2z^7+\frac{14}{15}z^6-2\frac{11}{80}z^5+11\frac{47}{150}z^4-13\frac{3}{8}z^3+1\frac{41}{68}z^2-\frac{6}{7}z+\frac{4}{7}$ by $\frac{4}{5}z^2-\frac{2}{3}z+2$.

41. $-3a^7-\frac{11}{40}a^6b+\frac{287}{120}a^5b^2+\frac{419}{120}a^4b^3-\frac{17}{12}a^3b^4+\frac{1}{8}a^2b^5+8\frac{1}{2}ab^6-6b^7$ by $1\frac{1}{3}a^4-\frac{1}{4}a^3b-\frac{2}{3}a^2b^2+1\frac{1}{2}ab^3-b^4$.

42. x^3-8a^2+2ax by $2a-x$.

43. $-73x^3-25+56x^4+95x-59x^2$ by $-11x+7x^2-3x^3+5$.

44. $26x^5y^3+x^8+6y^8-17xy^5-5x^5y-2x^4y^3-x^2y^4$ by x^2-3y^2-2xy.

45. $48x^5y^3+26xy^4-17x^3y^3+8x^5-22x^4y-8y^5$ by $2x^3-4y^3-3xy$.

46. $8ab^6-8a^6b+24a^2b^5+a^5b^2+14a^3b^4+12a^4b^3+5a^7$ by $7a^2b+2ab^3+5a^3$.

47. $62x^5y^3+43x^7y^5-55x^6y^4-55x^4y^3-6x^3-21x^3y^8+28x^3y$ by $4x^3y-2x-3x^3y^2$.

48. $4y^8z^6+4y^3z-38y^7z^5-42y^5z^3+51y^6z^4+52y^4z^3-15y^3$ by $-3y+4y^4z^3-2y^3z^3+5y^3z$.

49. $1\frac{1}{8}x^5+\frac{11}{144}a^4x+\frac{289}{480}a^2x^3-\frac{7}{8}ax^4-1\frac{1}{2}a^3x^2+\frac{5}{9}a^5$ by $\frac{5}{8}ax-1\frac{1}{2}x^2+\frac{2}{3}a^2$.

50. $7a^4b^3-19a^3b^5+a^7-9a^5b^2+13a^3b^4-b^7+8ab^6$ by $b^3-2ab^2+a^3$.

51. $2a^3-16a+6$ by $a+3$.

52. $49x^3-72xy^2+27y^3$ by $7x-3y$.

53. $4x^5y-4x^3y^3+4x^2y^4-xy^5$ by $2x^2-2xy+y^3$.

54. $4x^4-x^2y^2+6xy^3-9y^4$ by $2x^2-xy+3y^2$.

55. $x^6-5x^4+3x^3+6x^2-7x+2$ by x^3-3x^2+3x-1.

56. x^6y^6-64 by $xy-2$.

57. $y^{10}+y^9+y^8+2y^7+2y^6+2y^5+y^3+y+1$ by $y+1$.

58. a^4-b^4 by $a^3+a^2b+ab^2+b^3$.

59. $3x^5+4x^3y^2+4x^2y^3+xy^4+4y^5$ by $x^3+x^2y+xy^2+y^3$.

60. $12a^4-27a^3b^2+18ab^3-3b^4$ by $6a^2-9ab+3b^2$.

61. $a^5-5a^3b^2-5a^2b^3+b^5$ by $a^2-3ab+b^2$.

62. $a^4+4ab^3-4abc+3b^4+2b^2c-c^2$ by $a^2-2ab+3b^2-c$.

63. $4x^4-5x^3y^3-8x^2+y^4-4y^2+4$ by $2x^2-3xy+y^2-2$.

64. $a-4a^3+12a^4-9a^5$ by $1+2a-3a^2$.

65. $a^{m+2}-a^{m+3}+37a^{m+5}-55a^{m+6}+50a^{m+7}$ by $a^2-3a^3+10a^4$.

66. $6d^{x+3}-5d^{x+2}-6d^{x+1}+19d^x-21d^{x-1}+4d^{x-2}$
by $2d^3+d^2-4d$.

67. $4h^{x+1}-30h^x+19h^{x-1}+5h^{x-2}+9h^{x-4}$
by $h^{x-3}-7h^{x-4}+2h^{x-5}-3h^{x-6}$.

68. $6x^{m-n+2}+x^{m-n+1}-22x^{m-n}+19x^{m-n-1}-4x^{m-n-2}$
by $3x^{3-n}-4x^{2-n}+x^{1-n}$.

69. $6b^{x+y+2}+b^{x+y+1}-9b^{x+y}+11b^{x+y-1}-6b^{x+y-2}+b^{x+y-3}$
by $2b^{y+2}+3b^{y+1}-b^y$.

70. $6x^{8n+2}-23x^{7n+1}+18x^{6n}-x^{5n-1}-3x^{4n-2}+4x^{3n-3}-x^{2n-4}$
by $2x^{4n+1}-5x^{3n}-2x^{2n-1}+x^{n-2}$.

71. $a^{m+n}b^n-4a^{m+n-1}b^{2n}-27a^{m+n-2}b^{3n}+42a^{m+n-3}b^{4n}$
by $a^nb^n-7a^{n-1}b^{2n}$.

72. $x^{mn}-x^ny^{(n-1)m}-y^mx^{(m-1)n}+y^{mn}$ by x^n-y^m.

73. $x^{3n}-3x^{2n}y^n+3x^ny^{2n}-y^{3n}$ by x^n-y^n.

74. $a^{m+n}b^n-4a^{m+n-1}b^{2n}-27a^{m+n-2}b^{3n}+42a^{m+n-3}b^{4n}$
by $a^m+3a^{m-1}b^n-6a^{m-2}b^{2n}$.

Perform the following divisions, and continue the operation to the fifth term of the quotient:

75. $\frac{1}{1+x}$.

76. $\frac{1}{1-x}$.

77. $\frac{x}{x+1}$.

78. $\frac{x}{x^2-1}$.

79. $\frac{a}{1+x}$.

80. $\frac{m}{1-y}$.

81. $\frac{1+2x}{1-3x}$.

82. $\frac{1-2a}{1+2a}$.

83. $\frac{x+y}{x-y}$. 85. $\frac{ay-1}{by-1}$. 87. $\frac{1}{x-2}$. 89. $\frac{1}{-2+x}$.

84. $\frac{1-ax}{1+bx}$. 86. $\frac{1}{1-2x+x^2}$. 88. $\frac{1}{3+x}$. 90. $\frac{1+x}{1-2x+x^2}$.

Calculate to five decimals the numerical values of

91. $1+\frac{x}{1}+\frac{x^2}{1\times2}+\frac{x^3}{1\times2\times3}+\frac{x^4}{1\times2\times3\times4}$ ..... for $x=1$. (1)

92. $4(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}\cdots\cdots)$. (2)

93. $\frac{a}{1}-\frac{a^3}{3}+\frac{a^5}{5}-\frac{a^7}{7}+\frac{a^9}{9}$ ..... for $a=0.2$.

94. $x-\frac{x^3}{1\times2\times3}+\frac{x^5}{1\times2\times3\cdots\cdots5}-\frac{x^7}{1\times2\times3\cdots\cdots7}+\frac{x^9}{1\times2\times3\cdots\cdots9}\cdots\cdots$
for $x=1.5708$ (6 terms). (3)

95. $2\left(\frac{1}{2n+1}+\frac{1}{3(2n+1)^3}+\frac{1}{5(2n+1)^5}+\frac{1}{7(2n+1)^7}+\cdots\cdots\right)$ (4)
for $n=1$.

(1) This series is the development of e^x; and, if $x=1$, it gives the value of e, the base of the system of Napierian logarithms.

(2) This series, known by the name of the Leibnitz series, gives the value of π; but a great number of terms must be calculated to obtain a near approximation.

(3) This series gives the value of the sine of an angle in the function of the arc x, reduced to the radius.

(4) This series, if $n=1$, gives the Napierian logarithm of 2.

Ex. 6.

Parentheses.

Simplify:

1. $(7a-2b)-[(3a-c)-(2b-3c)]$.
2. $(9a-4c)-\{[(3b-4c)+5a]-3b\}$.
3. $2a-(3b+2c)-\{5b-3a-(a+b)+5c-[2a-(c+2b)]\}$.

4. $a-\{2b+[3x-3a-(a+b)]+[2a-(b+c)]\}$.

5. $16-x-\{7x-[8-9x-(3-6x)]\}$.

6. $2a-\{3b+(2b-c)-4c+[2a-(3b-(c-2b))]\}$.

7. $x^4-\{4x^3-[6x^2-(4x-1)]\}-(x^4+4x^3+6x^2+4x+1)$.

8. $ab-[(3bce-2ab)-(5bce-bef)+(3ab-3bef)]$.

9. $1-[1-(2-x)]+[4x-(3-6x)]+[4-(6x-5)]$.

10. $4x-3y-\{(2x+4y)+3x+[y-9x-(2y-x)+(x-y)]\}$.

11. $11x-\{7x-[8x-(9x-(12a-6x))]\}$.

12. $[(2a-3)b^2+(3a-4)b-(2a-5)]\times[(a+1)b-(1-2a)]$.

13. $[(a^2+ab+b^2)m^3-(a+b)m^2-abm]\times[(a-b)m^2+2m-1]$.

14. $[2x^3-(a+b)x^2+abx-(a-b)]\times[(a-b)x^2+abx+(a+b)]$.

15. $[x^4-(a-1)x^3y+(a-2)x^2y^2-(a+2)xy^3]\times[x^2+(a+1)xy+(a-2)y^2]$.

16. $[(b-a)m^3+(a-b)m^2-2(a-b)m]\times[(a+b)m^2-2(b-a)m-2a]$.

17. $[(a+2b+1)x^2-(3a-2b)x+(2a-1)]\times[(a-2b+1)x^2+(3a+2b)x-2b]$.

18. $[(b^2-1)x^2-(b-2)x+(1-b)]^2$.

19. $[9a^2-(1-m)a-(1-m^2)]^3$.

20. $[(2a-3c)y^2-(a-c)y+(2a+c)]\times[(2a+3c)y^2+(a+c)y-(2a-c)]$.

21. $[x^3+(a+1)x^2-(a^2-2a-3)x+(a^3-5a^2+8a-7)]\times[x^2+(a-1)x+(a^2-3a+1)]$.

22. $[y^3+(a+b)y^2+(a^2-b^2)y+(a^3-3a^2b+3ab^2-b^3)]\times[y^2-(a-b)y+(a^2-2ab+b^2)]$.

23. $(x+y+z)(x+y-z)-(x-y+z)(-x+y+z)$.

24. $(15a^2-12b)(2a+3b)-(4a^2+15b)(5a^2-4b)$.

25. $(9a^2-4b^2)(a-1)-(3a-2b)(3a^2+ab-3a-2b)$.

26. $(5x-2y)(x^2-2xy+3y^2-8)+(2x^2+2xy-5y^2+10)$
$(5x-2y)-(3x^2-2y^2+2)(5x-2y)$.

27. $(a^2-b^2)(2b-3a)+(a+b)(8b-7a)a$
$-(a+b)(8b-7a)b$.

28. $15x^2+24y^2-(3x+2y)(5x+6y)$.

29. $26ab-(9a-8b)(5a+2b)-(4b-3a)(15a+4b)$.

30. $12ab+5b^2-(3a-4b)(2a+8b)-(3a-2)(a-b)$.

31. $(4x-3y)(7x+8y)-(8x-9y)(5x+7y)$
$-(3x-2y)(5x+8y)$.

32. $(a^2-b^2)(2a-3b+5c)+(b-a)(3a^2+4bc-5ac)$
$+(b^2-a^2)(4a-3b+c)$.

33. $(a^2-2ab+b^2)(3a^3-5a^2b+3b^3+a)$
$-(a^3-a^2b-ab^2+b^3)(5a^2-5ab+4b^2+1)$
$+(a-b)^2(2a^3+15a^2b+b^3)$.

34. $(x^3+3x^2y+3xy^2+y^3)(a^3-2a^2b+3b^4+1)$
$+(x+y)^3(2a^3+2a^2b-3b^4)-(x+y)^2(3a^3x+3a^2y-2a^2x)$.

35. $[2a^3+(2x-9)a^2-(9x+6)a+27]\div(2a-9)$.

36. $[a^4-pa^3+(q-1)a^2+pa-q]\div(a^2-1)$.

37. $[z^4-2a(a-b)z^3+(a^2+b^2)(a-b)z-a^2b^2]$
$\div[z^2-(a-b)z+b^2]$.

38. $[(a+2b)^2-(c-3d)^2]\div(a+2b+c-3d)$.

39. $(x^3+3xy+y^3-1)\div[x+(y-1)]$.

40. $[a(a-1)x^3+(a^3+2a-2)x^2+(3a^2-a^3)x-a^4]$
$\div(ax^2+2x-a^2)$.

41. $[(x^3-1)y^3-(x^3+x^2-2)y^2+(4x^2+3x+2)y$
$-3(x+1)]\div[(x-1)y^2-(x-1)y+3]$.

42. $(x^4-2x^3y+x^2y^2-a^2x^2+2a^2xy-a^2y^2)\div(x-a)$.

43. $[z^4-(m+n)^2z^2+2mn(m+n)z-m^2n^2] \div [z^2-(m+n)z+mn]$.

44. $[my^6+(m+n)y^5+(m+n+o)y^4+(m+n+o+p)y^3 +(n+o+p)y^2+(o+p)y+p] \div (y^3+y^2+y+1)$.

45. $[20(a+b)^5-46(a+b)^4x+84(a+b)^3x^2-78(a+b)^2x^3 +64(a+b)x^4-32x^5] \div [5(a+b)^3-4(a+b)^2x +5(a+b)x^2-4x^3]$.

46. $[12x^5-31x^4y+(20y^2-29z^2)x^3+(37yz^2+18z^3)x^2 +(15z^4-24yz^3)x-30z^5] \div (3x^2-4xy-5z^2)$.

Ex. 7.

Multiplication and Division by Inspection.

$(a+b)^2=a^2+2ab+b^2$.
$(a-b)^2=a^2-2ab+b^2$.
$(a+b)^3=a^3+3a^2b+3ab^2+b^3$.
$(a-b)^3=a^3-3a^2b+3ab^2-b^3$.
$(a+b)(a-b)=a^2-b^2$.
$(x+a)(x+b)=x^2+(a+b)x+ab$.
$(x+a)(x-b)=x^2+(a-b)x-ab$.
$(x-a)(x-b)=x^2-(a+b)x+ab$.
$(a-b)(a^2+ab+b^2)=a^3-b^3$.
$(a+b)(a^2-ab+b^2)=a^3+b^3$.
$(a+b+c+d)^2=a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd$.
x^n-y^n is divisible by $x-y$ if n is a positive integer.
x^n-y^n is divisible by $x+y$ if n is an *even* positive integer.
x^n+y^n is divisible by $x+y$ if n is an *odd* positive integer.
x^n+y^n is not divisible by $x-y$.

Write by inspection:

1. $(x+y)^2$.
2. $(b-c)^2$.
3. $(m+n)^3$.
4. $(a-b)^3$.
5. $(m+2n)^2$.
6. $(2x-a)^2$.
7. $(3a-b)^3$.
8. $(4a-5b)^2$.
9. $(3x+5y)^2$.
10. $(8z-5)^2$.
11. $(3+2d)^2$.
12. $(-4m+5n)^2$.

13. $(1+2x^2)^2$.
14. $(3+5m^5)^2$.
15. $(4a-3a^2)^2$.
16. $(2x^3+5)^2$.
17. $(7c-4d)^2$.
18. $(8m+4n)^2$.
19. $(9p-3q)^2$.
20. $(5a^3-2b^4)^2$.
21. $(a^2-2b)^2$.
22. $(y+2z)^2$.
23. $(2u+v)^2$.
24. $(5a-b)^2$.
25. $(x^3+y^3)^2$.
26. $(9m^3-5n)^2$.
27. $(bc^3-ab^2)^2$.
28. $(m^2n+p^2q)^2$.
29. $(10-x^2)^2$.
30. $(6+5a^3)^2$.
31. $(7d^2-2)^2$.
32. $(8z^4-9)^2$.
33. $(m+n)(m-n)$.
34. $(1+x)(1-x)$.
35. $(3-y)(3+y)$.
36. $(3x-2y)(3x+2y)$.
37. $(3dy-1)(3dy+1)$.
38. $(5x^3-2y^3)(5x^3+2y^3)$.
39. $(3ab^2-5a^2b)(3ab^2+5a^2b)$.
40. $(4x^2y-6xy^2)(4x^2y+6xy^2)$.
41. $(a^2+3x)[-(3x-a^2)]$.
42. $(5-bx^3)(bx^3+5)$.
43. $(6m+7n^4)(7n^4-6m)$.
44. $(12-ab)(ab+12)$.
45. $(a^4x+ax^4)(ax^4-a^4x)$.
46. $(a+b-c)(a+b+c)$.
47. $(a-b+c)(a-b-c)$.
48. $(x^2+y^2-xy)(x^2+y^2+xy)$.
49. $(2x-y-3z)(2x-y+3z)$.
50. $(x-2y+3z)(x+2y+3z)$.
51. $(a+2b+3c+d)(a-2b+3c-d)$.
52. $(1+x-3x^3-2x^2)(1+x+2x^2+3x^3)$.
53. $(2+a^2+3a^3+d^2)(2-a^2+3a^3-d^2)$.
54. $(a-x)(a+x)(a^2+x^2)$.
55. $(m+1)(m+2)$.
56. $(b^2+2)(b^2+4)$.
57. $(x+2)(x+3)$.
58. $(3y^2+8)(3y^2+12)$.
59. $(x-2)(x+5)$.

43. $[z^4-(m+n)^2z^2+2mn(m+n)z-m^2n^2]$
$\div[z^2-(m+n)z+mn]$.

44. $[my^6+(m+n)y^5+(m+n+o)y^4+(m+n+o+p)y^3$
$+(n+o+p)y^2+(o+p)y+p]\div(y^3+y^2+y+1)$.

45. $[20(a+b)^5-46(a+b)^4x+84(a+b)^3x^2-78(a+b)^2x^3$
$+64(a+b)x^4-32x^5]\div[5(a+b)^3-4(a+b)^2x$
$+5(a+b)x^2-4x^3]$.

46. $[12x^5-31x^4y+(20y^2-29z^2)x^3+(37yz^2+18z^3)x^2$
$+(15z^4-24yz^3)x-30z^5]\div(3x^2-4xy-5z^2)$.

Ex. 7.

Multiplication and Division by Inspection.

$(a+b)^2=a^2+2ab+b^2$.
$(a-b)^2=a^2-2ab+b^2$.
$(a+b)^3=a^3+3a^2b+3ab^2+b^3$.
$(a-b)^3=a^3-3a^2b+3ab^2-b^3$.
$(a+b)(a-b)=a^2-b^2$.
$(x+a)(x+b)=x^2+(a+b)x+ab$.
$(x+a)(x-b)=x^2+(a-b)x-ab$.
$(x-a)(x-b)=x^2-(a+b)x+ab$.
$(a-b)(a^2+ab+b^2)=a^3-b^3$.
$(a+b)(a^2-ab+b^2)=a^3+b^3$.
$(a+b+c+d)^2=a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd$.
x^n-y^n is divisible by $x-y$ if n is a positive integer.
x^n-y^n is divisible by $x+y$ if n is an *even* positive integer.
x^n+y^n is divisible by $x+y$ if n is an *odd* positive integer.
x^n+y^n is not divisible by $x-y$.

Write by inspection:

1. $(x+y)^2$.
2. $(b-c)^2$.
3. $(m+n)^3$.
4. $(a-b)^3$.
5. $(m+2n)^2$.
6. $(2x-a)^2$.
7. $(3a-b)^3$.
8. $(4a-5b)^2$.
9. $(3x+5y)^2$.
10. $(8z-5)^2$.
11. $(3+2d)^2$.
12. $(-4m+5n)^2$.

13. $(1+2x^2)^2$.
14. $(3+5m^5)^2$.
15. $(4a-3a^2)^2$.
16. $(2x^2+5)^2$.
17. $(7c-4d)^2$.
18. $(8m+4n)^2$.
19. $(9p-3q)^2$.
20. $(5a^3-2b^4)^2$.
21. $(a^2-2b)^2$.
22. $(y+2z)^2$.
23. $(2u+v)^2$.
24. $(5a-b)^2$.
25. $(x^2+y^2)^2$.
26. $(9m^3-5n)^2$.
27. $(bc^2-ab^2)^2$.
28. $(m^2n+p^2q)^2$.
29. $(10-x^2)^2$.
30. $(6+5a^2)^2$.
31. $(7d^2-2)^2$.
32. $(8z^4-9)^2$.
33. $(m+n)(m-n)$.
34. $(1+x)(1-x)$.
35. $(3-y)(3+y)$.
36. $(3x-2y)(3x+2y)$.
37. $(3dy-1)(3dy+1)$.
38. $(5x^2-2y^2)(5x^2+2y^2)$.
39. $(3ab^2-5a^2b)(3ab^2+5a^2b)$.
40. $(4x^2y-6xy^2)(4x^2y+6xy^2)$.
41. $(a^2+3x)[-(3x-a^2)]$.
42. $(5-bx^2)(bx^2+5)$.
43. $(6m+7n^4)(7n^4-6m)$.
44. $(12-ab)(ab+12)$.
45. $(a^4x+ax^4)(ax^4-a^4x)$.
46. $(a+b-c)(a+b+c)$.
47. $(a-b+c)(a-b-c)$.
48. $(x^2+y^2-xy)(x^2+y^2+xy)$.
49. $(2x-y-3z)(2x-y+3z)$.
50. $(x-2y+3z)(x+2y+3z)$.
51. $(a+2b+3c+d)(a-2b+3c-d)$.
52. $(1+x-3x^3-2x^2)(1+x+2x^2+3x^3)$.
53. $(2+a^2+3a^3+d^2)(2-a^2+3a^3-d^2)$.
54. $(a-x)(a+x)(a^2+x^2)$.
55. $(m+1)(m+2)$.
56. $(b^2+2)(b^2+4)$.
57. $(x+2)(x+3)$.
58. $(3y^2+8)(3y^2+12)$.
59. $(x-2)(x+5)$.

60. $(3a-7)(3a+4)$.
61. $(2a-11)(2a+5)$.
62. $(4x-3)(4x-9)$.
63. $(5d-6)(5d-11)$.
64. $(m+2)(m+n)$.
65. $(1+3a)(1+b)$.
66. $(4a+1)(4a+b)$.
67. $(3a+4)(3a-n)$.
68. $(5x-a)(5x+1)$.
69. $(3x^2-m)(3x^2+p)$.
70. $(5y^4+3x)(5y^4+2a)$.
71. $(m^3+2)(m^3-2x)$.
72. $(x^2-xy+y^2)(x+y)$.
73. $(y^2-3yz+9z^2)(y+3z)$.
74. $(m^2+mn+n^2)(m-n)$.
75. $(4+2a+a^2)(2-a)$.
76. $(d^2-5d+25)(d+5)$.
77. $(a^6+3a^3+9)(a^3-3)$.
78. $(16m^4+4m^2y^4+y^8)(4m^2-y^4)$.
79. $(49x^6+56x^3y+64y^2)(7x^3-8y)$.
80. $(9m^2+3am+a^2)(3m-a)$.
81. $(2a+4b-5c+2d)^2$.
82. $(3x-2y+4z-5)^2$.
83. $(2p^2+3x^4-2xy-y^2)^2$.
84. $(a+2b^2+3c^3+4d^4)^2$.
85. $(a^2-5b^3+2a-3b^2)^2$.
86. $(\frac{1}{2}x^2-4y+\frac{2}{3}y^2+6z^3)^2$.
87. $(\frac{2}{5}a-\frac{3}{2}a^2-8b^2+\frac{1}{2}bc)^2$.
88. $(0.6m^2-\frac{1}{2}n+0.8p^3-3x^5)^2$.
89. $(0.3ax+x^3-2a^2+4x)^2$.
90. $(x^3-ax^2+2a^2x-a^3)^2$.
91. $(m^3+3dm^2+3d^2m+d^3)^2$.
92. $(y^4-2xy^3+3x^2y^2-4x^3y)^2$.
93. $(x^3+y^3)\div(x+y)$.
94. $(x^3-y^3)\div(x-y)$.
95. $(a^2-y^2)\div(a+y)$.
96. $(h^2-l^2)\div(h-l)$.
97. $(1+x^3)\div(1+x)$.
98. $(y^3-1)\div(y-1)$.
99. $(u^2-v^2)\div(u+v)$.
100. $(x^3-y^3)\div(x-y)$.
101. $(a^4-m^4)\div(a-m)$.
102. $(a^4b^4-x^4y^4)\div(ab-xy)$.
103. $(h^6-m^6)\div(h-m)$.
104. $(x^7-y^7)\div(x-y)$.
105. $(m^8-n^8)\div(m+n)$.
106. $(a^9+b^9)\div(a+b)$.

107. $(4h^2-m^2)\div(2h-m)$.
108. $(1+x^7)\div(1+x)$.
109. $(x^5-1)\div(x-1)$.
110. $(b^5x^5+c^5y^5)\div(bx+cy)$.
111. $(16-x^4)\div(2+x)$.
112. $(81-y^4)\div(3-y)$.
113. $(32a^5+b^5)\div(2a+b)$.
114. $(27m^3-n^3)\div(3m-n)$.
115. $(16x^2-y^2)\div(4x+y)$.
116. $(625u^4-v^4)\div(5u-v)$.
117. $(a^6b^6-x^6)\div(ab-x)$.
118. $(d^4-x^4y^4)\div(d+xy)$.
119. $(1+a^5y^5)\div(1+ay)$.
120. $[(a+b)^2-c^2]\div(a+b-c)$.
121. $[x^2-(a-b)^2]\div(x-a+b)$.
122. $[(a+b)^2-(c-d)^2]\div(a+b+c-d)$.
123. $[(x+y)^5+z^5]\div(x+y+z)$.
124. $[a^7+(y+z)^7]\div(a+y+z)$.
125. $[(m+n)^3-p^3]\div(m+n-p)$.
126. $[(a-b)^4-x^4]\div(a-b+x)$.
127. $[d^6-(e-h)^6]\div(d-e+h)$.
128. $[f^4-(x-y)^4]\div(f+x-y)$.
129. $[m^4n^4-(x+y)^4]\div[mn-(x+y)]$.
130. $(\frac{1}{4}a^2-\frac{1}{9}d^2)\div(\frac{1}{2}a+\frac{1}{3}d)$.
131. $(\frac{1}{8}x^3+y^3)\div(\frac{1}{2}x+y)$.
132. $(\frac{1}{243}a^5-m^5)\div(\frac{1}{3}a-m)$.
133. $(\frac{1}{1250}d^3+\frac{1}{64}h^3)\div(\frac{1}{5}d+\frac{1}{4}h)$.
134. $(1-\frac{8}{27}x^3)\div(1-\frac{2}{3}x)$.
135. $(x^6+y^3)\div(x^2+y)$.
136. $(a^{10}-m^{15})\div(a^2-m^3)$.
137. $(y^{12}-z^4)\div(y^3+z)$.
138. $(m^8-n^{12})\div(m^2-n^3)$.

Ex. 8.

Resolution into Factors.

Resolve into lowest factors:

1. a^2+2ab.
2. $3x^2+2x-1$.
3. $36x^2-49a^2$.
4. x^4-64.
5. $\dfrac{x^2}{a^2}+\dfrac{a^2}{x^2}-2$.
6. x^2-x-6.
7. $(x+1)^6-1$.
8. x^5+1.

9. x^2+8x+7.
10. $4x^2+32x+39$.
11. $4(ab+cd)^2-(a^2+b^2-c^2-d^2)^2$.
12. $a^2-b^2+2bc-c^2+d^2-2ad$.
13. $x^3+(a+b+c)x^2+(bc+ca+ab)x+abc$.
14. $x^5-x^4y+x^3y^2-x^2y^3+xy^4-y^5$.
15. $(a^2+ab+b^2)x^2+2(a^2+b^2)xy+(a^2-ab+b^2)y^2$.
16. $a^3-2a^2b+a^2-4a+8b-4$.
17. $(a+b)^4+4a^2(a+b)^2+16a^4$.
18. $(x+1)^2-5x-29$.
19. $1-7x-30x^2$.
20. x^6-64.
21. $3a^3+6a^2b$.
22. x^2+x-6.
23. $15x^2+19x-56$.
24. $81a^4b^4-256c^4$.
25. x^2-4y^2.
26. $x^4+\frac{288}{169}x^2+1$.
27. $6x^2-13x+6$.
28. $64x^6-729y^6$.
29. $x^2+23x+22$.
30. $9x^2-25y^2$.
31. x^2-6x+5.
32. $x^4+x^2+\frac{1}{4}$.
33. $x^2+24x+144$.
34. x^3-8.
35. $7x^6-21x^4$.
36. x^2-12x.
37. x^4-4.
38. $x^2-16x+63$.
39. $ac-ad+bc-bd$.
40. $1+4x-96x^2$.
41. $4x^2-(y-z)^2$.
42. x^3+27.
43. a^3b^3-1.
44. $(x+y)^7-x^7-y^7$.
45. $-4x^7+64x^6$.
46. $x^{2n}+(a+b)x^ny^n+aby^{2n}$.
47. $x^4-(4a^2+b^2)a^2x^2+4a^6b^2$.
48. $x^3-27y^3+6y(x^2+3xy+9y^2)$.
49. $a^5-a^4b+a^3b^2-a^2b^3+ab^4-b^5$.
50. $x^2-b(c-a)x-ca(b^2+bc+ba+ca)$.
51. $(x+y)^3+z^3-3xy(x+y+z)$.

52. $(a^2+ab+b^2)x^2-2abxy-(a^2-ab+b^2)y^2$.

53. $(x^2-yz)^2+8y^3z^3$.

54. $a^4+b^4-c^4-2a^2b^2$.

55. $x^4-(a+b)x^3+ab(a+b)x-a^2b^2$.

56. $(a+b)^2+(c+d)^2+2(ac+ad+bc+bd)$.

57. $x^4y^6+20x^2y^3z+96z^2$.

58. $a^2cx-abdx-abcy+b^2dy$.

59. $a^2+b^2+c^2+2ab-2ac-2bc$.

60. $x^2+(a+b)x-2a^2-ab$.

61. $36x^4+96x^2+55$.

62. $(x+y+z)^3-x^3-(y^3+z^3)$.

63. $x^4-2xy^3+y^4$.

64. $x^2+\frac{1}{x^2}-\frac{19}{9}$.

65. ax^3+x^2+ax+1.

66. $x^{15}+1024y^{10}$.

67. $(a+b)^3-a^3-b^3$.

68. x^4-11x^2+1.

69. $5x^3-10x^2-5x$.

70. $x^{2m}+2+\frac{1}{x^{2m}}$.

71. $\frac{x^2}{a^2}+\frac{2a^2}{x^2}+3$.

72. $x^4+x^2y^2+y^4$.

73. $x^2-9xy+20y^2$.

74. $6xy-x^2-9y^2$.

75. a^8-81.

76. $(a-2b)^2-4b^2$.

77. $x^9+x^6+x^3-3$.

78. $3x^2+9xy-\frac{10}{3}y^2$.

79. $x^2+ax+bx+ab$.

80. $x^4+y^4-\frac{x^2y^2}{4}$.

81. $x^2+26x+120$.

82. x^4+x^2+1.

83. $a^4b^{12}-c^8$.

84. x^6+x^3-2.

85. $x^4-18x^2y^2+y^4$.

86. $4x^2+4ax$.

87. $x^2+24x+143$.

88. $6a^2x-a^2x^2-9a^4$.

89. a^6-b^6.

90. $x^4-(2a^2+b^2+c^2+2ca+2ab)x^2+[a^2+a(b+c)+bc]^2$.

91. $ax^4+bx^3+mbx-am^2$.

92. $ax^{m+2}y^{n+3} + bx^{m+1}y^{n+2} + cx^m y^{n+1}$.

93. $x^2 + c(a+b)x - ab(a-c)(b+c)$.

94. $(a+b)^2 + (a-b)^2 - 2(a^2-b^2)$.

95. $(1+a)^2(1+b^2) - (1+b)^2(1+a^2)$.

96. $x^3 - (3bc + ca + ab)x + 3abc(b+c)$.

97. $11x^5 + 209x^3y + 528xy^2$.

98. $(a^2-b^2)x^2 + 4abxy + (b^2-a^2)y^2$.

99. $x^3 - y^3 + 2x(x^2-y^2) + 3y^2(x-y)$.

100. $(1-abc)(1+2abc) - (a^3+b^3+c^3)(1-abc)$.

101. $x^3 + x^2y + x^2z - xyz - y^2z - yz^2$.

102. $a^4 - 2(b^2+c^2)a^2 + (b^2-c^2)^2$.

103. $(a^2-b^2)x^2 + 2(a^2+b^2)xy + (a^2-b^2)y^2$.

104. $xy(a^2+b^2) - ab(x^2+y^2)$.

105. $abx^2 + (a^2+b^2)x + ab$.

106. $x^2 + (a+b-c)x - ac - bc$.

107. $2x^{10}y^4 - 60x^5y^2 + 450$.

108. $4abx^2 - 2(a^2+b^2)xy + aby^2$.

109. $(x+y)^2(b+c) - (x+y)(b+c)^2$.

110. $x^2 - 2(a^2+b^2)x + (a^2-b^2)^2$.

111. $x^2 - (y+2)x - 2(y^2+7y+12)$.

112. $6a^4x^2 + a^3x - 6a^3x^3 - a^2x^2$.

113. $8 - 50a^2b^2$.

114. $x^2 + 4xy - 12y^2$.

115. $(a+b)^2 - 4b^2$.

116. $x^2 + 17x + 72$.

117. $x^2 + \frac{5}{6}x + \frac{1}{6}$.

118. $9x^2 + 36$.

119. $\frac{x^2}{a^2} + \frac{a^2}{x^2} + 1$.

120. $x^6 + 19x^3 - 216$.

121. $x^2 - 22x + 121$.

122. $60x^2 - 4x - 45$.

123. $ax^2 + (a+b)x + b$.

124. $4a^2b^2 - (a^2+b^2-c^2)^2$.

125. $20x^2 - 9xy + y^2$.

126. $x^4 + \frac{289}{169}x^2 + 1$.

127. $x^2 + x - 56$.

128. $x^4 - 2x^3y + 3xy^3$.
129. $x^{16} - a^{16}$.
130. $49x^2 - 81y^4$.
131. $x^4 - 13x^2 + 40$.
132. $x^2 + \frac{9}{20}x + \frac{1}{20}$.
133. $x^2 + x - 132$.
134. $x^3 - 20x$.
135. $4x^2y^4 - 16x^4y^2$.
136. $3 - 12x^2$.
137. $x^4 - 2a^2x^2 + a^4x^2$.
138. $4x^2 - 4x - 3$.
139. $a^3 + a^2b + ab^2 + b^3$.
140. $x^{16} - 25^6$.
141. $x^3 + (a - b + c)x^2 - (bc - ca + ab)x - abc$.
142. $(x + y)(b + c - a) - (x + y)(a + b + c)$.
143. $a^2 + 2a(b + c) + b^2 + 2bc + c^2$.
144. $256x^4y^4 - 288x^2y^2 + 81$.
145. $9x^4 + 3x^2y^2 + 4y^4$.
146. $x^2 + (a + 2b)x + ab + b^2$.
147. $ax^3 + (a + b)x^2 + (a + b)x + a$.
148. $b(x^3 - a^3) + ax(x^2 - a^2) + a^2(x - a)$.
149. $a^4 + b^4 + c^4 - 2b^2c^2 - 2c^2a^2 - 2a^2b^2$.
150. $(a + b)x^2 + (a - 2b)xy - 3by^2$.
151. $(a^2 + b^2 - 20)^2 - (a^2 - b^2 + 12)^2$.
152. $(x - y)^3 - z^3 + 3xy(x - y - z)$.
153. $a^2b - bx^2 + a^2x - x^3$.
154. $b^2c^2x^2 - bca^2x - bcx + a^2$.
155. $8x^3 - 6xy(2x + 3y) + 27y^3$.
156. $x^2 + (a - b)x - 2a^2 - 2ab$.
157. $abxy + acx + bcy + b^2y^2$.
158. $5x^4 + 85x^3y + 210x^2y^2$.
159. $x^2 - (a + b + c)x + ab + ac$.
160. $80x^{22} - 11x^{11}y^2 - 60y^4$.
161. $1 - a^2 + x - ab + \frac{1}{4}(x^2 - b^2)$.

162. $a^2+(ac-b^2)x^2-bcx^3$.

163. $x^3-(2a-1)x-6-a+a^3$.

164. x^3+xy.

165. $(x^3-yz)^3+8y^3z^3$.

166. $30x^3y+3x^4+75y^3$.

167. x^2-2x-3.

168. x^4-7x^2+1.

169. $16x^4-81y^4$.

170. $x^6-16a^4x^4$.

171. x^4+64y^4.

172. x^6-x^3-110.

173. $16+44x-42x^2$.

174. $2x^2+x-1$.

175. $a^3x^2y+27x^3y^4$.

176. $7x^3+96x^2-103x$.

177. x^2-6x+8.

178. $1+15x+14x^2$.

179. $x^2+15x+54$.

180. x^4-14x^2+1.

181. $x^4+\frac{697}{841}x^2+1$.

182. x^2-x-a^2-3a-2.

183. $63x^3-21x-42$.

184. $2x^3-x^2-4x+2$.

185. $x^3+x^2y+xy^2+y^3$.

186. $3x^3-15x-42$.

187. $75x^2-35x^3-10x^4$.

188. $x^2-30x+200$.

189. $(a+b)^2-(a-b)^2$.

190. $x^2+11xy+30y^2$.

191. $9x^2+27xy-70y^2$.

192. $x^2-(a^2-b^2)^2$.

193. $(a+b)^2+2(ac+bc)+c^2$.

194. $ab(x^2-y^2)-(a^2-b^2)xy$.

195. $2x^5+2x^4+3x^3+3x^2+4x+4$.

196. $x^2+(a+2b)x+2ab$.

197. $b(x^3+a^3)+ax(x^2-a^2)+a^3(x+a)$.

198. $x^4-16a^2x^2+32a^3x-16a^4$.

199. $2x^2+4ax+6bx+12ab$.

200. $x^4-2(a^2+b^2)x^2-c^4+2(a^2-b^2)c^2+4a^2b^2$.

201. $3(1-x^2)+\frac{a^2}{3}-2a$.

202. $abx^2-(a^2-b^2)x-ab$.

203. $(x+y)(b+c)+(x+y)(b-c)$.

204. $ab(x^2+y^2)+(a^2+b^2)xy$.

205. $a^4-2(b^2+c^2)a^2+b^4-2b^2c^2+c^4$.

206. $x^3-(2a-b)x^2-(2ab-a^2)x+a^2b$.

207. $x^2+4y^2-9z^2-4xy$.

208. $8x^2+12ax+10bx+15ab$.

209. $81(a-b)^2-63(a-b)x-60x^2$.

210. $(x^2+x+1)^2-2x^2-4x-3$.

211. $4x^4+30x^2y^2-100y^4$.

212. $x^2+2ax+3bx+6ab$.

213. $16a^2b^2-(a^2+4b^2-c^2)^2$.

214. $3x^4y^3z^2-6x^3y^4z^3+12x^3y^3z^4$.

215. $x^2-(2a+b)x+2ab$.

216. $28x^2-84x+35$.

217. $x^4-11x^2y^2+y^4$.

218. x^4-171x^2+1.

219. $(a^2+b^2)^2-4a^2b^2$.

220. $(a-b)^3-a^3+b^3$.

221. $x^2+13x-300$.

222. $5-70x^2y^2+245x^4y^4$.

223. $5x^2-15xy-\frac{5}{9}0y^2$.

224. x^4-167x^2+1.

225. $(x^2+3)^2-16x^2$.

226. $9x^3y^3-3xy^5-6y^4$.

227. $x^2+2ax+a^2-b^2$.

228. $(3x-2)^2-(2x-3)^2$.

229. $x^2+9xy-10y^2$.

230. $28x^2y^2+3xy-1$.

231. x^3+2x^2+4x+8.

232. $(ax+by)^2-(ax-by)^2$.

233. $49x^2+49xy+6y^2$.

234. $18a^2b^2+19ab+1$.

235. $ax^2+(a-b)x-b$.

236. $x^2-8xy+12y^2$.

237. $x^8-82x^4y^4+81y^8$.

238. x^6+11x^3-620.

239. $a^3+ab^2-2a^2b-2b^3$.

240. $28x^2+14x-84$.

241. $7x^{10}-21x^5+14$.

242. $21x^2+26xy-15y^2$.

243. $x^2-b(a-c)x-ac(a+b)(b+c)$.

244. $a^2bc+b^2c^2-ab^3-ac^3$.

245. $13x^4-118x^2y^2+240y^4$.

246. $a(x^2+y^2)+(a^2+1)xy$.

247. $x^4+(2-m^2)x^2y^2+y^4$.

248. $x^2(x-a)-bx(x-a)$.

249. $(x^2+5x)^2-8x^2-40x-84$.

250. $x^4+2x^2y^2+(1-m^2)y^4$.

251. $99x^4-13x^2y-132y^3$.

252. $x^4y^2 - 17x^2yz^2 + 16z^4$.

253. $x^2 - c(7x + ab) - a^2b^2 + 12c^2$.

254. $x^4 - (2 + m^2)x^2y^2 + y^4$.

255. $ab(x^2 + y^2) - (a^2 + b^2)xy$.

256. $a^3 - 4a^2b + 4ab^2 - 9ax^2$.

257. $a^2x^4 - (b^2 - 2ac)x^2 + c^2$.

258. $x^4 - (a^2 + b^2)x^2 + a^2b^2$.

259. $xy(1 + z^2) + z(x^2 + y^2)$.

260. $(x + y)^2 - 7(x + y)z - 30z^2$.

261. $abx^2 + c(a - b)x - c^2$.

262. $(x^2 + 6x + 7)^2 - (x + 3)^2$.

263. $9(x + 1)^2 - 3(x + 1) - 6$.

264. $x^4 - 2abx^2 - a^4 - a^2b^2 - b^4$.

265. $45x^4 + 98x^2y^6 - 75y^{12}$.

266. $2a^3 - a^2b + 4abx - 2b^2x$.

267. $x^2 + 2bx - a^2 + b^2$.

268. $x^2 - 2bx + b^2 - a^2$.

269. $99x^2 - 297x + 220$.

270. $a^3b - 2a^2b^2 - 2ab^3$.

271. $x^8y^4 + 15x^4y^2 + 36$.

272. $192x^4 - 480x^2 - 132$.

273. $x^4 - 63x^2y^2 - 64y^4$.

274. $11x^2 - 54xy^2 + 63y^4$.

275. $x^4 + 13x^2y + 36y^2$.

276. $3x^2 + 51xy + 90y^2$.

277. $a^3 - a^2b - 2ab^2 + 2b^3$.

278. $\frac{3}{5}(a + b)^2 - \frac{5}{8}(a - b)^2$.

279. $24a^2x^2 - 14ax^3 - 3x^4$.

280. $3x^4 + 2x^3 - 21x - 14$.

281. $a^7 - a^5b^2 + 2a^4b^3 - a^3b^4$.

282. $abx^2 + (b - a)xy - y^2$.

283. $x^3 - a^2x^2 - b^2x + a^2b^2$.

284. $a^{12} - a^8b^4 - a^4b^8 + b^{12}$.

285. $x^3 + bx^2 - a^2x - a^2b$.

286. $(x - 2y)x^3 - (y - 2x)y^3$.

287. $2a + (a^2 - 4)x - 2ax^2$.

288. $5x^4 + 4x^3 - 20x - 125$.

289. $a^2 + (ac - b^2)x^2 + bcx^3$.

290. $36x^4 - 12x^3 - 35x^2$.

291. $(a + b)^3 + (a - b)^3$.

Ex. 9.

Highest Common Factor.

Find the highest common factor of:

1. $4(x+1)^3$ and $6(x^3-1)$.
2. $18(a^3-b^3)$ and $27(a+b)^3$.
3. $6(x+1)^3$ and $9(x^3-1)$.
4. $12(x^3+y^3)^3$ and $8(x^4-y^4)$.
5. (a^3-x^3) and a^3-x^3.
6. $(a^2x-ax^2)^2$ and $ax(a^2-x^2)^2$.
7. a^6-b^6 and a^4-b^4.
8. x^5-y^5 and x^3-y^3.
9. $12a^4c^2d-48b^4d$ and $15f(a^3c+2b^3)^3$.
10. $6ac+9bc-2ad-3bd$ and $10af+16ag+15bf+24bg$.
11. $bc-2b^2-ac+2ab$ and $2b^2-2ab+3bc-3ac$.
12. $2a^2+5a-4ab-10b$ and $4a^2-6ac+10a-15c$.
13. $16a^2-9b^2$ and $16ab+12b^2+12ac+9bc$.
14. $25x^2-1$ and $35x^2-7x+5ax-a$.
15. $21a^6x^5y^3-28a^4x^7y^5-42a^3x^5y^4$ and $33a^3x^5-44ax^7y^3-66x^5y$.
16. $x^2+2xy+y^2$ and x^3-y^3.
17. $a^3+3a^2b+3ab^2+b^3$ and $a^2+2ab+b^2$.
18. $4x^2-12xy+9y^2$ and $4x^2-9y^2$.
19. $a^2+2ab+b^2$ and a^2-b^2.
20. $x^2+8x+15$ and $x^2+9x+20$.
21. $x^2-9x+14$ and $x^2-11x+28$.
22. $x^2+2x-120$ and $x^2-2x-80$.

23. $x^2-15x+36$ and $x^2-9x-36$.

24. x^2-1 and x^2+4x+3.

25. $6x^2+5x-6$ and $6x^2-13x+6$.

26. $12x^2-17ax+6a^2$ and $9x^2+6ax-8a^2$.

27. $6x^2+13x+6$ and $8x^2+6x-9$.

28. x^3+4x^2-5x and x^3-6x+5.

29. x^3+2x^2-3x and $2x^3+5x^2-3x$.

30. $14-34x+12x^2$ and $42a-4ax-6ax^2$.

31. x^3+2x^2+2x+1 and x^3-2x-1.

32. $x^3-3x^2+7x-21$ and $2x^4+19x^2+35$.

33. $6x^3-6xy^2+2x^2y-2y^3$ and $12x^2-15xy+3y^2$.

34. $a^4+7a^3+7a^2-15a$ and $a^3-2a^2-13a+110$.

35. $3x^3-3x^2y+xy^2-y^3$ and $4x^3-x^2y-3xy^2$.

36. $3a^3-3a^2b+ab^2-b^3$ and $4a^2-5ab+b^2$.

37. $x^4+a^3x-ax^3-a^4$ and x^3-a^3.

38. $x^4-2x^3y+2xy^3-y^4$ and $x^4-2x^3y+2x^2y^2-2xy^3+y^4$.

39. $20x^4+x^2-1$ and $75x^4+15x^3-3x-3$.

40. $x^4-6x^3+13x^2-12x+4$ and $x^5-4x^4+8x^3-16x^2+16x$.

41. $2x^4-11x^2y^2+12y^4$ and $3x^6-48x^2y^4$.

42. $2a^5-4a^4+8a^3-12a^2+6a$ and $3a^5-3a^4-6a^3+9a^2-3a$.

43. $36x^6-18x^5-27x^4+9x^3$ and $27x^2y^2-18x^4y^2-9x^3y^2$.

44. $3x^4-x^3y^3-2y^4$ and $10x^4+15x^3y-10x^2y^2-15xy^3$.

45. $a^6+4a^5-3a^4-16a^3+11a^2+12a-9$
and $6a^5+20a^4-12a^3-48a^2+22a+12$.

46. $2x^3-16x+6$ and $5x^6+15x^5+5x+15$.

47. $a^6-b^4a^2-ba^4+b^5$ and $a^8+b^2a^6+ba^2+b^3$.

48. $4a^5b-4a^3b^3+4a^2b^4-ab^5$ and $24a^5+16a^4b$
$-36a^3b^2-12a^2b^3+8ab^4$.

49. $35x^3+47x^2+13x+1$ and $42x^4+41x^3-9x^2-9x-1$.

50. $x^6-3x^5+6x^4-7x^3+6x^2-3x+1$
and $x^6-x^5+2x^4-x^3+2x^2-x+1$.

51. $2x^4-6x^3+3x^2-3x+1$ and $x^7-3x^6+x^5-4x^4+12x-4$.

52. x^3-1 and $x^{10}+x^9+x^8+2x^7+2x^6+2x^5+x^4+x+1$.

53. $2a^4+3a^3x-9a^2x^2$ and $6a^4x-17a^3x^2+14a^2x^3-3ax^4$.

54. $x^5+3x^4-8x^3-9x-3$ and $x^5-2x^4-6x^3+4x^2+13x+6$.

55. $6x^5-4x^4-11x^3-3x^2-3x-1$
and $4x^4+2x^3-18x^2+3x-5$.

56. $x^4-ax^3-a^2x^2-a^3x-2a^4$ and $3x^3-7ax^2+3a^2x-2a^3$.

57. $x^2+(a-b)x-ab$ and $x^2+(a+b)x+ab$.

58. $x^4-(a^2+b^2)x^2+a^2b^2$
and $x^4-(a+b)^2x^2+2ab(a+b)x-a^2b^2$.

59. $2x^3+(2a-9)x^2-(9a+6)x+27$ and $2x^2-13x+18$.

60. $x^4-px^3+(q-1)x^2+px-q$
and $x^4-qx^3+(p-1)x^2+qx-p$.

61. $x^4-2a(a-b)x^2+(a^2+b^2)(a-b)x-a^2b^2$
and $x^4-(a-b)x^3+(a-b)b^2x-b^4$.

62. $ax^6+(a+b)x^5+(a+b+c)x^4+(a+b+c+d)x^3$
$+(b+c+d)x^2+(c+d)x+d$ and $ax^3+(a+b)x^4$
$+(a+b+c)x^3+(a+b+c)x^2+(b+c)x+c$.

63. $3x^3-(3c+d-3)x-3c-d$
and $2x^2+(2a+b+2)x+2a+b$.

64. x^4-a^4, x^3+a^3, and x^3-a^2.

65. $a^3+a^2b-ab^2-b^3$, $a^3-3ab^2+2b^3$,
and $a^3-2a^2b-ab^2+2b^3$.

66. x^3-2a^2-ax, x^3-6a^2+ax, and x^3-8a^2+2ax.

67. a^5+b^5, a^4-b^4, and a^3+b^3.

68. $3x^3-7x^2y+5xy^2-y^3$, $x^2y+3xy^2-3x^3-y^3$,
and $3x^3+5x^2y+xy^2-y^3$.

Ex. 10.

Lowest Common Multiple.

Find the lowest common multiple of:

1. $25a^3b^4c^5$ and $20a^5b^2c^6$.
2. $a(a+b)$ and $b(a+b)$.
3. $12a(b-1)$ and $12b(b-1)$.
4. $15b(a+b)$ and $18b(a-b)$.
5. $(a+b)(c+d)$ and $(a+b)(c-d)$.
6. a^2-b^2 and $(a-b)^2$.
7. $3(a+x)$ and $4(a^2-x^2)$.
8. a^2bx-ab^2y and $abx+b^2y$.
9. b^2 and $b(b+ax)$.
10. x^2-3x-4 and x^2-x-12.
11. x^2-1 and x^2+4x+3.
12. $6x^2+5x-6$ and $6x^2-13x+6$.
13. $12x^2+5x-3$ and $6x^3+x^2-x$.
14. x^3+5x^2+7x+2 and x^2+6x+8.
15. $12x^2-17ax+6a^2$ and $9x^2+6ax-8a^2$.
16. $a^3-9a^2+23a-15$ and a^2-8a+7.
17. $a^3+2a^2b-ab^2-2b^3$ and $a^3-2a^2b-ab^2+2b^3$.
18. $15x^5+10x^4y+4x^3y^2+6x^2y^3-3xy^4$ and $12x^3y^2+38x^2y^3+16xy^4-10y^5$.
19. $a^5+a^4b+a^3b^2+a^2b^3+ab^4+b^5$ and $a^5-a^4b+a^3b^2-a^2b^3+ab^4-b^5$.
20. mn, mp, and pq.
21. $4x^3y^2z$, $6xy^3z^2$, and $18x^2yz^3$.
22. $x+y$, $x-y$, and x^2-y^2.
23. x^2-y^2, $(x-y)^2$, and $(x+y)$.
24. $6a$, $2(a+1)$, and $2(a+2)$.
25. a, $(a+2b)$, and a^2-4b^2.
26. a, $2a-1$, and $4a^2-1$.
27. $(a+b)$, $2a$, and $2a(a-b)$.
28. $x-1$, x^2+x+1, and x^3-1.

29. x^2+3x+2, x^2+4x+3, and x^2+5x+6.

30. $x^2+5x+10$, $x^3-19x-30$, and $x^3-15x-50$.

31. x^3+2x-3, x^3+3x^2-x-3, and x^3+4x^2+x-6.

32. $6x^2-13x+6$, $6x^2+5x-6$, and $9x^2-4$.

33. $a^3-6a^2+11a-6$, $a^3-9a^2+26a-24$, and $a^3-8a^2+19a-12$.

34. x^2-1, x^2+1, x^4+1, and x^8-1.

35. x^2-1, x^3+1, x^3-1, and x^6+1.

36. $x^2-(a+b)x+ab$, $x^2-(b+c)x+bc$, and $x^2-(c+a)x+ac$.

37. $2x^3+(2a-3b)x^2-(2b^2+3ab)x+3b^2$, and $2x^3-(3b-2c)x-3bc$.

Ex. 11.

Reduction of Fractions to Lowest Terms.

Simplify the following fractions:

1. $\dfrac{(15m^2n^2p)(7p^2n)}{(14p^3)(5m^2q)}$.

2. $\dfrac{10a^2-2ab}{15ab-3b^2}$.

3. $\dfrac{39a^2b^3-36ab^3}{65a^3bc-60a^2bc}$.

4. $\dfrac{42a^3-30a^2x}{35ax^2-25x^3}$.

5. $\dfrac{16a^3b-20a^2c}{28a^2b^3c^2-35ab^2c^3}$.

6. $\dfrac{156a^3b^2-104a^2b^3}{351a^2bx-234ab^2x}$.

7. $\dfrac{21a^2b^2-35b^3c-12a^3c+20abc^2}{18a^2c^2-21a^3b-30bc^3+35ab^2c}$.

8. $\dfrac{40a^2b^4-32a^3bx^2-5b^3x^3+4ax^5}{4ab^2x^3-36a^4x^2-5b^5x+45a^3b^3}$.

9. $\dfrac{14a^3x^5+9a^2b^6-6b^4x^4-21a^5b^2x}{12a^4b^4-28a^7x+21a^3bx^4-9b^5x^3}$.

10. $\dfrac{a^3+(1+a)\,ab+b^3}{a^3+(1-3c^3)\,ab-3a^3c^3}$.

11. $\dfrac{ab(ab-3c^3)-c(3a^3-b^3)}{ac(ac-4b^3)-b(4a^3-c^3)}$.

12. $\dfrac{6x^3-3y-2x^4+x^3y}{9x^3-3y^3-3x^4+x^3y^3}$.

13. $\dfrac{2x^3z^3-3x^3y^3-2y^3z^3+3y^4}{3x^3z^3+2y^4-2x^3y^3-3y^3z^3}$.

14. $\dfrac{6a^3c^3-2a^4+18bc^3-6a^3b}{4a^4+2a^3c^3+12a^3b+6bc^3}$.

15. $\dfrac{15a^3-28bc+12a^3b-35ac-18a^3c+42c^3}{45ab^3+36b^3+20a^3-54b^3c+16a^3b-24a^3c}$.

16. $\dfrac{35a^5+24a^3b^3-21a^3b+15a^3b^3-40a^4b-9ab^3}{28a^3b^3-42a^4-18a^3b^3-32a^3b^3+48a^3b+12ab^4}$.

17. $\dfrac{2x-1}{4x^3-1}$.

18. $\dfrac{3x+1}{9x^3-1}$.

19. $\dfrac{8x^3+1}{64x^6-1}$.

20. $\dfrac{x^3+3x^3}{x^3-9}$.

21. $\dfrac{x^3y-xy}{x^3-1}$.

22. $\dfrac{x^4-y^3}{x^3+xy}$.

23. $\dfrac{9x^5-4x}{6x^3y^3-4y^3}$.

24. $\dfrac{x^6y^4z^3-x^4y^3}{x^6y^3z^4-x^3y^{10}}$.

25. $\dfrac{3ax+9x^3}{4a^3-36x^3}$.

26. $\dfrac{x^3+x^3y}{x^3+2xy+y^3}$.

27. $\dfrac{x^3+2x^3}{x^3+4x+4}$.

28. $\dfrac{12x^3+8xy}{9x^3+12xy+4y^3}$.

29. $\dfrac{3a^3b^3-6b^3c}{a^4-4a^3c+4c^3}$.

30. $\dfrac{7x^3+3}{245x^5+210x^3+45x}$.

31. $\dfrac{15a^3-27ab}{25a^3-90ab+81b^3}$.

32. $\dfrac{a^4-b^4}{a^3+2ab+b^3}$.

33. $\dfrac{x-xy+z-zy}{1-3y+3y^3-y^3}$.

34. $\dfrac{a^3x+abx}{a^3+3a^3b+3ab^3+b^3}$.

35. $\dfrac{20a^5x^3+16a^3bx^3}{75a^4b+120a^3b^2+48b^3}$.

36. $\dfrac{x^4+(2b^2-a^2)x^2+b^4}{x^4+2ax^3+a^2x^2-b^4}$.

37. $\dfrac{x^2+(a+b+c)x+(a+b)c}{a^2+2ab+b^2-x^2}$.

38. $\dfrac{x^2+(b-c-a)x-(b-c)a}{x^2-b^2+2bc-c^2}$.

39. $\dfrac{ax^2+(bc-ab-ac)x+abc-b^2c}{b^2c^2-2abc^2+a^2c^2-a^2x^2}$.

40. $\dfrac{a^2bc-b^3c+2b^2c^2-bc^3}{4a^2b^2-(a^2+b^2-c^2)^2}$.

41. $\dfrac{x^2+y^2+z^2+2xy+2xz+2yz}{x^2-y^2-z^2-2yz}$.

42. $\dfrac{x^2+3x+2}{x^2+6x+5}$.

43. $\dfrac{x^2+10x+21}{x^2-2x-15}$.

44. $\dfrac{x^2-4x+4}{x^2-5x+6}$.

45. $\dfrac{x^2-3x+2}{x^2-1}$.

46. $\dfrac{x^2-7x+12}{x^2-8x+15}$.

47. $\dfrac{x^2-x-20}{x^2+x-30}$.

48. $\dfrac{a^2-9ab+14b^2}{a^2-ab-2b^2}$.

49. $\dfrac{2a^2-ab-3b^2}{2a^2-5ab+3b^2}$.

50. $\dfrac{x^2-9}{x^2-6x+9}$.

51. $\dfrac{2x^3+5x^2-12x}{7x^3+25x^2-12x}$.

52. $\dfrac{4a^2-7ab+3b^2}{5a^2-3ab-2b^2}$.

53. $\dfrac{2x^3+5x^2-3x}{x^3+2x^2-3x}$.

54. $\dfrac{3x^2-10xy+8y^2}{5x^2-13xy+6y^2}$.

55. $\dfrac{12y^3-7y^2-12y}{9y^3+6y^2-24y}$.

56. $\dfrac{6a^2x+ax-x}{3a^2+2a-1}$.

57. $\dfrac{x^4-x^3-x+1}{x^4-2x^3-x^2-2x+1}$.

58. $\dfrac{a^5-a^4b-ab^4+b^5}{a^4-a^3b-a^2b^2+ab^3}$.

59. $\dfrac{bx+2}{2b+(b^2-4)x-2bx^2}$.

60. $\dfrac{(x+y)^7-x^7-y^7}{(x+y)^5-x^5-y^5}$.

Ex. 12.

Addition and Subtraction of Fractions.

1. $\dfrac{a}{m}+2m$.

2. $\dfrac{2x}{x-a}+1$.

3. $2-\dfrac{6a}{3a+2b}$.

4. $\dfrac{a}{2b}-\dfrac{b}{3a}$.

5. $\dfrac{5a}{x^2}-\dfrac{2b}{xy}$.

6. $\dfrac{x}{10y}-\dfrac{3y}{15x}$.

7. $\dfrac{n}{a^2}+\dfrac{x}{5a}$.

8. $\dfrac{a+b}{a}+\dfrac{a-b}{a}$.

9. $\dfrac{a+b}{a}-\dfrac{a+b}{b}$.

10. $\dfrac{a}{b}+\dfrac{a+b}{2b}$.

11. $\dfrac{a+b}{3a}-\dfrac{b}{12a}$.

12. $\dfrac{5a-2b}{2}+\dfrac{3a-5b}{3}$.

13. $\dfrac{15a+4b}{12}-\dfrac{3b-22a}{9}$.

14. $\dfrac{3a+2b}{a}+\dfrac{2a^2-2b^2}{ab}$.

15. $\frac{7}{12}x+\frac{10}{21}y-\frac{1}{7}y-\frac{1}{4}x$.

16. $\frac{4}{5}a-\frac{3}{10}a-\frac{4}{7}b+\frac{1}{14}b$.

17. $\dfrac{2x}{15}+\dfrac{5y}{12}-\dfrac{x}{5}-\dfrac{3y}{4}$.

18. $\dfrac{4a-23b}{4}-\dfrac{4a-25b}{6}+\dfrac{19b-3a}{12}$.

19. $\dfrac{3x-2y}{3}-\dfrac{4y+2x}{5}+\dfrac{22y-9x}{15}$.

20. $\dfrac{29a}{90}-\dfrac{3a-20b}{60}+\dfrac{3a-10b+5c}{20}-\dfrac{4a-2b+3c}{12}$.

21. $\frac{4a-3b+2c}{9}-\frac{b+c-3a}{4}+\frac{b-2a}{2}-\frac{2a-b}{3}+\frac{c-b+2a}{6}$.

22. $\frac{a-4b-3c}{4}-\frac{c-a}{2}-\frac{3a-4b-6c}{3}+\frac{2a-b-3c}{7}$
$-\frac{6b+6c-2a}{21}$.

23. $\frac{3a+2b}{a}+\frac{2a^2-2b^2}{ab}-\frac{2a+3b}{b}$.

24. $\frac{a-b}{a}+\frac{a^2(b+1)+b^2(a+1)}{ab}-\frac{a+b}{b}$.

25. $\frac{7b-5a}{3b}-\frac{c-7a}{a}+\frac{a+5c}{5a}+\frac{11a}{6b}$.

26. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$.

27. $\frac{a}{m}+\frac{a}{n}+\frac{a}{p}$.

28. $\frac{a}{xy}-\frac{b}{xz}-\frac{c}{yz}$.

29. $\frac{2}{x^2}+\frac{3}{ax}-\frac{4}{bx}$.

30. $\frac{1}{2b}-\frac{1}{3b}-\frac{1}{4b}+\frac{1}{5b}$.

31. $\frac{4a}{3x}-\frac{5a}{6x}-\frac{4a}{5x}+\frac{7a}{15x}$.

32. $\frac{3a}{bc}-\frac{5a}{bd}+\frac{4a}{bf}$.

33. $\frac{2a}{5b}+\frac{8a}{3bg}-\frac{2c}{9bd}$.

34. $\frac{a-3b}{6a}+\frac{4a-b}{2b}+\frac{5a+3x}{9x}-\frac{a^2-bx}{2ax}-\frac{2a}{b}$.

35. $\frac{3a-5b}{15ab}-\frac{a-7c}{12ac}+\frac{5b-4c}{20bc}+\frac{3}{4a}+\frac{1}{b}+\frac{5}{6c}$.

36. $\frac{6a+c}{6bc}-\frac{5a-4b}{4ac}-\frac{3b-5c}{5ab}+\frac{3}{5a}-\frac{1}{6b}+\frac{5}{4c}$.

37. $\frac{2}{3a}-\frac{1}{2b}-\frac{2a+3}{6a^2}+\frac{1}{2x^2}+\frac{3a-2b}{6ab}$.

38. $\frac{1}{a-b}+\frac{1}{a+b}$.

39. $\frac{a}{a-b}+\frac{b}{a+b}$.

40. $\frac{c}{a-b}-\frac{c}{a+b}$.

41. $\frac{1}{x+y}+\frac{2y}{x^2-y^2}$.

42. $\frac{1+3a}{1-3a}-\frac{1-3a}{1+3a}$.

43. $\frac{a}{x(a-x)}-\frac{x}{a(a-x)}$.

44. $\frac{a}{2a-2b}-\frac{b}{2b-2a}$.

45. $\frac{2a+3x}{2a-3x}+\frac{2a-3x}{3x-2a}$.

46. $\frac{a-b}{2(a+b)}+\frac{a^2+b^2}{a^2-b^2}$.

47. $\frac{5x-3}{x+1}-\frac{2x^2-13x+1}{x^2-1}$.

48. $\frac{x(a+x)}{a-x}-\frac{5ax-x^2}{x-a}$.

49. $\frac{x}{a^n+1}+\frac{x}{a^n-1}$.

50. $\frac{a}{a-x}+\frac{3a}{a+x}-\frac{2ax}{a^2-x^2}$.

51. $\frac{2b-a}{x-b}+\frac{b-2a}{x+b}+\frac{3x(a-b)}{x^2-b^2}$.

52. $\frac{a-b}{b}+\frac{2a}{a-b}-\frac{a^3+a^2b}{a^2b-b^3}$.

53. $\frac{3}{x}-\frac{5}{2x-1}-\frac{2x-7}{4x^2-1}$.

54. $\frac{5a}{6b}-\frac{10a^2-17ab}{12ab-6b^2}-\frac{b}{b-2a}$.

55. $\frac{2}{x}+\frac{3}{1-2x}-\frac{2x-3}{4x^2-1}$.

56. $\frac{1}{x-2}-\frac{3}{x+2}+\frac{2x}{(x+2)^2}$.

57. $\frac{a+x}{a-x}+\frac{a-x}{a+x}-\frac{a^2-x^2}{a^2+x^2}$.

58. $\frac{1}{x+1}-\frac{2}{x+2}+\frac{1}{x+3}$.

59. $\frac{x}{x-1}-\frac{2x}{x+1}+\frac{x}{x-2}$.

60. $\frac{5}{2(x+1)}-\frac{1}{10(x-1)}-\frac{24}{5(2x+3)}$.

61. $\frac{x-1}{x+1}+\frac{x+1}{x-1}+\frac{3x-2}{x+3}+\frac{3x-4}{x-2}$.

62. $\frac{a+x}{a-x}+\frac{a-x}{a+x}+\frac{a^2+x^2}{a^2-x^2}+\frac{4ax}{a^2+x^2}$.

63. $\frac{1}{1+x}-\left[\frac{6}{1-x}-\left(\frac{2}{1+2x}-\frac{16}{2x-1}\right)\right]$.

64. $\frac{1}{x^2(x^2+y^2)}-\frac{1}{2x^3(y-x)}+\frac{1}{2x^3(x+y)}$.

65. $\frac{3}{2a-3}-\frac{2}{3+2a}-\frac{2a+15}{4a^2+9}$.

66. $\frac{a+c}{(x-a)(b-a)}-\frac{b+c}{(x-b)(b-a)}$.

67. $\frac{1}{(n+1)(n+2)}-\frac{1}{(n+1)(n+2)(n+3)}-\frac{1}{(n+1)(n+3)}$.

68. $\frac{b+c}{(a-b)(a-c)}+\frac{a+c}{(b-c)(b-a)}+\frac{a+b}{(c-a)(c-b)}$.

69. $\frac{1}{x^2-a^2}+\frac{1}{(x+a)^2}-\frac{1}{(x-a)^2}$.

70. $\frac{1}{(x-2)(x-3)}+\frac{2}{(x-1)(3-x)}+\frac{1}{(x-1)(x-2)}$.

71. $\frac{a^2-bc}{(a+b)(a+c)}+\frac{b^2-ac}{(b+c)(b+a)}+\frac{c^2-ab}{(c+a)(c+b)}$.

72. $\frac{bc}{(c-a)(a-b)}+\frac{ac}{(a-b)(b-c)}+\frac{ab}{(b-c)(c-a)}$.

73. $\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-c)(b-a)}+\frac{1}{c(c-a)(c-b)}$.

74. $\frac{x-y}{x+y}+\frac{y-z}{y+z}+\frac{z-x}{z+x}+\frac{(x-y)(y-z)(z-x)}{(x+y)(y+z)(z+x)}$.

75. $\frac{2}{x+4}-\frac{x-3}{x^2-4x+16}+\frac{x^2}{x^3+64}$.

76. $\frac{x^2+ax+a^2}{x^3-a^3}-\frac{x^2-ax+a^2}{x^3+a^3}$.

77. $\frac{2}{(x-1)^3}+\frac{1}{(x-1)^2}+\frac{2}{x-1}-\frac{1}{x}$.

78. $\frac{x^2+y^2}{xy}-\frac{x^2}{xy+y^2}-\frac{y^2}{x^2+xy}$.

WESTFIELD SCHOOL PROPERTY
Purchased 19

79. $\frac{x^2-2x+3}{x^3+1}+\frac{x-2}{x^2-x+1}-\frac{1}{x+1}$.

80. $\frac{x-y}{x^2-xy+y^2}+\frac{1}{x+y}+\frac{xy}{x^3+y^3}$.

81. $\frac{1}{x-y}+\frac{x-y}{x^2+xy+y^2}+\frac{xy-2x^2}{x^3-y^3}$.

82. $\frac{1-2x}{3(x^2-x+1)}-\frac{2x-3}{2(x^2+1)}+\frac{1}{6(x+1)}$.

83. $\frac{x+1}{x^2+x+1}+\frac{x-1}{x^2-x+1}+\frac{2}{x^4+x^2+1}$.

84. $\frac{1}{x-3}+\frac{x-1}{x^2+3x+9}+\frac{x^2+x-3}{x^3-27}$.

85. $\frac{2a}{a^4-a^2+1}-\frac{1}{a^2-a+1}+\frac{1}{a^2+a+1}$.

86. $\frac{1}{a^2-7a+12}+\frac{2}{a^2-4a+3}-\frac{3}{a^2-5a+4}$.

87. $\frac{2+3x-5x^2}{x^3-3x^2+3x-1}-\frac{2x+5}{x^2-2x+1}+\frac{1}{x-1}$.

88. $\frac{7+3x^2}{4-x^2}-\frac{5-2x^2}{4+4x+x^2}-\frac{3-2x+x^2}{4-4x+x^2}$.

89. $\frac{x+3}{2x-1}-\frac{x^2-5}{4x^2-4x+1}-\frac{2x^2-x-3}{8x^3-12x^2+6x-1}$.

90. $\frac{3a^2-2ax+x^2}{a^3-6a^2x+12ax^2-8x^3}+\frac{a+2x}{2a^2-8ax+8x^2}$.

91. $\frac{x^2+8x+15}{x^2+7x+10}-\frac{x-1}{x+2}$.

92. $\frac{x^2-5ax+6a^2}{x^2-8ax+15a^2}-\frac{x-7a}{x-5a}$.

93. $\dfrac{(x-3a)(x-7a)}{x^2-10ax+24a^2}-\dfrac{x-5a}{x-4a}$.

94. $\dfrac{2+x}{3+x^2}-\dfrac{3-x}{9+6x+x^2}+\dfrac{1-2x^2}{(3+x)(3+x^2)}$.

95. $\dfrac{2-3x}{6-4x}+\dfrac{3-4x}{6+4x}-\dfrac{1-2x+x^2}{9-12x+4x^2}+\dfrac{2-5x^2}{36-16x^2}$.

96. $\dfrac{z^4-2z^2-3}{15z^6-17z^3-18+25z^4}-\dfrac{z^2-4z+1}{12z^4-z^3-6}$.

97. $\dfrac{5x-2}{x^2-3x-4}+\dfrac{2x+1}{x^2-x-12}-\dfrac{3x-1}{x^2+x-20}$.

98. $\dfrac{x-1}{x^2-7x+10}-\dfrac{x+2}{x^2-9x+14}-\dfrac{x-3}{x^2-12x+35}$.

99. $\dfrac{a-5x}{4a^2-20ax+25x^2}-\dfrac{3a^2-4ax+x^2}{24a^3-180a^2x+450ax^2-375x^3}$.

100. $\dfrac{a^2-bc}{(a-b)(a-c)}+\dfrac{b^2+ac}{(b-a)(b+c)}+\dfrac{c^2+ab}{(c-a)(c+b)}$.

Ex. 13.

Multiplication and Division of Fractions.

1. $\dfrac{6p^2q^2r^3}{7mx^5y^6}\times\dfrac{3q^7r^6}{5mx^6y^7}$.

2. $\dfrac{12x^3}{5y^2}\times\dfrac{10xy}{9z^2}$.

3. $\dfrac{3a^3c^2d}{4pqm}\times\dfrac{16pm}{27a^4c^3}$.

4. $\dfrac{a^2}{bc}\times\dfrac{b^2}{ac}\times\dfrac{c^2}{ab}$.

5. $\dfrac{a^2b}{x^2y}\times\dfrac{b^2c}{y^3z}\times\dfrac{x^2y^3z}{abc}$.

6. $\dfrac{5a^2b}{3cd}\times\dfrac{4b^2c}{10a^3}\times\dfrac{9c^2d}{16b^3}$.

7. $\dfrac{2a^3x^7}{3b^8}\times\dfrac{5a^4b^5}{4c^4x^6}\times\dfrac{18ab^2c^3}{25a^4x}$.

8. $\frac{5a^3b^2}{7m^2n^4} \times \frac{14a^9m^7}{25n^5b^{11}} \times \frac{5n^{11}m^6}{6a^{15}b^{13}} \times \frac{6am}{b^3n}$.

9. $\left(\frac{a}{b}\right)^2$.

10. $\left(\frac{2a}{5b}\right)^3$.

11. $\left(\frac{5xy}{7ab}\right)^4$.

12. $\left(\frac{2ab}{3cx}\right)^5$.

13. $\left(\frac{2a^2b^3x^4}{3y^5}\right)^3$.

14. $\left(\frac{a^2y^3x^{n-3}}{2b^{2-1}z^4}\right)^5$.

15. $\frac{3(2a^2x^5)^2}{4(3a^3x^4)^3}$.

16. $\left(\frac{3a^2x^2}{4bc^3}\right)^2 \times \left(\frac{2a^2c^4}{b^3x}\right)^3$.

17. $\frac{(3a^4b)^2}{(5cd)^3} \times \frac{(5c)^4}{(6a)^5} \times \frac{(4b)^6}{40a^3b^5c}$.

18. $\frac{3(a^2-b^2)}{5bc} \times \frac{10c}{9(a+b)}$.

19. $\frac{x^2-y^2}{x^2+y} \times \frac{4x}{x+y}$.

20. $\frac{(-5b^3c^5)^2}{[(-2a^2)^5]^2}$.

21. $\frac{a+b}{5} \times \frac{2}{a+b}$.

22. $\frac{x+1}{y} \times \frac{4y^3}{x^3-1}$.

23. $\frac{x^2-y^2}{x^2+y^2} \times \left(-\frac{3x^3}{4x+4y}\right)$.

24. $\frac{ab+ac}{bd+cd} \times \frac{ab-ac}{bd-cd}$.

25. $\frac{4x+4y}{3x^2-3y^2} \times \frac{5x^2-5y^2}{7x+7y}$.

26. $\frac{3abc-3abd}{5fc-5fd} \times \frac{4(a+b)}{a^2-b^2}$.

27. $\frac{3x^2-x}{a} \times \frac{2a}{2x^2-4x}$.

28. $\frac{3x^2}{5x-10} \times \frac{3x-6}{4x^2}$.

29. $\frac{(x-y)^2}{x+y} \times \frac{y}{x(x-y)}$.

30. $\frac{x^4-y^4}{x^2-2xy+y^2} \times \frac{x-y}{x^2+xy}$.

31. $\frac{x(a-x)}{a^2+2ax+x^2} \times \frac{a(a+x)}{a^2-2ax+x^2}$.

32. $\frac{a^6-b^6}{a^4+2a^2b^2+b^4} \times \frac{a^2+b^2}{a^2-ab+b^2} \times \frac{a+b}{a^3-b^3}$.

33. $\dfrac{x^2-(m+n)x+mn}{x^2-(m+p)x+mp}\times\dfrac{x^2-p^2}{x^2-n^2}$.

34. $\dfrac{a^3-b^3}{a^3+b^3}\times\dfrac{a+b}{a-b}\times\left(\dfrac{a^2-ab+b^2}{a^2+ab+b^2}\right)^2$.

35. $\dfrac{a^3+3a^2b+3ab^2+b^3}{a^2-b^2}\times\dfrac{a+b}{a^2+2ab+b^2}$.

36. $\dfrac{2a^3+13a^2x-15ax^2-126x^3}{a^2x+6ax^2-7x^3}\times\dfrac{2a^3+19a^2x+35ax^2}{a^3-a^2x-4ax^2-6x^3}$.

37. $(a+b)\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$.

38. $(2a+3b)\left(\dfrac{b}{a}-\dfrac{a}{b}\right)$.

39. $\left(\dfrac{1}{a}+\dfrac{1}{b}\right)\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$.

40. $\left(\dfrac{3a}{4b}-\dfrac{2c}{3d}\right)\left(\dfrac{4d}{3c}-\dfrac{3b}{2a}\right)$.

41. $\dfrac{x^2+xy}{x^2+y^2}\left(\dfrac{x}{x-y}-\dfrac{y}{x+y}\right)$.

42. $\left(1-\dfrac{a-b}{a+b}\right)\left(2+\dfrac{2b}{a-b}\right)$.

43. $\dfrac{a^2-x^2}{a+b}\times\dfrac{a^2-b^2}{ax+x^2}\times\left(a+\dfrac{ax}{a-x}\right)$.

44. $\left(\dfrac{a}{bc}-\dfrac{b}{ac}-\dfrac{c}{ab}-\dfrac{2}{a}\right)\left(1-\dfrac{2c}{a+b+c}\right)$.

45. $\left(\dfrac{x^2}{a^2}+\dfrac{a^2}{x^2}-\dfrac{x}{a}-\dfrac{a}{x}+1\right)\left(\dfrac{x}{a}-\dfrac{a}{x}\right)$.

46. $\left(\dfrac{x}{a}-\dfrac{a}{x}+\dfrac{y}{b}-\dfrac{b}{y}\right)\left(\dfrac{x}{a}-\dfrac{a}{x}-\dfrac{y}{b}+\dfrac{b}{y}\right)$.

47. $\dfrac{2a^2b^3c}{3x^2y^3}\times\dfrac{a^mb^nc^r}{x^my^n}\times\dfrac{6x^{m-1}y^{n-2}}{a^{m+1}b^{n+2}c^{r+3}}$.

48. $\dfrac{9a^2b^3}{8x^3y^n}\times\dfrac{10a^{n-1}x^{n+2}}{21b^{m+3}y^{m-n}}\times\dfrac{x}{a}$.

49. $\dfrac{3a^3c^m}{7x^3b^n}\times\dfrac{49x^{m-1}b^{n+2}}{9a^{m+5}c^{m+1}}\times\dfrac{a^2x}{c}$.

50. $\frac{a^{x-1}b^{x-2}c^{x-3}}{x^{n-1}y^{n+2}z^{n-3}} \times \frac{x^{n+1}y^{n-2}z^{n+4}}{a^{x-2}b^{x+1}c^{x-1}} \times \frac{a^2c^2}{b^2x^2}$.

51. $\frac{a^{m-n}b^{n-p}c^{p-m}}{x^{n-p}y^{p-m}z^{m-n}} \times \frac{a^{n-p}b^{p-m}c^{m-n}}{x^{p-1}y^{m-2}z^{n-3}} \times \frac{x^ny^pz^m}{a^mb^nc^p}$.

52. $\frac{18xy}{25zu} \div \frac{6mn}{35pq}$.

53. $\frac{7ab}{3mn} \div \frac{5pq}{11xy}$.

54. $\frac{14a^2b^3c}{39d^2f^5g^6} \div \frac{35d^7f^4g^8}{9a^4b^5c^2}$.

55. $\frac{25p^4q^5r^6}{49x^4y^5z^6} \div \frac{30p^7qr^8}{77xy^7z^2}$.

56. $\frac{m+n}{m-n} \div \frac{n+m}{n-m}$.

57. $\frac{3p-3q}{5p+5q} \div \frac{9q-9p}{10q+10p}$.

58. $\frac{x^2+y^2}{x^2-y^2} \div \frac{3x^2+3y^2}{x+y}$.

59. $\frac{6ab-6b^2}{a(a+b)} \div \frac{2b^2}{a(a^2-b^2)}$.

60. $\frac{45(x-y)}{32(z+y)} \div \frac{27(x-y)^2}{128b(z+y)^2}$.

61. $\frac{a^2-4x^2}{a^2+4ax} \div \frac{a^2-2ax}{ax+4x^2}$.

62. $\frac{8a^3}{a^3-b^3} \div \frac{4a^3}{a^2+ab+b^2}$.

63. $\frac{x^3+3x^2y+3xy^2+y^3}{a^3+b^3} \div \frac{(x+y)^2}{a^2-ab+b^2}$.

64. $\frac{y^2+(a+c)y+ac}{y^2+(b+c)y+bc} \div \frac{y^2-a^2}{y^2-b^2}$.

65. $\frac{x^2+y^2+2xy-z^2}{z^2-x^2-y^2+2xy} \div \frac{x+y+z}{y+z-x}$.

66. $\frac{a^2+ab+b^2}{a^3+b^3} \div \frac{a^3-b^3}{a^2-ab+b^2}$.

67. $\frac{a^2-3a+2}{a^2-6a+9} \div \frac{a^2-2a+1}{a^2-5a+6}$.

68. $\frac{x^3+3x^2y+3xy^2+y^3}{a^3-b^3} \div \frac{(x+y)^2}{a^2+ab+b^2}$.

69. $\dfrac{m^3-y^3}{m^2y^2-m^4} \div \dfrac{m^2+my+y^2}{my^2+y^3}.$

70. $\dfrac{a^2+ab}{a^2+b^2} \div \dfrac{ab\,(a+b)^2}{a^4-b^4}.$

71. $\dfrac{a^4-3a^3x+3a^2x^2-ax^3}{a^3b-b^4} \div \dfrac{a^4-2a^2x+a^2x^2}{a^2b^2+ab^3+b^4}.$

72. $\dfrac{ax-a}{bx+b} \div \dfrac{c-cx}{2b+bx}.$

73. $\dfrac{64m^3-96m^2n+36mn^2}{36m^2n-729n^3} \div \dfrac{48m^2n-27n^3}{8m^3-72m^2n+162mn^2}.$

74. $\dfrac{p^3+3p+9}{p^4-3p^2+9} \div \dfrac{p^3-27}{p^6+27}.$

Ex. 14.

Complex Fractions.

Simplify the following expressions:

1. $\dfrac{\dfrac{x}{m}+\dfrac{y}{m}}{\dfrac{z}{m}}.$

2. $\dfrac{\dfrac{m}{x}-\dfrac{c}{x}}{\dfrac{a}{x}+\dfrac{d}{x}}.$

3. $\dfrac{\frac{2}{5}mn-\frac{1}{5}mp}{np}.$

4. $\dfrac{\frac{3}{4}mn-\frac{5}{8}mn}{\frac{3}{8}m}.$

5. $\dfrac{x-\frac{2}{3}y}{z-\frac{1}{3}u}.$

6. $\dfrac{x-\dfrac{y}{z}}{z-\dfrac{y}{x}}.$

7. $\dfrac{\dfrac{ab}{c}-3d}{3c-\dfrac{ab}{d}}.$

8. $\dfrac{1+\dfrac{1}{a-1}}{1-\dfrac{1}{a+1}}.$

9. $\dfrac{1+\dfrac{y}{x-y}}{1-\dfrac{y}{x+y}}.$

10. $\dfrac{m+\dfrac{mn}{m-n}}{m-\dfrac{mn}{m+n}}$

11. $\dfrac{\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}}{\dfrac{x-y}{x+y}+\dfrac{x+y}{x-y}}$

12. $\dfrac{\dfrac{m}{n}+\dfrac{p}{q}}{\dfrac{o}{n}-\dfrac{t}{q}}$

13. $\dfrac{x+y+\dfrac{y^2}{x}}{x+y+\dfrac{x^2}{y}}$

14. $\dfrac{x+\dfrac{y-x}{1+xy}}{1-\dfrac{(y-x)x}{1+xy}}$

15. $\dfrac{a-1+\dfrac{6}{a-6}}{a-2+\dfrac{3}{a-6}}$

16. $\dfrac{\dfrac{2m+n}{m+n}-1}{1-\dfrac{n}{m+n}}$

17. $\dfrac{\dfrac{x^2+y^2}{x^2-y^2}}{\dfrac{x^2-xy+y^2}{x-y}}$

18. $\dfrac{3}{x+1}-\dfrac{2x-1}{x^2+\dfrac{x}{2}-\dfrac{1}{2}}$

19. $\dfrac{y-a}{y-\dfrac{(y-b)(y-c)}{y+a}}$

20. $1-\dfrac{1}{1+\dfrac{1}{a}}$

21. $1+\dfrac{a}{1+a+\dfrac{2a^2}{1-a}}$

22. $\dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{a}}}$

23. $\dfrac{1}{1+\dfrac{a}{1+a+\dfrac{2a^2}{1-a}}}$

24. $\dfrac{1}{a+\dfrac{1}{1+\dfrac{a+1}{3-a}}}$

25. $\dfrac{a}{b+\dfrac{c}{d+\dfrac{e}{f}}}$

26. $\dfrac{x+\dfrac{1}{y}}{x+\dfrac{1}{y+\dfrac{1}{z}}} - \dfrac{1}{y(xyz+x+z)}$.

27. $\dfrac{3abc}{bc+ca-ab} - \dfrac{\dfrac{a-1}{a}+\dfrac{b-1}{b}+\dfrac{c-1}{c}}{\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{1}{c}}$.

28. $\left(\dfrac{m+n}{m-n}+\dfrac{m^2+n^2}{m^2-n^2}\right) \div \left(\dfrac{m-n}{m+n}-\dfrac{m^2-n^2}{m^2+n^2}\right)$.

29. $\left(\dfrac{x-y}{x+y}-\dfrac{x^2-y^2}{x^2+y^2}\right) \div \left(\dfrac{x+y}{x-y}+\dfrac{x^2+y^2}{x^2-y^2}\right)$.

30. $\left(\dfrac{a^2+b^2}{a^2-b^2}-\dfrac{a^2-b^2}{a^2+b^2}\right) \div \left(\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}\right)$.

31. $\left(\dfrac{m+n}{m-n}+\dfrac{m-n}{m+n}\right) \div \left(\dfrac{m^2+n^2}{m^2-n^2}-\dfrac{m^2-n^2}{m^2+n^2}\right)$.

32. $\dfrac{\dfrac{a^2+b^2}{b}-a}{\dfrac{1}{b}-\dfrac{1}{a}} \times \dfrac{a^2-b^2}{a^3+b^3}$.

33. $\dfrac{x}{x-y}-\dfrac{x}{x+y}-\dfrac{\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}}{\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}}$

34. $\dfrac{\dfrac{1}{a}+\dfrac{1}{b+c}}{\dfrac{1}{a}-\dfrac{1}{b+c}}\left(1+\dfrac{b^2+c^2-a^2}{2bc}\right)$.

35. $\left(\dfrac{9m^2-3n^2}{4mn}-\dfrac{m-4n}{5n}\right) \div \left(\dfrac{2m+n}{3m}-\dfrac{5n^2-3m^2}{16m^2}\right)$.

36. $\left(\dfrac{10xy-3y^2+10x-3y}{15y^2+10xy^2+30y+20xy} \div \dfrac{10x-3y}{45y+30xy}\right)+\dfrac{3}{y+2}$.

37. $\left(\dfrac{x^3-y^3}{x^3-y^3}-\dfrac{10x^2-13xy-3y^2}{10x^2-3xy-y^2}-\dfrac{2x^3+xy^2+xy+y^2}{2x^3+xy-y^2}\right) \div \dfrac{xy-y^2-2x+2y}{2x-y}$.

38. $\dfrac{\left[\dfrac{(a+b)^2}{4ab}-1\right]\left[\dfrac{(a-b)^2}{4ab}+1\right]}{(a+b)^3-3a^2b-3ab^2} \times \dfrac{[(a+b)^2-ab][(a-b)^2+ab]}{(a-b)^3+3ab(a-b)}$.

39. $\dfrac{\dfrac{x^3-y^3}{x^2+y^2} \times \dfrac{x^2-y^2}{x^3+y^3}\left(\dfrac{1}{x^2}+\dfrac{1}{y^2}\right) \div \left(\dfrac{1}{y}-\dfrac{1}{x}\right)}{\dfrac{(x+y)^2-xy}{(x-y)^2+xy}}$.

40. $\dfrac{\left[\dfrac{(m+n)^3}{3mn}-m-n\right]\left[\dfrac{(m-n)^3}{3mn}+m-n\right]}{\left[\dfrac{(m+n)^2}{4mn}-1\right][(m+n)^2-mn]} \times \dfrac{(n-m)^3-(n-m)(m^2+mn+n^2)}{(m+n)^3-3mn(m+n)}$.

41. $\dfrac{\dfrac{1+x}{1-x}+\dfrac{4x}{1+x^2}+\dfrac{8x}{1-x^4}-\dfrac{1-x}{1+x}}{\dfrac{1+x^2}{1-x^2}+\dfrac{4x^2}{1+x^4}-\dfrac{1-x^2}{1+x^2}}$.

42. $\dfrac{\dfrac{1}{1+x}}{1-\dfrac{1}{1+x}}+\dfrac{\dfrac{1}{1+x}}{\dfrac{x}{1-x}}+\dfrac{\dfrac{1}{1-x}}{\dfrac{x}{1+x}}$.

43. $\left(\dfrac{1}{a+\dfrac{1}{b+\dfrac{1}{c}}} \div \dfrac{1}{a+\dfrac{1}{b}}\right)-\dfrac{1}{b(abc+a+c)}$.

Ex. 15.

The Symbols $\frac{0}{m}$, $\frac{m}{0}$, $\frac{0}{0}$.

1. What will be the quotient when the dividend is equal to 0, and the divisor is any assigned quantity?

2. What will be the quotient $\frac{1}{b}$ when b is greater than any assignable quantity and increases without limit?

3. What does $\frac{n}{k}$ become when k decreases and approaches indefinitely to 0?

4. What is the value of the expression $\frac{4x-3}{24x-18}$ if $x=\frac{3}{4}$?

5. What is the value of $\frac{3x}{7x}$ if $x=0$?

6. What will the fraction $\frac{3x}{7x}$ become if x has any other value than 0?

The symbol $\doteq$ is used for the word *approaches*.

Find the *limits* of the values of the following fractions:

7. $\frac{x^2-8x+15}{x^2-10x+21}$; if $x \doteq 3$.

8. $\frac{a^2-a-2}{a^2-4}$; if $a \doteq 2$.

9. $\frac{x^2-5xy+4y^2}{x^2+xy-2y^2}$; if $x \doteq y$.

10. $\frac{16-a^4}{2-a}$; if $a \doteq 2$.

11. $\frac{x^2-y^2}{2x^2-xy-y^2}$; if $x \doteq y$.

12. $\frac{m^2-2mn+n^2}{m^2-n^2}$; if $m \doteq n$.

13. $\frac{p^2-pq-2q^2}{p^2-4pq+4q^2}$; if $p \doteq 2q$.

14. $\frac{m^4-5m^2n^2+4n^4}{m^4-3m^2n^2-4n^4}$; if $m \doteq 2n$.

15. $\frac{a^2-6ab+9b^2}{2a^2-ab-15b^2}$; if $a=3b$. 16. $\frac{x^3-a^3}{x^2-a^2}$; if $x=a$.

17. $\frac{x^3-15x^2+74x-120}{x^3-12x^2+41x-30}$; 1st, when $x=5$; 2d, when $x=6$.

18. $\frac{m^2+2mn+n^2-4}{m+n-2}$; if $m=1$, and $n=1$.

19. $\frac{x^2-4}{x^2-3x+2}$; if $x=2$. 21. $\frac{2x^2-3x-9}{3x^2-10x+3}$; if $x=3$.

20. $\frac{x^2+2x-3}{x^2+3x-4}$; if $x=1$. 22. $\frac{x^2-2x+1}{x^2-5x+4}$; if $x=1$.

23. $\frac{x^3-6x^2+6x-5}{x^3-2x^2-19x+20}$; if $x=5$.

24. $\frac{2x^2+5x-3}{x^2+x-6}$; if $x=-3$.

25. $\frac{3a^3-3\frac{1}{2}a^2+a}{2a^3-6a^2+\frac{5}{3}a}$; if $a=\frac{1}{2}$.

26. $\frac{5a^2-9a-2}{10a^2-13a-3}$; if $a=-\frac{1}{5}$.

27. $\frac{2x^3-5ax^2+3a^2x}{3x^3-8ax^2+5a^2x}$; if $x=a$.

28. $\frac{2}{x^2-1}-\frac{1}{x-1}$; if $x=1$.

29. $\frac{x^3+2x^2-14x-3}{x^2-x-6}$; if $x=3$.

30. $\frac{4x^6-6ax^5+2ax^5-3a^2x}{6x^5-4ax^3+3ax^4-2a^2x}$; if $x=-\frac{a}{2}$

31. $\frac{a(x^2+c^2)-2acx}{b(x^2+c^2)-2bcx}$; if $x=c$.

32. $\frac{x^3+4x^2+x-6}{x^3+7x^2+14x+8}$; if $x=-2$.

Ex. 16.

Various Exercises in Fractions.

Find the value of the following expressions:

1. $\frac{m-x}{n-x}$; if $x=\frac{mn}{m+n}$.

2. $\frac{x-m}{n}-\frac{x-n}{m}$; if $x=\frac{m^2}{m-n}$.

3. $\frac{x}{a}+\frac{x}{b-a}-\frac{a}{a+b}$; if $x=\frac{a^2(b-a)}{b(b+a)}$.

4. $\frac{x+2a}{2b-x}+\frac{x-2a}{2b+x}-\frac{4ab}{4b^2-x^2}$; if $x=\frac{ab}{a+b}$.

5. $\frac{x+y-1}{x-y+1}$; if $x=\frac{a+1}{ab+1}$ and $y=\frac{a(b+1)}{ab+1}$.

6. $\left(\frac{x-a}{x-b}\right)^2-\frac{x-2a+b}{x+a-2b}$; if $x=\frac{a+b}{2}$.

7. If $y=\frac{1-z^2}{1+z^2}$, $z=\frac{1-x}{1+x}$, express the value of y in terms of x.

8. What will the expression $(a^2-x)a+bx$ equal, when $x=a^2+ab+b^2$?

9. What will $\frac{x^2+2x-8}{x^2-x-4}$ equal, when $x=0$?

10. What will $\frac{ab^2}{mb^2-b}$ equal, when $b=\infty$?

Simplify the following expressions:

11. $\dfrac{(1-xy)(1+xy)-(x-y)(x+y)}{(1+xy)^2+(x-y)^2}$.

12. $\dfrac{a^2-bc}{(a+b)(a+c)}+\dfrac{b^2-ac}{(b+a)(b+c)}+\dfrac{c^2-ab}{(c+a)(c+b)}$.

13. $\dfrac{a^2}{(a-b)(a-c)}+\dfrac{b^2}{(b-a)(b-c)}+\dfrac{c^2}{(c-a)(c-b)}$.

The first member of the following equalities being given, through what transformations can it be changed to the form indicated by the second member?

14. $(2x^2-1)\left[\dfrac{x}{1-x}+\dfrac{1+x}{x}\right]=\dfrac{x}{1-x}-\dfrac{1+x}{x}$.

15. $\dfrac{a^6+2a^3b^3+b^6}{a^2+2ab+b^2}=(a^2-ab+b^2)^2$.

16. $\dfrac{a-c}{(a-b)(x-a)}-\dfrac{b-c}{(a-b)(x-b)}=\dfrac{x-c}{(x-a)(x-b)}$.

17. $\dfrac{1}{x(x-a)(x-b)}=\dfrac{1}{abx}+\dfrac{1}{a(a-b)(x-a)}-\dfrac{1}{b(a-b)(x-b)}$.

18. $\dfrac{x}{1-x^2}+\dfrac{y}{1-y^2}+\dfrac{z}{1-z^2}=\dfrac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$, when $xy+yz+xz=1$.

19. Change $1-\dfrac{b^2+c^2-a^2}{2bc}$ into a product of two factors.

20. What is, in the development of $\dfrac{1}{1-x}$, the nth term, and what is the fraction that remains?

21. When, in the preceding example, we have $x < \frac{1}{2}$, we also have $\frac{x^n}{1-x} < x^{n-1}$. If, then, $2x^{n-1}$ is small enough not to affect the degree of exactness proposed, we can stop at the $n-1$th term, and write:

$$\frac{1}{1-x} = 1 + x + x^2 \cdots\cdots + x^{n-2}.$$

Application: change $\frac{1}{0.97}$ into a decimal fraction, exact to the seventh figure.

22. When $\frac{1}{x}$ is small, since $\frac{a}{x+b} = \frac{a}{x} - \frac{ab}{x^2} + \frac{ab^2}{x^3} \cdots\cdots$ a quantity in a decimal fraction can be developed by approaching rapidly with the aid of this formula to a given degree of approximation. Ex.: reduce $\frac{3}{107}$ to a decimal fraction, exact to the units of the sixth decimal place.

23. Reduce $\frac{3}{104}$ to a decimal fraction, exact to the ten millionths.

24. Change $\frac{5}{0.98}$ to a decimal fraction, exact to the eighth figure.

25. Change $\frac{7}{0.995}$ to a decimal fraction, exact to the twelfth figure.

26. Change $\frac{3}{1001}$ to a decimal fraction, exact to the thirteenth figure.

27. Reduce to a decimal fraction: $\dfrac{1+\dfrac{2}{3\frac{4}{5}}}{1-\dfrac{2}{3\frac{4}{5}}} \div \left(1+\dfrac{4}{9-\dfrac{3}{1-\frac{3}{5}}}\right)$.

54. $\frac{2x+1}{3}+\frac{3x+1}{4}=28-\frac{5x-2}{7}$.

55. $\frac{5x-6}{7}-\frac{3x-4}{9}-\frac{4x}{11}=0$.

56. $\frac{x+1}{2}+\frac{3x-4}{5}+\frac{1}{8}=\frac{6x+7}{8}$.

57. $\frac{5x-11}{4}-\frac{x-1}{10}=\frac{11x-1}{12}$.

58. $\frac{8-x}{6}+\frac{3x-5}{3}=\frac{x+6}{2}-\frac{x}{3}$.

59. $\frac{x+3}{2}-\frac{1}{8}(x-2)=\frac{1}{12}(3x-5)+\frac{1}{4}$.

60. $11-\left(\frac{3x-1}{4}+\frac{2x+1}{3}\right)=10-\left(\frac{2x-5}{3}+\frac{7x-1}{8}\right)$.

61. $\frac{x-1}{3}+\frac{16x-3}{20}-\frac{7x-6}{8}=\frac{x+2}{2}+\frac{3(x-3)}{10}$.

62. $\frac{x-3}{4}-\frac{2x-5}{6}-\frac{41}{60}-\frac{3x-8}{5}+\frac{5x+6}{15}=0$.

63. $\frac{3x+5}{8}-\frac{5x-4}{9}+\frac{7x-5}{6}=\frac{9x-53}{5}-\frac{3x}{17}$.

64. $\frac{4x-2}{9}-\frac{3x-5}{8}+\frac{7x-1}{5}=\frac{2(5x-13)}{7}+\frac{2x}{23}$.

65. $\frac{7x-4}{9}+\frac{3x-1}{5}-\frac{5(x-1)}{6}=\frac{3(3x-1)}{20}-\frac{9x}{7}$.

66. $15x-\frac{5(2x+5)}{7}=80-2\frac{1}{2}(7x+19)-\frac{5}{8}(2x+1)$.

67. $6x-\frac{27x-1}{4}-\frac{2(4x-1)}{5}=\frac{9x-5}{4}-\frac{11x-2}{3}+22$.

68. $\frac{6(x+3)}{13} - 3\frac{1}{6} - \frac{11-3x}{36} = 5x - 48 - \frac{13-x}{12} - \frac{21-2x}{18}$.

69. $\frac{3x+4}{7} - \frac{9x+44}{5} + \frac{3(3x+10)}{4} = \frac{5x+12}{3}$.

70. $\frac{102x-1}{4} - \frac{8(x-1)}{5} = \frac{18x-1}{4} - \frac{2(11x-1)}{3} + 21$.

71. $\frac{10x+11}{6} - \frac{14x-13}{3} - 4 = \frac{7-6x}{4}$.

72. $\frac{11x-13}{25} + \frac{19x+3}{7} - \frac{5x-25\frac{1}{4}}{4} = 28\frac{1}{7} - \frac{17x+4}{21}$.

73. $\frac{4x-21}{7} + \frac{47}{6} + \frac{7x-28}{3} = \frac{4x+15}{4} - \frac{9-7x}{8} + \frac{1}{12}$

74. $\frac{x-3\frac{2}{3}}{2} - \frac{5-4x}{7} = x + 21\frac{6}{7} - \frac{3x-\frac{1}{3}(3-2x)}{6}$.

75. $\frac{4x}{15} - \frac{9}{10} - \frac{16x-81}{24} = \frac{2x-3}{15} - \frac{4x-9}{20} + \frac{9}{40}$.

76. $\frac{19(13.4-2.5x)}{4} + 7.1 = 39.7625 - 9(0.45+8x)$.

77. $\frac{1+2x}{3} - \frac{5-3x}{3} + 4 - \frac{8x}{5} = \frac{x}{1.5} + \frac{x}{1.05}$.

78. $\frac{3x}{8} - \frac{0.25x}{0.4} + 3.642 - \frac{x}{0.8} + 0.01 = 0$.

79. $\frac{5(3x-2)}{4} + \frac{3x}{2} - 23\frac{5}{6} = \frac{x - \frac{4x-9}{3}}{6} + 5$.

80. $\frac{x - \frac{2(x-18)}{9}}{8} - \frac{x-18}{6} = x + 9 - \frac{5x - \frac{2(x+10)}{23}}{4}$.

81. $\frac{2\frac{1}{3}x-2}{4}-\frac{\frac{10x-1}{2}}{3}=\frac{\frac{x-8}{4}}{5}-3\frac{1}{3}$.

82. $\frac{3}{y-4}+\frac{5}{2(y-4)}+\frac{9}{2(y-4)}=\frac{1}{2}$.

83. $\frac{2}{z-1}-\frac{5}{2z-2}=\frac{8}{3z-3}-\frac{z}{z-1}+\frac{5}{18}$.

84. $\frac{5}{m+3}+\frac{3}{2(m+3)}+\frac{42}{12(m+3)}=0.5$.

85. $\frac{2x-3}{2x-4}-6=\frac{x+5}{3x-6}-\frac{11}{2}$.

86. $\frac{2x-13}{2x-16}+\frac{2(x-6)}{x-8}=\frac{7}{8}+\frac{2(5x-39)}{3x-24}$.

87. $\frac{10-7x}{6-7x}=\frac{5x-4}{5x}$.

88. $\frac{29-10y}{9-5y}=\frac{5+36y}{18y}$.

89. $\frac{5+8z}{3+2z}=\frac{45-8z}{13-2z}$.

90. $\frac{47+6x}{11+2x}=\frac{2(8+3x)}{3+2x}$.

91. $\frac{5u-16}{u-5}=\frac{5u-3}{u+5}$.

92. $\frac{5x-1}{2x+3}=\frac{5x-3}{2x-3}$.

93. $\frac{3z+7}{4z+5}=\frac{3z+5}{4z+3}$.

94. $\frac{x}{3}-\frac{x^2-5x}{3x-7}=\frac{2}{3}$.

95. $\frac{4y+3}{9}-\frac{8y+19}{18}=-\frac{7y-29}{5y-12}$.

96. $\frac{11m-13}{14}-\frac{22m-75}{28}=\frac{13m+7}{2(3m+7)}$.

97. $\frac{16a+7}{24}-\frac{2a+1}{3}=\frac{a-16}{3(3a-59)}$.

98. $\frac{3x+2}{x-1}+\frac{2x-4}{x+2}=5.$

99. $\frac{3x-1}{2x-1}-\frac{2(2x-1)}{3x-2}=\frac{1}{6}.$

100. $\frac{a^2-a+1}{a-1}+\frac{a^2+a+1}{a+1}=2a.$

101. $\frac{4x+17}{x+3}+\frac{3x-10}{x-4}=7.$

102. $\frac{4}{1+x}+\frac{x+1}{1-x}=\frac{x^2-3}{1-x^2}.$

103. $\frac{5-2b}{b-1}-\frac{2-7b}{b+1}=\frac{5b^2+4}{b^2-1}.$

104. $\frac{6}{x+2}-\frac{x+2}{x-2}+\frac{x^2}{x^2-4}=0.$

105. $\frac{9y+5}{6(y-1)}+\frac{3y^2-51y-71}{18(y^2-1)}=\frac{15y-7}{9(y+1)}.$

106. $\frac{2u+1}{2u-1}-\frac{8}{4u^2-1}=\frac{2u-1}{2u+1}.$

107. $\frac{1+3m}{5+7m}-\frac{9-11m}{5-7m}=\frac{14(2m-3)^2}{25-49m^2}.$

108. $\frac{2y+1}{3y-3}=\frac{7y-1}{6y+6}-\frac{2y^2-3y-45}{4y^2-4}.$

109. $\frac{4u+3}{15u-35}-\frac{11u-5}{9u+21}=\frac{375u-86u^2-35}{10(9u^2-49)}.$

110. $\frac{2z+3}{5z-7}-\frac{3z+4}{7z-5}=\frac{25z-3(2z^2-73)}{6(35z^2-74z+35)}.$

Ex. 18.

LITERAL EQUATIONS WITH ONE UNKNOWN QUANTITY.

Solve:

1. $\frac{m}{px}+n=\frac{k}{px}+q.$

2. $\frac{a}{bz}+c=\frac{d}{gz}+3.$

3. $\frac{c+d}{x}-a=\frac{c-d}{x}+b.$

4. $\frac{m+n}{px}-\frac{c-d}{qx}=1.$

5. $\frac{m+1}{m+x}=\frac{n}{m-x}.$

6. $\frac{p}{m(n+x)}=\frac{x}{n+x}.$

7. $\frac{z}{a+az}=\frac{b}{c+cz}.$

8. $\frac{c+d}{ab+by}=\frac{m-y}{an+ny}.$

9. $\frac{1+z}{1-z}=\frac{1}{m}.$

10. $\frac{m+ny}{p+qy}=b.$

11. $\frac{my+n}{my-n}=\frac{a}{b}.$

12. $\frac{m+z}{n+3z}=\frac{3}{4}.$

13. $\frac{p(q+x)}{p-x}=q.$

14. $\frac{m}{m-x}=\frac{n}{n-x}.$

15. $\frac{z+2}{z-2}=\frac{m+n}{m-n}.$

16. $\frac{m+n}{p+x}=\frac{m-n}{p-x}.$

17. $\frac{a+bx}{a+b}=\frac{c+dx}{c+d}.$

18. $\frac{a+bx}{a+b}=\frac{c-dx}{c-d}.$

19. $\frac{m-z}{n-z}=\frac{m+z}{n+z}.$

20. $\frac{6x+a}{4x+b}-\frac{3x-b}{2x-a}=0.$

21. $\frac{x}{a}+\frac{x}{b}+\frac{x}{c}=d.$

22. $\frac{ax}{b}-\frac{cx}{d}+\frac{fx}{g}=h.$

23. $\frac{ax-b}{c}-\frac{bx+c}{a}=abc.$

24. $\frac{a-x}{b}-\frac{b-x}{a}+\frac{c-x}{c}=a^2-b.$

25. $\frac{x+a}{b}-x=b-\frac{x-b}{c}+\frac{c-bx}{b}.$

26. $\frac{3-x}{a}-\frac{5+2x}{b}=c-\frac{dx}{3}+\frac{(2b-7a)x+5b}{3ab}.$

27. $\frac{20p-qx}{5p}+\frac{9n-px}{3n}+\frac{6m-nx}{2m}=10.$

28. $\frac{3b(x-a)}{5a}+\frac{x-b^2}{15b}=-\frac{(4a+cx)b}{6a}.$

29. $\frac{mx}{n}-\frac{n-x}{2p}+\frac{m(n-x)}{3q}=m.$

30. $\frac{3ax-2b}{3b}-\frac{ax-a}{2b}=\frac{ax}{b}-\frac{2}{3}.$

31. $\frac{a^2-x}{b}-\frac{ab-x}{c}+\frac{c^2-x}{a}+\frac{a^2b+cx}{ab}=\frac{a^2}{b}+\frac{c^2}{a}$

32. $\frac{b+c}{x}+a=\frac{a}{x}-b+\frac{a(a-1)-b(b-1)+c}{x}.$

33. $\frac{(bd+1)x}{bd}+\frac{3x}{bc^2}-\frac{0.6x}{cd^2}-\frac{x}{bd}=x+0.84.$

34. $\frac{b-2x}{3a}-\frac{5a^2}{2b^2}=\frac{x}{a}-2+\frac{a(x-a)}{b^2}.$

35. $\frac{3a^2}{4b}+2x-\frac{(a-b)^2}{2a}=\frac{3bx}{5a}+b.$

36. $\frac{m}{n}:\frac{p}{q}=\frac{q}{p}:x.$ 37. $\frac{m}{n}:\frac{p}{q}=a:\frac{1}{x}.$

38. $\dfrac{mx+n}{p} : \dfrac{ax-b}{c} = d : f.$

39. $\dfrac{a+1}{x} : \dfrac{b}{x} = (a+x) : (b+x).$

40. $\dfrac{\dfrac{m+n}{x}}{\dfrac{1}{m}} = \dfrac{p}{q}.$

41. $\dfrac{\dfrac{mx-n}{x}}{\dfrac{m}{n}} = \dfrac{\dfrac{mx+n}{x}}{\dfrac{n}{m}}.$

42. $\dfrac{\dfrac{cx+d}{m}}{\dfrac{cx}{d}} = \dfrac{2d}{m}.$

43. $\dfrac{\dfrac{a+x-8}{3}}{\dfrac{a-x+3}{3}} = \dfrac{b}{c}.$

44. $p - \dfrac{x-np}{m} = \dfrac{x-mp}{n} - \dfrac{x-mn}{p} - p.$

45. $\dfrac{x}{mn} + \dfrac{x}{mp} + \dfrac{x}{np} - 1 = mnp - (m+n+p)x.$

46. $\dfrac{5ax}{a-b} - 3a = 8x.$

47. $\dfrac{5x}{3a+b} - 2 = \dfrac{8b}{5a}.$

48. $\dfrac{3a+x}{5a-b} = \dfrac{4x-b}{3a+2b} - 1.$

49. $\dfrac{x}{a-b} - \dfrac{5a}{a+b} = \dfrac{2bx}{a^2-b^2}.$

50. $\dfrac{z-2a}{z+3a} = 3 - \dfrac{2z^2-13a^2}{z^2-9a^2}.$

51. $\dfrac{c^2-3dy}{c^2+3dy} + \dfrac{d^2+2cy}{d^2-2cy} = 2.$

52. $\dfrac{1}{n} + \dfrac{n}{x+n} = \dfrac{x+n}{nx}.$

53. $\dfrac{1}{ab-ax} + \dfrac{1}{bc-bx} = \dfrac{1}{ac-ax}.$

54. $\dfrac{x}{a} + \dfrac{x}{b-a} = \dfrac{a}{b+a}.$

55. $\dfrac{z+a}{z+b} = \left(\dfrac{2z+a+c}{2z+b+c}\right)^2.$

56. $\dfrac{m}{z-m} - \dfrac{n}{z-n} = \dfrac{m-n}{z-p}.$

57. $\dfrac{m}{z+m} + \dfrac{n}{z+n} = \dfrac{m+n}{z+p}.$

58. $\frac{1}{y-m}-\frac{1}{y-n}=\frac{m-n}{y^2-mn}$.

60. $\frac{x-c}{c-d}-\frac{x+c}{c+d}=\frac{2cx}{c^2-d^2}$.

59. $\frac{mz-a-b}{nz-c-d}=\frac{mz-a-c}{nz-b-d}$.

61. $\frac{y-a}{y-b}=\left(\frac{2y-a}{2y-b}\right)^2$.

62. $\frac{z}{a-b}-\frac{5a}{a+b}=\frac{2bz}{a^2-b^2}$.

63. $\frac{6a+5b}{6a}-\frac{4bx}{3a^2}=1-\frac{bx}{a^2+ab}$.

64. $\frac{7z}{9m^2}-\frac{3}{2m}=\frac{9n+14z}{18m^2-24mn}$.

65. $\frac{3y}{a+3b}+\frac{18by}{a^2-9b^2}=1$.

66. $\frac{y}{3a+y}+\frac{y}{3a-y}=\frac{a^2}{9a^2-y^2}$.

67. $\frac{m(2x-m)}{m+2n}+\frac{n(2x-n)}{n+2m}=x$.

68. $\frac{z}{p+q}+pqz=p+q+\frac{1}{pq}$.

69. $p^2=\frac{p^3-q^3}{p-q}-\frac{pq(p+q)}{x}$.

70. $q^2-\frac{p^3-q^3}{p-q}=-\frac{pq(p+q)}{x}$

71. $\frac{m+nx}{p+qx}=\frac{qrx^2}{p(p+qx)}+\frac{s-rx}{p}$.

72. $\frac{3def}{d+e}+\frac{d^2e^2}{(d+e)^2}+\frac{(2d+e)e^2x}{d(d+e)^2}=3fx+\frac{ex}{d}$.

73. $\frac{k}{z}+\frac{z}{k}+\frac{k(z-k)}{z(z+k)}-\frac{z(z+k)}{k(z-k)}=\frac{kz}{k^2-z^2}-2$.

74. $\frac{f+3y}{4f^2+6fg}-\frac{2y-f}{6fg-9g^2}=\frac{2f+y}{4f^2-9g^2}$.

75. $\frac{3bx}{2a^2}-\frac{x-b}{a+b}-\frac{bx-a^2}{a^2-b^2}+\frac{x}{4a}=0$.

Ex. 19.

PROBLEMS WITH ONE UNKNOWN QUANTITY.

1. If the population of a city increases 5 per cent annually, and is now 194,481, what was it 4 years ago?

2. In order to dig a ditch A and B work together 40 days, or B and C 48 days, or A and C 36 days. How many days would it take for each of them to do the work alone, and how many if they all work together?

3. The two arms of the beam of a false balance are respectively 6 inches and 6.2 inches. It is necessary to put in the pan suspended to the longer arm weights of $2\frac{1}{2}$ pounds in order to make equilibrium with a body placed in the other pan. What is the exact weight of the body?

4. A bamboo 10 feet high was bruised at a certain height. When the upper part was bent and touched the ground the top of the bamboo was 3 feet from the base. At what height was it bruised?

5. The fore and hind wheels of a carriage are respectively 3.25 yards and 3.8 yards in circumference. What distance has the carriage passed over when one of the fore wheels has made 143 revolutions more than one of the hind wheels?

6. A train which travels 32 miles an hour is $\frac{3}{4}$ of an hour in advance of a second train which travels 42 miles an hour. In how long a time will the last overtake the first?

7. A train travelling b miles per hour is m hours in advance of a second train which travels a miles per hour. In how long a time will the last overtake the first? Discuss the result when $a > b$; $a = b$; $a < b$.

8. Two travellers set out from A, taking the same route and in the same direction. The first walks 9 hours per day, travels $5\frac{1}{2}$ miles per hour, and starts 2 days before the other. The second walks 12 hours per day, and travels $4\frac{1}{2}$ miles per hour. At what distance from A will the first be overtaken?

9. An express train which travels 42 miles per hour starts 50 minutes after a freight train, which it overtakes in 2 hours 5 minutes. What is the velocity of the freight train?

10. A messenger starts to carry a despatch; 5 hours after his departure a second messenger is sent, who ought to overtake the first in 8 hours, and to do this he is obliged to travel $2\frac{1}{2}$ more miles per hour. How many miles per hour did the first travel?

11. Two trains move in a line in the same direction. The first starts from A at 2 o'clock in the morning, and travels 8 miles per hour. The second starts from B at 9.30 o'clock in the morning, and travels $10\frac{1}{2}$ miles per hour. The distance between A and B is 20 miles. When and where do the two trains come together?

12. Two friends, at a distance of 78 miles, agree to meet in an intermediate locality, and set out at the same moment; one from A, travelling $5\frac{1}{2}$ miles per hour, the other from B, travelling $7\frac{1}{2}$ miles per hour. When and where do they meet?

13. At 12 o'clock the hands of a watch are together. At what time will they be opposite to each other?

14. How many times and at what hours will the two hands of a watch be together during the time between midnight and noon? The coincidence of the hands at midnight or at noon is not taken into account.

15. A hare is 50 leaps in advance of a fox; 3 leaps of the hare are equal to 2 leaps of the fox, but the fox makes only 3 leaps while the hare is making 4. How many leaps must the fox make to overtake the hare?

16. A company of foot-soldiers is 565 steps in advance of a squadron of cavalry. Allowing that a soldier takes 5 steps while a horse takes 4, but that 3 steps of a horse are equal to 4 of a man, how many steps must the horses of the squadron take to overtake the foot-soldiers?

17. If A, who is travelling, makes $\frac{1}{2}$ of a mile more per hour, he will employ only $\frac{4}{5}$ of the time; but if he makes $\frac{1}{2}$ of a mile less per hour, he will be on the route $2\frac{1}{2}$ hours more. Find the length of the route and the speed.

18. A boatman moves 5 miles in $\frac{3}{4}$ of an hour, rowing with the tide, and in returning it takes him $1\frac{1}{2}$ hours, rowing against a tide one-half as strong. What is the velocity of the stronger tide?

19. A boatman, rowing with the tide, moves n miles in t hours. Returning, he uses t' hours to accomplish the same distance, rowing against a tide m times as strong as the first. What is the velocity of the second tide?

20. A and B can accomplish a piece of work in m days; A and C can accomplish it in n days, and B and C in p days. How many days will it take each to do the work alone, and how many if they work together?

21. The circumference of a fore wheel of a carriage is a feet; that of a hind wheel, b feet. What distance will the carriage have passed over, when a fore wheel has made n more revolutions than a hind wheel?

22. Find a number such that if its pth part is added to it, the sum obtained will be equal to the qth part of the number, plus r.

23. A traveller wishes to accomplish a journey of k miles in n days; at the end of d days he receives news which obliges him to hasten by a days his return. How many more miles must he travel each day? Find the result if $k=360$; $n=18$; $d=7$; $a=5$.

24. A sum s is divided among A, B, C, in the following manner: the part given to A is to that given to B as $a:b$; that of B to that of C as $d:h$. What is the part given to each?

25. A number is divided by d, and the sum of the quotient, dividend, and divisor is b. What is this number? Find the result if $d=12$; $b=129$.

26. A wine merchant has two kinds of wine which cost him, one a dollars, and the other b dollars per gallon. He wishes to make a mixture of l gallons, which shall cost him on the average m dollars a gallon. How many gallons must he take of each?

Discuss the question (i.) when $a=b$; (ii.) when a or $b=m$; (iii.) when $a=b=m$; (iv.) when $a>b$ and $<m$; (v.) when $a>b$ and $b>m$.

27. A square pond, whose centre is distant from each side a meters, is surrounded by a fence. If the owner wishes to change the pond to a circular piece of water, what should be the radius that the fence may be just as long?

28. The population of a city increases each year t per cent of that of the preceding year; to-day it has A inhabitants. What was its population n years ago?

29. Three friends, A, B, and C, wish to divide a profit g that they have made in an enterprise. A has furnished a dollars during m months; B, b dollars during n months; and C, c dollars during p months. What will be the share of each?

30. Divide the number a into four parts such that, if n is added to the first, n subtracted from the second, the third multiplied by n, and the fourth divided by n, results will be equal. What are the results if $a=90$, and $n=2$.

31. If A can do a piece of work in $2m$ days, B and A together in n days, and A and C in $m+\frac{n}{2}$ days, how much time will it take them to do the work together?

32. If I divide a number by a, subtract b from the quotient, and multiply the remainder by m, I obtain the same result as if, from the number in question, I subtract n times its pth part. What is this number?

33. A man, setting out on a journey, drives at the rate of a miles per hour to the nearest railway station, distant b miles from his house. On arriving at the station, he finds that the express train for his place of destination has left c hours before. At what rate should he have driven, in order to reach the station just in time for the express train?

What does the answer become in the following cases: (i.) when $c=0$; (ii.) when $c=\frac{b}{a}$; (iii.) when $c=-\frac{b}{a}$. In case (ii.) how much time does the man have to drive from his house to the station? In case (iii.) what is the meaning of the *negative* value of c?

Ex. 20.

SIMPLE NUMERICAL EQUATIONS WITH TWO UNKNOWN QUANTITIES.

Solve:

1. $72x+14y=330$,
   $63x+7y=273$.

2. $2x+7y=52$,
   $3x-5y=16$.

3. $7x-4y=40$,
   $15x-13y=68$.

4. $4x+9y=79$,
   $7x-17y=40$.

5. $29x-14y=175$,
   $87x-56y=497$.

6. $171x-213y=642$,
   $114x-326y=244$.

7. $5x+9y=188$,
   $13x-2y=57$.

8. $5y-2x=21$,
   $13x-4y=120$.

9. $15x-7y=9$,
   $9y-7x=13$.

10. $12x+7y=176$,
    $3y-19x=3$.

11. $2x+5y=91$,
    $7y-3x=139$.

12. $4x+9y=106$,
    $8x+17y=198$.

13. $2x-7y=8$,
    $4y-9x=19$.

14. $17x+12y=59$,
    $19x-4y=153$.

15. $8x+3y=3$,
    $12x+9y=3$.

16. $69y-17x=103$,
    $14x-13y=-41$.

17. $2x+7y-34=0$,
    $5x+9y-51=0$.

18. $12x+13y-37=0$,
    $17x-19y-15=0$.

19. $3y-4x-1=0$,
    $18-3x=4y$.

20. $x=16-4y$,
    $y=34-4x$.

21. $2x=11+9y$,
    $3x-15=12y$.

22. $3y=104+x$,
    $5x=2y+78$.

23. $5x-3y=12,$
$19x-5y=73\frac{1}{2}.$

24. $5+3x=16y,$
$5x+28y=19.$

25. $21x+8y=-66,$
$28x-23y=13.$

26. $\frac{2x}{3}+\frac{3y}{2}=16\frac{1}{2},$
$\frac{3x}{2}-\frac{2y}{3}=16\frac{1}{2}.$

27. $\frac{7x}{4}+\frac{5y}{8}=20,$
$\frac{3x}{5}-\frac{y}{4}=5.$

28. $4+y=\frac{3x}{4},$
$x-8=\frac{4y}{5}.$

29. $\frac{2x}{3}=10-\frac{1}{2}y,$
$4\frac{1}{4}y=5x-7.$

30. $2\frac{1}{3}y-\frac{5}{6}x=90,$
$2\frac{1}{2}x+\frac{2}{3}y=90.$

31. $\frac{x}{4}+\frac{y}{6}=3\frac{1}{2},$
$\frac{x}{3}-\frac{y}{8}=\frac{1}{2}.$

32. $\frac{x}{3}+\frac{y}{2}=\frac{4}{3},$
$\frac{x}{2}+\frac{y}{3}=\frac{7}{6}.$

33. $\frac{x}{4}+\frac{y}{4}=14,$
$\frac{x}{3}-\frac{y}{6}=1\frac{1}{2}.$

34. $\frac{x+y}{3}+\frac{y-x}{2}=9,$
$\frac{x}{2}+\frac{x+y}{9}=5.$

35. $\frac{x+y}{3}+x=15,$
$\frac{x-y}{5}+y=6.$

36. $\frac{x+y}{8}+\frac{x-y}{6}=5,$
$\frac{x+y}{4}-\frac{x-y}{3}=10.$

37. $\frac{x-1}{8}+\frac{y-2}{5}=2,$
$2x+\frac{2y-5}{3}=21.$

38. $\frac{3x-5y}{2}+3=\frac{2x+y}{5},$
$8-\frac{x-2y}{4}=\frac{x}{2}+\frac{y}{3}.$

39. $\frac{4x+5y}{40}=x-y,$
$\frac{2x-y}{3}+2y=\frac{1}{2}$

40. $\frac{x-2}{5}-\frac{10-x}{3}=\frac{y-10}{4},$
$\frac{2y+4}{3}=\frac{4x+y+13}{8}.$

41. $2+\frac{5x-6y}{13}=4y-3x,$
$12+\frac{5x-6y}{6}=2y+\frac{3x-2y}{4}.$

42. $\frac{5x-3}{4}-\frac{3x-19}{4}=2-\frac{3y-x}{6},$
$\frac{2x+y}{2}-\frac{9x-7}{8}=\frac{3y+9}{4}-\frac{4x+5y}{16}$

43. $\frac{7+x}{5}-\frac{2x-y}{4}=3y-5,$
$\frac{5y-7}{2}+\frac{4x-3}{6}=18-5x.$

47. $\frac{1}{x}+\frac{1}{y}=6,$
$\frac{2}{x}-\frac{1}{y}=0.$

44. $\frac{1}{x}:\frac{1}{y}=13:11,$
$\frac{5x+3}{2}=\frac{7y-4}{3}.$

48. $\frac{3}{x}+\frac{8}{y}=3,$
$\frac{15}{x}-\frac{4}{y}=4.$

45. $\frac{3}{x+1}:\frac{4}{y+4}=1:\frac{2}{3},$
$\frac{3}{x}=\frac{4}{y}.$

49. $\frac{1}{x}+\frac{2}{y}=10,$
$\frac{4}{x}+\frac{3}{y}=20.$

46. $\frac{1}{x}+\frac{1}{y}=\frac{11}{30},$
$\frac{1}{x}-\frac{1}{y}=\frac{1}{30}.$

50. $\frac{7}{x}+\frac{5}{y}=19,$
$\frac{8}{x}-\frac{3}{y}=7.$

51. $\frac{5}{3x}+\frac{2}{5y}=7,$
$\frac{7}{6x}-\frac{1}{10y}=3.$

52. $\frac{4}{5x}+\frac{5}{6y}=5\frac{11}{15},$
$\frac{5}{4x}-\frac{4}{5y}=\frac{11}{20}.$

53. $\frac{1}{2x}+\frac{2}{3y}=3,$
$\frac{3}{4x}+\frac{4}{5y}=3\frac{9}{10}.$

54. $\frac{5x}{0.7}+\frac{0.15}{y}=6-\frac{0.15}{y},$
$\frac{10x}{3.5}+\frac{18}{y}=62.$

55. $\frac{x+y}{y-x}=5,$
$\frac{15x-4y}{6}-3=\frac{4x-3y}{5}+\frac{1}{5}.$

56. $x-\frac{2y-x}{23-x}=20-\frac{59-2x}{2},$
$y+\frac{y-3}{x-18}=30-\frac{73-3y}{3}.$

57. $\frac{6y+5}{8}-\frac{4x-5y+3}{4x-2y}=\frac{9y-4}{12},$
$\frac{8x+3}{4}-\frac{x-9y}{3-x}=\frac{6x-1}{3}.$

58. $\frac{7+8x}{10}-\frac{3(x-2y)}{2(x-4)}=\frac{11+4x}{5},$
$\frac{3(2y+3)}{4}=\frac{13}{4}+\frac{3y+4}{2}-\frac{3y+5x}{2(2y-3)}$

59. $\frac{4x+7}{3}+\frac{5x-4y}{2x+1}=\frac{17+8x}{6},$
$\frac{5x-12}{4}-\frac{4x-6y-13}{2x-3y}=\frac{10x-53}{8}.$

60. $\frac{31-11y-7x}{12}+\frac{5x-3y+4}{3x-4y+5}=\frac{5y-4x}{3}-\frac{31y-9x-79}{12}$
$21x+11y=202.$

61. $\frac{2(5-11x)}{11(x-1)}+\frac{11-7y}{3-y}=5,$
$\frac{7+2x}{3-x}-\frac{125-144y}{36(y+5)}=2.$

62. $\dfrac{7-6x}{10y-19}=\dfrac{4-3x}{5y-11}$,

$\dfrac{6x-10y-17}{3x-5y+2}=\dfrac{4x-14y-5}{2x-7y+12}$

63. $\dfrac{12(7+6y)}{1-y}+\dfrac{432x-125}{3x+5}=72$,

$\dfrac{6(5-33y)}{3y-1}+\dfrac{11(11-21x)}{1-x}=165$.

64. $8y-\dfrac{4(4+15y)}{3x-1}=\dfrac{16xy-107}{2x+5}$,

$2+6x+9y=\dfrac{27y^2-12x^2+38}{3y-2x+1}$

65. $x-\dfrac{2xy}{2y+5}=\dfrac{15x+4y}{6y-2x}+\dfrac{5x^3+4y^3+105}{(x-3y)(2y+5)}$,

$3-\dfrac{7x+2y}{5x}=5-\dfrac{5y+9}{3x}$.

66. $\dfrac{6+x}{6-y}=\dfrac{9}{4}$,

$\dfrac{4(x-2y)+5}{2}-2=\dfrac{2(5x^2-6y^2-7xy+x)}{5x+3y+3}$.

67. $\dfrac{1}{1-x+y}-\dfrac{1}{x+y-1}=\dfrac{2}{3}$,

$\dfrac{1}{\dfrac{1}{1-x+y}-\dfrac{1}{1-x-y}}=\dfrac{3}{4}$.

68. $\dfrac{1}{x+\dfrac{1}{y-\dfrac{5}{x}}}=\dfrac{1}{x-\dfrac{1}{y-\dfrac{7}{x}}}$,

$\dfrac{1}{y}\left(1-\dfrac{1}{x}\right)=1$.

Ex. 21.

LITERAL EQUATIONS WITH TWO UNKNOWN QUANTITIES.

Solve:

1. $x+y=a,$
   $x-y=b.$

2. $x+y=3a-2b,$
   $x-y=2a-3b.$

3. $2x-3y=5b-a,$
   $3x-2y=a+5b.$

4. $3x-2y=(a+b)^2+3ab,$
   $3y-2x=(a-b)^2-3ab.$

5. $ax+by=c,$
   $px+qy=r.$

6. $ax-by=m,$
   $cx+ey=n.$

7. $mx-ny=r,$
   $m'x+n'y=r'.$

8. $abx+cdy=2,$
   $ax-cy=\frac{d-b}{bd}.$

9. $a(3a+x)=b(b+y),$
   $ax+2by=d.$

10. $x-y=ab,$
    $cx+afy=ag.$

11. $\frac{x}{a}+\frac{y}{b}=c,$
    $\frac{x}{d}=y.$

12. $\frac{x}{a}-\frac{y}{b}=c,$
    $\frac{x}{g}-\frac{y}{f}=0.$

13. $\frac{x}{m}+\frac{y}{n}=p,$
    $\frac{x}{r}+\frac{y}{s}=q.$

14. $\frac{mx}{n}+\frac{py}{q}=t,$
    $\frac{x}{n}-\frac{ry}{s}=v.$

15. $\frac{ax}{b}+\frac{cy}{d}=\frac{f}{g},$
    $\frac{mx}{p}+\frac{ny}{q}=\frac{s}{t}.$

16. $\frac{x}{a}+\frac{y}{b}=1-\frac{x}{c},$
    $\frac{y}{a}+\frac{x}{b}=1+\frac{y}{c}.$

17. $\frac{x}{m+n}-\frac{y}{m-n}=\frac{1}{m+n},$
    $\frac{x}{m+n}+\frac{y}{m-n}=\frac{1}{m-n}.$

18. $\frac{x}{m-a}+\frac{y}{m-b}=1,$
$\frac{x}{n-a}+\frac{y}{n-b}=1.$

19. $ax+by=2a^2-2b^2,$
$\frac{y}{a-b}-\frac{x}{a+b}=\frac{a^2+b^2}{ab}.$

20. $(m+n)x-(m-n)y=4mn,$
$\frac{x}{m+n}+\frac{y}{m-n}=2.$

21. $\frac{x}{q^2-1}-\frac{y}{p^2-1}=p^2-q^2,$
$\frac{x}{p^2+1}+\frac{y}{q^2+1}=p^2+q^2-2.$

22. $\frac{x+y-1}{x-y+1}=m,$
$\frac{y-x+1}{x-y+1}=mn.$

23. $\frac{x-m}{y}=\frac{p}{q},$
$\frac{m}{x}=\frac{n}{y}.$

24. $\frac{x+m}{y-n}=\frac{p}{q},$
$qx+py=s.$

25. $\frac{a}{b+y}=\frac{b}{a-x},$
$\frac{c}{d-x}=\frac{d}{c+y}.$

26. $\frac{1}{x}+\frac{1}{y}=m,$
$\frac{1}{x}-\frac{1}{y}=n.$

27. $\frac{m}{x}+\frac{1}{y}=p,$
$\frac{n}{x}+\frac{1}{y}=q.$

28. $\frac{m}{x}+\frac{n}{y}=p,$
$\frac{r}{x}+\frac{s}{y}=t.$

29. $\frac{x}{m}+\frac{y}{n}=p,$
$\frac{r}{x}=\frac{q}{y}.$

30. $\frac{dy}{bx}=\frac{c}{a},$
$bx+dy=a+c.$

31. $\frac{x+y}{x-y}=\frac{a}{b-c},$
$\frac{x+c}{y+b}=\frac{a+b}{a+c}.$

32. $\dfrac{x-m}{y-m}=\dfrac{m-n}{m+n}$,
$\dfrac{x}{y}=\dfrac{m^2-n^2}{m^2+n^2}$.

34. $\dfrac{x+y+1}{x-y+1}=\dfrac{p+1}{p-1}$,
$\dfrac{x+y+1}{x-y-1}=\dfrac{1+m}{1-m}$.

33. $\dfrac{x+1}{y+1}=\dfrac{m+n+p}{m-n+p}$,
$\dfrac{x-1}{y-1}=\dfrac{m+n-p}{m-n-p}$.

35. $\dfrac{b}{x-a+c}=\dfrac{c}{y-a+b}$,
$\dfrac{x+c}{a+b}=\dfrac{y+b}{a+c}$.

36. $\dfrac{bx+ay}{a+b+2}-x=\dfrac{b-a-1}{b}$,
$bx-a=ay-b$.

37. $\dfrac{1+x}{2}=\dfrac{y-1}{2}+\dfrac{(a-b)^2-2b^2}{a^2-b^2}$,
$by-ax=\dfrac{(3a+b)ab}{a^2-b^2}+\dfrac{ab}{a+b}-(a+b)$.

38. $3x+5y=\dfrac{2(4b-m)bm}{b^2-m^2}$,
$b^2x-\dfrac{bcm^2}{b+m}+(b+c+m)my=m^2x+(b+2m)bm$.

39. $(p-q)x=2p-(p+q)y$,
$2x-\dfrac{5p^2-2pq+5q^2}{p^2-q^2}=-3y$.

40. $2x+3y=\dfrac{2m^2+mn+3n^2}{m^2-n^2}$,
$3x-\dfrac{3m^2-5mn-2n^2}{m^2-n^2}=2y$.

41. $0.6x+y=\dfrac{a}{b}+\dfrac{b(3b-2a)}{5ab}$,
$x-0.6y=\dfrac{5b^2+a(8b-3a)}{5ab}$.

42. $1.5x+y=\dfrac{m^2+n^2}{2mn}$,

$2x-3y=\dfrac{(2m+3n)(2m-3n)}{6mn}+2.166.....$

43. $x=\dfrac{q(q-ry)+(2pq-r)r}{qr}$,

$rx+qy=q(p+1)+r(p-1)$.

44. $qx-py=2m$,

$3y=\dfrac{pq[5(p-x)+3q]+(5p-3q)m}{pq}$.

45. $\dfrac{(c+d)x}{5}-\dfrac{(c-d)y}{3}=4cd$,

$3x+\dfrac{5(c+d)y}{c-d}=30(c+d)$.

46. $x+y=\dfrac{2bc(a^3-2a^2b+3a^2c)}{abc-2b^2c+3bc^2}$,

$a(x-a^2)+b(y+b^2)=ab(a+b)+(a-b)^2$.

47. $x+y=\dfrac{4(a^2-2ab+ac+b^2-bc)}{a-b}$,

$3a(x-a)=3ac-6ab+4bc+2c(y-c)$.

48. $x+y=\dfrac{2[2m(m+p)-n(m+n+2p)]}{m-n}$,

$m(x-m)=5mp+3mn+2p^2-p(n+y)$.

49. $\dfrac{2(m^2-n^2)}{x}-m=\dfrac{ny}{x}$,

$\dfrac{1}{(m-n)x}-\dfrac{1}{(m+n)y}=\dfrac{m^2+n^2}{mnxy}$.

50. $2+x-y=\dfrac{2[(c-d)^2-2d^2]}{c^2-d^2}$,

$dy-\dfrac{cd(3c+d)}{c^2-d^2}=cx-(c+d)+\dfrac{cd}{c+d}$.

51. $x^2-y^2=m$,
$x-y=n$.

52. $x^2-y^2=m^2$,
$x-y=n$.

53. $\dfrac{x}{y}=\dfrac{c+d-\dfrac{cd}{c+d}}{c-d+\dfrac{cd}{c-d}}$,
$x+y=2c^2$.

54. $\dfrac{x}{y}=\dfrac{a-b+\dfrac{b^2}{a-b}\left(1-\dfrac{b(a+b)}{a^2+ab+b^2}\right)}{\dfrac{a^2}{a+b}+\dfrac{b^2}{a^2+ab+b^2}}$,
$x-y=2b^5$.

Ex. 22.

Numerical Equations with More than Two Unknown Quantities.

Solve:

1. $x+y=5$,
$y+z=7$,
$x+z=6$.

2. $2x+y=5$,
$x+3z=16$,
$5y-z=10$.

3. $2x+3y=21$,
$3x-2z=-5$,
$4y+7z=69$.

4. $5x-3y=-1$,
$9y-2z=12$,
$8x+3z=17$.

5. $0.5x+0.3y=2.2$,
$1.2z-8.5y=-24.4$,
$3.3z+1.3x=29$.

6. $0.25x+0.125y=5$,
$0.3y-0.9z=-21$,
$1.4x+1.2z=50$.

7. $2.5x+1.5y=6.5$,
$3.5x-1.5z=4$,
$0.6y+z=2.2$.

8. $0.65x-0.95y=0.5$,
$5.1y-3.3z=6$,
$20.3z-14.7x=21$.

9. $\frac{x}{2}+\frac{y}{3}=12,$
$\frac{z}{5}-\frac{y}{6}=4,$
$\frac{x}{12}+\frac{z}{7}=6.$

10. $\frac{2}{5}x-y=0,$
$x-\frac{3}{2}z=3,$
$\frac{1}{2}z-\frac{1}{3}y=2.$

11. $\frac{x}{2}+\frac{3y}{4}=3,$
$\frac{3x}{2}+z=5\frac{1}{2},$
$\frac{3y}{8}+\frac{z}{2}=1\frac{1}{4}.$

12. $\frac{2x}{3}+\frac{3y}{4}=10,$
$\frac{5y}{24}-\frac{z}{5}=-\frac{1}{3},$
$\frac{5x}{6}+\frac{7z}{10}=12.$

13. $x+y+z=36,$
$3x+6=3z,$
$6y-7=5z.$

14. $x-y+z=34,$
$2y=5x+8,$
$3z-12=13x.$

15. $x+y+z=100,$
$0.7x-4=y,$
$0.3x+4=z.$

16. $\frac{5x}{2}+\frac{10y}{3}+\frac{17z}{4}=64,$
$3\frac{3}{4}x=2\frac{1}{2}y,$
$3\frac{1}{3}y=2\frac{1}{2}z.$

17. $x+6=2\frac{1}{3}y,$
$y+1=3.5z,$
$z+8=1.25x.$

18. $x=21-4y,$
$z=9-\frac{3}{8}x,$
$y=64-7.5z.$

19. $1.7x-0.6y=-4.08,$
$2.8x+0.6z=-6.72,$
$2.8y-1.7z=19.04.$

20. $x+y+z=35,$
$y-x+z=-5,$
$z-y-x=-25.$

21. $x+y-z=17,$
$x+z-y=13,$
$y+z-x=7.$

22. $x+y+z=6,$
$5x+4y+3z=22,$
$15x+10y+6z=53.$

23. $x+y+z=30,$
$8x+4y+2z=50,$
$27x+9y+3z=64.$

24. $x+y+z=24,$
$x:y=7:8,$
$\frac{y}{z}=\frac{8}{9}.$

25. $2x+3y-z=130$,
$x:y=1:5$,
$\frac{y}{z}=\frac{5}{7}$.

26. $7x-5y+3z=66$,
$x:y=2:3$,
$\frac{y}{z}=\frac{3}{4}$.

27. $x+y+z=12$,
$x\div z=3\div 5$,
$y\div z=12\div 15$.

28. $5x+7y-2z=13$,
$8x+3y+z=17$,
$x-4y+10z=23$.

29. $4x-5y+2z=6$,
$2x+3y-z=20$,
$7x-4y+3z=35$.

30. $8x+4y-3z=6$,
$x+3y-z=7$,
$4x-5y+4z=8$.

31. $12x+5y-4z=29$,
$13x-2y+5z=58$,
$17x-y-z=15$.

32. $4x-3y+2z=40$,
$5x+9y-7z=47$,
$9x+8y-3z=97$.

33. $3x+2y+z=23$,
$5x+2y+4z=46$,
$10x+5y+4z=75$.

34. $4x-2y+5z=18$,
$2x+4y-3z=22$,
$6x+7y-z=63$.

35. $2x+5y-7z=-288$,
$5x-y+3z=227$,
$7x+6y+z=297$.

36. $x+y+z=6$,
$8x+4y+2z=22$,
$27x+9y+3z=54$.

37. $0.1x+0.2y+0.3z=14$,
$0.4x+0.5y+0.6z=32$,
$0.7x-0.8y+0.9z=18$.

38. $1.5x-2.5y+2z=2.5$,
$3.5x+y-1.5z=1$,
$2x+1.5y-0.5z=3.5$.

39. $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=62$,
$\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=47$,
$\frac{x}{4}+\frac{y}{5}+\frac{z}{6}=38$.

40. $258-\frac{x}{5}=\frac{y}{7}+\frac{z}{9}$,
$304-\frac{z}{5}=\frac{x}{7}+\frac{y}{9}$,
$296-\frac{z}{7}=\frac{y}{5}+\frac{x}{9}$.

41. $3x-\frac{1}{4}y+z=7.5$,
$2x-\frac{y-3z}{3}=5\frac{1}{3}$,
$2x-\frac{1}{2}y+4z=11$.

WESTFIELD SCHOOL PROPERTY
Purchased 190

42. $\frac{1}{x+y}=1,$
$\frac{2}{x+z}=1,$
$\frac{3}{y+z}=1.$

43. $\frac{2(5y-x)}{3}=11\frac{1}{3},$
$\frac{x-3y}{2x+z}+4=\frac{35}{11},$
$\frac{7y+8z}{2}=34.$

44. $\frac{2(3y-2x)}{3z-7}=1,$
$\frac{5z-x}{2y-3z}=1,$
$\frac{y-2z}{3y-2x}=1.$

45. $x+2y:3y+4z=7:8,$
$3y+4z:5x+6z=8:9,$
$x+y-z=126.$

46. $3y+7z:5x-4y=107:5,$
$5x-4y:7x-2z=5:41,$
$3x-5y+4z=21.$

47. $\frac{8x+5y-7}{5y-z+8}=\frac{14}{19},$
$\frac{3x+5y-7}{7z+3y-21}=\frac{7}{8},$
$2x+3y+4z=29.$

48. $\frac{5-4x}{6y+1}=\frac{55-2x}{3y+74},$
$\frac{3+x}{3z-3}=\frac{2x+9}{6z},$
$\frac{3y-1}{3z+1}=\frac{7y-3}{7z-1}.$

49.* $\frac{12}{2x+3y}-\frac{7.5}{3x+4z}=1,$
$\frac{30}{3x+4z}+\frac{37}{5y+9z}=3,$
$\frac{222}{5y+9z}-\frac{8}{2x+3y}=5.$

50.† $115(113-x)+719(y-219)-590(337-z)=27,$
$\frac{5(113-x)+2}{2(y-219)}=2,$ $\frac{337-z}{y-221}=4.$

51. $\frac{7x-3y+6}{20}-\frac{7y+2z-10}{45}=\frac{5z-11x-32}{36}+2,$
$71z-10x+235+27y=345x-189y+185,$
$5y+3z=44.$

---

* Put $\frac{1}{2x+3y}=m;$ $\frac{1}{3x+4z}=n;$ $\frac{1}{5y+9z}=p.$

† Put $113-x=m;$ $y-219=n;$ $337-z=p.$

52. $\frac{5x-7y+2}{12}-\frac{8x+3z-4}{21}=\frac{11y-5z-4x+18}{14}$,

$\frac{11x-5z+12}{14}-\frac{3y+7z-2x}{18}=\frac{8z-3x+82}{21}$,

$3x-y-2z=16$.

53. $\frac{6x-7y+8z-7}{25}-\frac{8x-3z+44}{35}$

$=\frac{13y-7z+1}{15}+\frac{4x-5y+23}{21}$,

$\frac{4x+5z-1}{33}-\frac{3y-5x+60}{22}=\frac{9z-7y+2}{4}+\frac{7x-6y-5z-8}{6}$,

$2x+y-z=2$.

54. $\frac{3x-7y+5}{18}+\frac{7-4y-11z}{9}=\frac{5x-7z}{8}-\frac{18x+27y+11z-30\frac{1}{2}}{36}$,

$\frac{4x-9y+11}{22}-\frac{5y-3x-9}{4}=\frac{6y+5z}{11}+\frac{7x-17y-3z+19\frac{6}{11}}{8}$,

$\frac{5y+6z-17}{54}-\frac{2y+7z+5}{9}=\frac{x+9y}{27}-\frac{2x+7y+11z+18\frac{1}{2}}{18}$.

55. $\frac{1}{x}+\frac{1}{y}=1$,

$\frac{1}{x}+\frac{1}{z}=2$,

$\frac{1}{y}+\frac{1}{z}=\frac{3}{2}$.

56. $\frac{2}{x}+\frac{1}{y}=\frac{3}{z}$,

$\frac{3}{z}-\frac{2}{y}=2$,

$\frac{1}{x}+\frac{1}{z}=\frac{4}{3}$

57. $\frac{1}{x}+\frac{2}{y}-\frac{3}{z}=1$,

$\frac{5}{x}+\frac{4}{y}+\frac{6}{z}=24$,

$\frac{7}{x}-\frac{8}{y}+\frac{9}{z}=14$.

58. $\frac{3}{x}-\frac{4}{5y}+\frac{1}{z}=7\frac{2}{5}$,

$\frac{1}{3x}+\frac{1}{2y}+\frac{2}{z}=10\frac{1}{6}$,

$\frac{4}{5x}-\frac{1}{2y}+\frac{4}{z}=16\frac{1}{10}$.

59.* $\frac{xy}{x+y}=\frac{1}{5}$,
$\frac{xz}{x+z}=\frac{1}{6}$,
$\frac{yz}{y+z}=\frac{1}{7}$.

60.† $\frac{xy}{4y-3x}=20$,
$\frac{xz}{2x-3z}=15$,
$\frac{yz}{4y-5z}=12$.

61. $x+2y=9$,
$3y+4z=14$,
$7z+u=5$,
$2u+5x=8$.

62. $2x-9y+3z-10u=121$,
$x+7y-z-u=583$,
$3x+2y+5z+2u=255$,
$4x-6y-2z-9u=516$.

63. $x+2y+3z+4u=2$,
$2x+3y+4z+5u=3$,
$6x+7y-8z+9u=3$,
$256x+64y+16z+4u=100$.

64. $3u-2y=2$,
$2x+3y=39$,
$5x-7z=11$,
$4y+3z=41$.

65. $2x-3y+2z=13$,
$4u-2x=30$,
$4y+2z=14$,
$5y+3u=32$.

66. $\frac{x}{3}+\frac{y}{5}+\frac{2z}{7}=58$,
$\frac{5x}{4}+\frac{y}{6}+\frac{z}{3}=76$,
$\frac{x}{2}+\frac{3z}{8}+\frac{u}{5}=79$,
$\frac{y}{5}+\frac{z}{2}+\frac{u}{25}=92$.

---

* By taking the reciprocal values, and performing the division, the equation will become $\frac{x+y}{xy}=\frac{5}{1}$, or $\frac{1}{y}+\frac{1}{x}=5$, etc.

† By taking the reciprocal values, and performing the division, the equation will become $\frac{4}{x}-\frac{3}{y}=\frac{1}{20}$, etc.

Ex. 23.

Literal Equations with More than Two Unknown Quantities.

Solve:

1. $x+y=a,$
   $x+z=b,$
   $y+z=c.$

2. $mx+y=n,$
   $z-y=p,$
   $qx+z=t.$

3. $x+y+z=m,$
   $x-y+z=n,$
   $x+y-z=p.$

4. $x+ay+a^2z+a^3=0,$
   $x+by+b^2z+b^3=0,$
   $x+cy+c^2z+c^3=0.$

5. $ax+by-cz=2ab,$
   $by+cz-ax=2bc,$
   $cz+ax-by=2ac.$

6. $x-mx+y=0,$
   $y-ny+z=0,$
   $x+z=p.$

7. $x+y+z=a,$
   $\frac{x}{y}=\frac{m}{n},$
   $\frac{y}{z}=\frac{p}{q}.$

8. $\frac{1}{x}=m-\frac{1}{y},$
   $\frac{1}{x}=n-\frac{1}{z},$
   $\frac{1}{y}=p-\frac{1}{z}$

9. $\frac{a}{x}+\frac{b}{y}-\frac{c}{z}=m,$
   $\frac{a}{x}-\frac{b}{y}+\frac{c}{z}=n,$
   $-\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=p.$

10. $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=3,$
    $\frac{a}{x}+\frac{b}{y}-\frac{c}{z}=1,$
    $\frac{2a}{x}-\frac{b}{y}-\frac{c}{z}=0.$

11. $\frac{1}{x+y}=m,$
    $\frac{1}{x+z}=n,$
    $\frac{1}{y+z}=p.$

12. $\dfrac{xy}{x+y}=a,$ $\dfrac{xz}{x+z}=b,$ $\dfrac{yz}{y+z}=c.$

13. $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1,$ $\dfrac{x}{a}+\dfrac{y}{c}+\dfrac{z}{b}=1,$ $\dfrac{x}{b}+\dfrac{y}{a}+\dfrac{z}{c}=1.$

14. $\dfrac{x}{m+n}+\dfrac{y}{n-p}+\dfrac{z}{m+p}=2p,$
$\dfrac{x}{m-n}-\dfrac{y}{n-p}+\dfrac{z}{p-m}=2m,$
$\dfrac{x}{m-n}-\dfrac{y}{n-p}-\dfrac{z}{m+p}=2m-2p.$

15. $\dfrac{x}{m+n}+m=n-\dfrac{y}{n+p},$
$\dfrac{y}{p-m}-m=p-\dfrac{z}{m+p},$
$\dfrac{x}{n-p}+p=n+\dfrac{z}{m-n}.$

16. $m-\dfrac{x}{n+p}=\dfrac{y}{p-m}-n,$
$n-\dfrac{y}{p+m}=\dfrac{z}{m-n}-p,$
$p-\dfrac{z}{m+n}=\dfrac{x}{n-p}-m.$

17. $\dfrac{m-n}{nx+my}=\dfrac{(n-p)(m-p)}{p},$
$\dfrac{n-p}{py+nz}=\dfrac{(p-m)(n-m)}{m},$
$\dfrac{p-m}{mz+px}=\dfrac{(m-n)(p-n)}{n}.$

18. $2np = (p+m)x-(p-m)y,$
$2mp = (m+n)y-(m-n)z,$
$2mn = (n+p)z-(n-p)x.$

19. *$(z+x)m-(z-x)n = 2yz,$
$(x+y)n-(x-y)p = 2xz,$
$(y+z)p-(y-z)m = 2xy.$

20. $y+z+u=m,$
$z+u+x=n,$
$u+x+y=p,$
$x+y+z=q.$

21. $x+y-u=m,$
$y+z-v=n,$
$z+u-x=0,$
$u+v-y=p,$
$v+x-z=q.$

22. $x+y+z=3m+n+p,$
$x+y+u=m+3n+p,$
$x-y-u=m-n+p,$
$y+z-u=m-n-p.$

23. $x+y+z+u+v=l,$
$x+y+z+v+w=m,$
$x+y+z+u+w=n,$
$x+y+v+u+w=0,$
$x+z+v+u+w=p,$
$y+z+v+u+w=q.$

Ex. 24.

PROBLEMS WITH TWO OR MORE UNKNOWN QUANTITIES.

1. A man has two vessels, and for the two a single cover worth \$0.90 If he puts the cover upon the first vessel it will be worth $1\frac{1}{2}$ times as much as the other. If he puts the cover upon the second vessel it will be worth $1\frac{1}{1\frac{1}{2}}$ times as much as the first. What is the value of each vessel?

* Do not determine the values of x, y, and z, but those of the known quantities, in terms of the unknown; then use the three new equations in order to deduce x, y, and z. In replacing the unknown quantities, x, y, and z, by the letters m, n, and p, and *vice versa* we can obtain the equations of Example 18.

2. Two numbers are such that 3 times the first plus 5 times the second equals 44; but 3 times the second plus 6 times the first equals 60. What are the numbers?

3. A book-seller sells 3 bound copies of a work and 7 stitched copies. He receives for them all $32.40. Another day he sells 2 bound copies and 5 stitched copies for $22.60. How much does a stitched copy cost, and how much a bound copy?

4. A man has two horses, and a saddle which is worth $150. If he puts the saddle upon the first horse, he is worth double the other; if he puts the saddle upon the second horse, he is still worth $300 less than the first. What is the value of each horse?

5. A fraction is such that if 1 is added to the numerator its value is 1; and if, after having doubled the numerator, 1 is added to the denominator, it still equals 1, what is the fraction?

6. A fraction is such that if 3 is added to each of its terms it equals $\frac{4}{5}$; and if 3 is subtracted from each of its terms it equals $\frac{1}{2}$. What is the fraction?

7. The sum of two numbers is 210, and their sum is to the first as 7 : 4. Find the numbers.

8. Find two numbers whose sum is 54, and whose sum and difference are in the ratio of 9 : 5.

9. A number consists of two digits whose difference is 3. If the order of the digits is changed, the number obtained is $\frac{4}{7}$ of the first number. What is the first number?

10. A number consists of two digits whose sum is 8. If the order of the digits is changed, the new number is $\frac{5}{4}$ of the sum of the digits plus 2 times the first number. What is the first number?

11. The sum of two figures of which a number is composed is 9. If 3 is subtracted from each of the figures, the result is half of the first number diminished by 6. What is the first number?

12. A number of two figures is such that divided by their difference the quotient is 12; but if the order of the figures is changed, and 9 added to the number thus formed, and the result divided by the sum of the figures, 8 is the quotient; find the number.

13. The sum of the two figures of a number equals 6 times their difference, and the number itself is 3 more than 6 times the sum of the figures. Find the number.

14. The sum of two numbers is 26. The third of the first and three-fourths of the second are equal. What are the numbers?

15. Two numbers of three figures each are such that their sum, increased by 1, is 1000. If the two sets of figures are written one after the other, the number obtained by placing the figures of the largest number first is 6 times that obtained by placing the figures of the smallest number first. What are the numbers?

16. Find two numbers such that the first added to 4 times the second equals 29; the second added to 6 times the first equals 36.

17. Two numbers are such that the smaller divided by the greater gives for a quotient $\frac{5}{11}$, and the greater divided by the smaller gives 2 for a quotient and 5 for a remainder. Find the numbers.

18. The smaller of two numbers divided by the larger is 0.21 with a remainder 0.04162. The greater divided by the smaller is 4 with 0.742 for a remainder. What are these numbers?

19. Find two numbers whose sum and difference are as 5 : 1, and their sum and product are as 5 : 8.

20. A servant receives $9 to buy 4 kilograms of butter and 7 of soap, and 15 cents ought to be returned. But she makes a mistake and buys 7 kilograms of butter and 4 of soap, and requires 30 cents more. What was the price of the butter and the soap?

21. If $28 are paid to 6 artificers and 2 laborers for a day's work; and if for another day's work the same sum is paid to 5 artificers and 4 laborers; what is a day's pay of an artificer and of a laborer?

22. A pedestrian has a certain distance to walk. After having passed over 20 miles, he increases his speed by 1 mile per hour. If he had walked the entire journey with this speed, he would have accomplished his walk in 40 minutes less time; but, by keeping his first pace, he would have arrived 20 minutes later than he did. What distance had he to walk?

23. A garden is of a rectangular form. If 3 meters are added to its length, and 2 meters to its width, the surface will be increased by 66 square meters; but if 2 meters are added to its length, and 3 to its width, the original surface will be increased by 71 meters. What are the two dimensions of the garden?

24. If 5 grams of an ingot are mixed with 11 grams of a second ingot, the alloy formed is at the standard of $0.868\frac{3}{4}$. But if 2 grams of the first and 1 gram of the second are taken, the alloy will be at the standard of $0.833\frac{1}{3}$. What is the standard of the two ingots?

25. A cask contains 12 gallons of wine and 18 gallons of water; a second contains 9 gallons of wine and 3 of water. How many gallons must be taken from each cask, so that when mixed there may be 14 gallons, consisting half of water, and half of wine?

26. Two persons owe $1200 each. The first said to the second, "If you give me $\frac{3}{4}$ of what you have, I shall have enough to pay my debt." The second replied, "If you give me $\frac{5}{6}$ of what your purse contains, I can pay my debt." How much do they each have?

27. A society contains a certain number of men and women. When 4 women leave, there are $1\frac{1}{2}$ times as many men as women. But when the women return with their husbands, the number of men is $\frac{1}{8}$ greater than that of the women. How many men and women did the society have?

28. Two friends, A and B, in reviewing their libraries, said that $\frac{1}{5}$ of A's library plus $\frac{1}{8}$ of B's would be 780 volumes; but that $\frac{1}{6}$ of A's library added to $\frac{1}{8}$ of B's would be 20 volumes less. How many volumes did each have?

29. Two women buy velvet and silk. One buys $3\frac{1}{2}$ yards of velvet and $12\frac{3}{4}$ yards of silk; the other took $4\frac{1}{2}$ yards of velvet and 5 yards of silk. They each pay $63.80. How much per yard did each cost?

30. Three brothers, M, N, P, have bought a house which cost $32,000. M could pay the whole sum if N would give him $\frac{5}{8}$ of what he has; N could pay it if P would give him $\frac{3}{5}$ of what he has; and P could pay the whole sum if he had $\frac{1}{2}$ of what M has plus $\frac{3}{16}$ of what N had. How much does each have?

31. Three women go to market to sell eggs. If the first gives $\frac{1}{7}$ of hers, and the third $\frac{1}{15}$ of hers to the second, they will all have an equal number; they all together carry 360. How many has each?

32. A farmer sold at the same prices, first, 5 hektoliters of wheat, 4 of rye, and 3 of oats for $24.20; second, 3 hektoliters of wheat, 5 of rye, and 4 of oats for $21.40; third, 1 hektoliter of wheat, 8 of rye, and 5 of oats for $22.20. What was the price per hektoliter of each kind of grain?

33. A number is composed of three figures whose sum is 17. The figure of the hundreds is double that of the units. When 396 is subtracted the order of the figures is reversed. What is the number?

34. Find three numbers such that the first plus half the sum of the other two is 120; that the second, added to $\frac{1}{3}$ of the difference between the third and the first, is 70; and half the sum of the three equals 95.

35. Find three numbers such that the first plus $\frac{1}{2}$ the second, the second plus $\frac{1}{3}$ of the third, and the third plus $\frac{1}{4}$ of the first are each 1000.

Ex. 25.

Square and Cube Roots.

Extract the square root to the fifth decimal place:

1. 326.	6. 1020.	11. 9.3434.....
2. 29.	7. 935.	12. 27.1919.....
3. 491.	8. 842.	13. 3.666.....
4. 726.	9. 719.	14. 3.1414.....
5. 391.	10. 145.	15. 1.315315.....

Extract the square root to the third decimal:

16. 120.	**20.** 101.	**24.** 1,440,009.
17. 133.	**21.** 10,004.	**25.** 809,995.
18. 3602.	**22.** 9997.	**26.** 6396.
19. 401.	**23.** 90,004.	**27.** 40,003.

Extract the square root of:

28. $4+9y^2-20x+25x^2+30xy-12y$.

29. $4x^4+9y^6-12x^2y^3+16x^2z^5+16z^{10}-24y^3z^5$.

30. $4+13a^2+9a^4-4a-6a^3$.

31. $a^2b^2+2a^2b+a^2-2ab^2-2ab+b^2$.

32. $18x^2+x^4+1-8x^3-8x$.

33. $9a^4+25b^2+64m^2+z^2-30a^2b+48a^2m-6a^2z$
$-80bm+10bz-16mz$.

34. $y^{10}+4u^6+9v^8+6v^4y^5-4u^3y^5+16q^{10}-8q^5y^5$
$-12u^3v^4+16q^5u^3-24q^5v^4$.

35. $a^4+16d^{4x}+9e^{2y}+9c^{10}-6a^2c^5-8a^2d^{2x}+24c^5d^{2x}$
$+6a^2e^y-18c^5e^y-24d^{2x}e^y$.

36. $9a^6-12a^5b-38a^4b^2+52a^3b^3+33a^2b^4-56ab^5+16b^6$.

37. $\frac{m^4}{4}+\frac{n^6}{9}+\frac{m^2p^4}{4}-\frac{m^2n^3}{3}+\frac{p^8}{16}-\frac{n^3p^4}{6}$.

38. $\frac{x^2}{y^2}-\frac{4xz}{uy}+\frac{4z^2}{u^2}+\frac{6qx}{vy}+\frac{9q^2}{v^2}-\frac{12qz}{uv}$.

39. $\frac{9}{25}+\frac{a^8}{25}+\frac{m^6}{36}-\frac{6a^4}{25}-\frac{m^3}{5}+\frac{4n^8}{49}+\frac{12n^4}{35}+\frac{a^4m^3}{15}-\frac{4a^4n^4}{35}$
$-\frac{2m^3n^4}{21}$.

40. $\frac{9m^6n^4}{25p^6q^8} - \frac{12m^5n^5}{35p^7q^9} - \frac{332m^4n^6}{735p^8q^{10}} + \frac{16m^3n^7}{63p^9q^{11}} + \frac{16m^2n^8}{81p^{10}q^{12}}$.

41. $25\frac{3}{7} - \frac{20x}{7y} + \frac{9y^2}{16x^2} - \frac{15y}{2x} + \frac{4x^2}{49y^2}$.

42. $x^3 - \frac{2x^{11}}{a^3} + 2a^4x^4 + \frac{x^{14}}{a^6} - 2ax^7 + a^8$.

43. $x^6y^6 - \frac{2x^3y^{12}}{3a^3} + \frac{y^{18}}{9a^6} + \frac{4x^3y^{18}}{a^9} - \frac{4y^{24}}{3a^{12}} + \frac{4y^{30}}{a^{18}}$.

Extract the cube root to the third decimal:

44. 867.	49. 4915.	54. 3.2525.....
45. 1241.	50. 8972.	55. 317.8181.....
46. 976.2	51. 12,761.	56. 15.4141.....
47. 517.	52. 56,431.	57. 690.
48. 1637.	53. 20,911.	58. 10,010.

Extract the cube root of

59. $8b^3 - 9a^2y + 6a^2b + a^3 + 27ay^2 + 12ab^2 - 27y^3 + 54y^2b - 36yb^2 - 36aby$.

60. $147x^2v - 126xuv + 343x^3 - 441x^2u - 27u^3 + v^3 + 189xu^2 + 21xv^2 + 27u^2v - 9uv^2$.

61. $125a^9 - 8b^6 + c^{12} - 150a^6b^2 + 75a^6c^4 + 60a^3b^4 + 15a^3c^8 + 12b^4c^4 - 6b^2c^8 - 60a^3b^2c^4$.

62. $36x^{10}y^3 - 24x^{10}z^3 + 54x^5y^4 + 8x^{15} + 27y^6 + 24x^5z^6 - 54y^4z^3 - 72x^5y^2z^3 - 8z^9 + 36y^2z^6$.

63. $162a^8d^7 - 108a^4b^5d^7 + 27a^{12} - b^{15} + 9a^4b^{10} + 324a^4d^{14} + 216d^{21} + 18b^{10}d^7 - 108b^5d^{14} - 27a^8b^5$.

64. $360m^3np + 1000m^6 - 900m^4n - 27n^3 - 600m^4p - 8p^3 + 270m^2n^2 + 120m^2p^2 - 54n^2p - 36np^2$.

65. $36a^5b^4 + 8a^6b^3 - 45a^3b^6 - 8b^9 + 30a^4b^5 - 30a^2b^7 + 36ab^8$.

66. $225x^7y^3 - 150x^8y^5 + 27x^3y^3 - 8x^6y^9 + 36x^5y^7 + 125x^9y^3 + 60x^7y^7 + 135x^5y^3 - 180x^6y^5 - 54x^4y^5$.

67. $125m^3n^6 - 192m^7n^{10} - 64m^9n^{12} + 240m^5n^8 - 504m^6n^9 + 450m^4n^7 + 288m^8n^{11}$.

68. $-172a^6x^9 + 27a^3x^6 + 54a^4x^7 - 125a^9x^{12} - 99a^5x^8 + 165a^7x^{10} + 150a^8x^{11}$.

69. $\frac{8a^6}{x^3} - \frac{36a^7}{x^4} + \frac{a^{12}}{x^9} + \frac{33a^{10}}{x^7} + \frac{66a^8}{x^5} - \frac{9a^{11}}{x^8} - \frac{63a^9}{x^6}$.

70. $\frac{3m^8}{y^{14}} - \frac{m^3}{y^9} + \frac{m^9}{y^{15}} - \frac{5m^6}{y^{12}} + \frac{3m^4}{y^{10}}$.

71. $\frac{300x^5}{y^8} - \frac{75x^4}{2y^7} + \frac{64x^9}{y^{12}} + \frac{3x^7}{y^{10}} - \frac{x^6}{8y^9} + \frac{15x^5}{4y^8} + \frac{125x^3}{y^6} + \frac{240x^7}{y^{10}} - \frac{60x^6}{y^9} - \frac{24x^8}{y^{11}}$.

72. $\frac{12}{a^7} - \frac{125}{a^{12}} - \frac{60}{a^8} + \frac{1}{a^9} - \frac{15}{a^{10}} + \frac{8}{a^6} + \frac{6}{a^8} + \frac{150}{a^{10}} - \frac{60}{a^9} + \frac{75}{a^{11}}$.

73. $\frac{135a^5}{33b^4} + \frac{125a^9}{8b^6} - \frac{9}{8b} - \frac{8b^3}{27a^6} + \frac{10}{3a} + \frac{27a^3}{64b^3} + \frac{b}{a^3} + \frac{225a^7}{16b^5} - \frac{15a^2}{2b^2} - \frac{25a^4}{2b^3}$.

74. $\frac{64a^9}{x^{12}} + \frac{171a^6}{x^9} + \frac{102a^5}{x^8} + \frac{8a^3}{x^6} + \frac{36a^4}{x^7} + \frac{204a^7}{x^{10}} + \frac{144a^8}{x^{11}}$.

75. $27b^6 + 296b^9 - 125b^{12} - 108b^7 + 9b^8 - 15b^{10} - 300b^{11}$.

Ex. 26.

Pure Quadratics.

1. $3(2x-3)^2 = 4x(2x-9)+43.$

2. $2x+\frac{17}{x} = \frac{7x-10}{2\frac{1}{2}}+4.$

3. $(x+9)(x-9) = 19-3x^2.$

4. $(x+4)(2x+9) = \left(2x+\frac{17}{2}\right)2x-13.$

5. $\frac{2x-1}{x-2} = \frac{x-5}{3x-2}.$

6. $\frac{4}{x+3}-\frac{4}{x-3} = -\frac{1}{3}.$

7. $\frac{1}{3x+11}-\frac{1}{3x-11} = -\frac{22}{23}.$

8. $\frac{1}{12-5x}+\frac{1}{5x-12} = 1+\frac{120x}{(12-5x)^2}.$

9. $\frac{5}{2+x}-\frac{5}{x-4} = \frac{2-x}{x-4}+\frac{2}{x^2-2x-8}.$

10. $\frac{3x^2}{4}-\frac{15x^2+8}{6} = 2x^2-3.$

11. $\frac{x+7}{x(x-7)}-\frac{x-7}{x(x+7)} = \frac{7}{x^2-73}.$

12. $\frac{x-3}{x^2-4}+1 = \frac{1}{x-2}.$

13. $\frac{2x}{x-2}+\frac{x-2}{x} = 2.$

14. $\frac{x+m}{x-m}+\frac{x-m}{x+m} = -\frac{4mn+2n^2}{x^2-m^2}.$

15. $\frac{x}{a^2}-\frac{b^2}{x}=\frac{x-2b^2}{a^2x}-\frac{1}{a^2}$. 16. $\frac{a^2}{x}-\frac{x}{b^2}=\frac{x}{a^2-b^2}$.

17. $(x+a)(x-a)+(x+b)(x-b)-(x+c)(x-c)=0$.

18. $(3a+b-x)(a+3b-x)=2(a+b-x)^2$.

19. $\frac{x^4-m^4}{x-m}-\frac{x^4-m^4}{x+m}=10m^3$. 21. $\frac{ab-x}{b-ax}=\frac{b-cx}{bc-x}$.

20. $\frac{n-x}{n+x}=\frac{x+p}{x-p}$. 22. $\frac{x+3a-b}{x+2a-b}=\frac{x-4a+5b}{3b-x+a}$.

23. $\frac{4a+b+x}{x+2a-b}=\frac{x-2a+b}{4a+b-x}$.

24. $\frac{17a+b-x}{a+17b-x}=\frac{a^2(a+17b+x)}{b^2(17a+b+x)}$.

25. $\frac{x^3-2bx+2ax-b^2}{x^3-a^3}+\frac{x+2b}{x^2+ax+a^2}=\frac{1}{x-a}$.

Ex. 27.

Affected Quadratics.

Numerical Quadratics of the form $x^2+px+q=0$:

1. $x^2-2x=48$.
2. $x^2-84=-8x$.
3. $10x=171-x^2$.
4. $x^2+12=7x$.
5. $14=x^2-5x$.
6. $60-x^2=-7x$.
7. $x(x-1)=380$.
8. $3x^2-5x=10+2x^2-2x$.
9. $5x^2-18x=630-9x+4x^2$.
10. $250+2x^2=3x^2-15x$.
11. $6x^2-\frac{7}{4}+4x=x+5x^2$.

12. $2-8x+3x^2=-4+2x^2-3x.$

13. $(3+5x)x-\frac{1}{9}=3x\left(1-\frac{5}{3x}\right)+x(15+4x).$

14. $(x+5)^2=2(x+3)^2-17.$

15. $(3x-2)^2=8(x+1)^2-100.$

16. $(x-5)^2-5=(2x+6)8-x^2.$

17. $(2x-8)^2=(3x+25)4+12.$

18. $\frac{x^2}{6}+\frac{2x}{3}=10.$

19. $3(2x-3)-\frac{22}{x}=\left(x-\frac{3}{2}\right)5.$

Equations with the form $ax^2+bx+c=0$:

20. $5x^2-3x-2=0.$

21. $3x^2+5x-42=0.$

22. $9x^2+15x=6.$

23. $5x^2-7x+\frac{6}{5}=0.$

24. $3x^2-50=25x.$

25. $7x^2-9x=10.$

26. $3x^2-7x-6=0.$

27. $2x^2-13x=-15.$

28. $2x^2-8=3x+12.$

29. $5x(x-2)+2=-4-x(4x-5).$

30. $3\left(x+\frac{33}{25}\right)-13x^2=3x^2.$

31. $11x^2+7x-\frac{3}{7}=4x(x+1)+1.$

32. $5x(x-2)+\frac{1}{4}=1-3x.$

33. $x(12x+0.7)-\frac{7}{11}=x(3x-0.2)+6\frac{4}{11}$.

34. $11(x-1)^2+x=2(1-x)^2+11$.

35. $3(z^2+2)^2-54=3z^4+z(5z-7)$.

36. $(1-3x)(x-6)=2(x+2)$.

37. $(2x-3)(x-2)-49\frac{2}{3}=(x-1)^2+\frac{16}{3}-2x^2$.

38. $(x-1)(1-3x)-\frac{1}{3}=\left(x-\frac{1}{3}\right)3-1$.

39. $(x+1)(2x+3)=4x^2-22$.

40. $5\left(\frac{1}{8}+x\right)^2+4x=(3\frac{7}{8}-x)^2-\frac{1}{2}$.

41. $(3x+1)(2x-3)+3=(x+3)^2-15$.

42. $\frac{x-3}{x+4}+\frac{x-4}{2(x-1)}=\frac{1}{2}$.

43. $x-3+\frac{x+6}{x-6}=2x-7$.

44. $\frac{2x-1}{3}-\frac{3}{x-8}=\frac{x-2}{x-8}+3$.

45. $\frac{3x+2}{2x-1}+\frac{7-x}{2x+1}=\frac{7x-1}{4x^2-1}+5$.

Equations with the form $ax^2+2bx+c=0$:

46. $x^2+6x-11\frac{1}{4}=0$.

47. $x^2-4x+2=-\frac{14}{25}$.

48. $3x^2-6x=-\frac{5}{3}$.

49. $9x^2+12x-5=7$.

50. $5x^2-8x+3=0$.

51. $7x^2-48=2x(x+7)$.

52. $(5x+1)^2-2\left(x+\frac{1}{5}\right)=10x^2+16\frac{2}{5}$.

53. $(5x-2)(x+1)=\left(x+\frac{3}{5}\right)5-5$.

54. $(3x+2)(x-3)=(6-x)5+4$.

55. $13x^2-30=6(1-x)^2+63$.

56. $(3x+2)(5x-2)=(5x-1)^2-17$.

57. $(1.2-x)^2+(x+0.8)^2=2(6x-0.2)^2$.

58. $(0.7+x)^2+(1.3-x)^2-(1.6-2x)^2=\frac{10x}{3}$.

59. $(3x+0.5)^2+(5x-0.5)^2=6(3x-0.5)^2+4x$.

60. $\frac{x-2}{x-1}-\frac{x-3}{x+3}=\frac{x+4}{x-1}-\frac{7}{4}$.

61. $\frac{6}{2x-6}-\frac{1}{x-3}=\frac{x-5}{4}$.

62. $\frac{x-5}{x+3}+\frac{x-8}{x-3}=\frac{80}{x^2-9}+\frac{1}{2}$.

63. $\frac{2x+1}{7-x}+\frac{4x+1}{7+x}=\frac{45}{49-x^2}+1$.

64. $(x+1)^2+(2x-1)(1-x)=(x+2)(2x-1)-9.27$.

65. $(x+0.9)^2+(x-1.1)(2x-0.2)=(3x-4.3)^2+3(x-2.1)$

66. $\frac{x^2}{2}-\frac{x}{3}+\frac{59}{8}=8$.

67. $\frac{x^2}{4}-\frac{3x}{8}-\frac{7}{6}=1\frac{1}{8}$.

68. $\frac{x+1}{3}+\frac{3(x-1)}{4}=(x-3)^2+1$.

69. $\frac{2x+5}{5}-\frac{x-4}{3}=\frac{2x(x-7)}{6}-\frac{7x}{10}$.

70. $\frac{x-2}{6}+\frac{(x-4)(x-6)}{5}-\frac{2(15-x)(x-5)}{3}=0.$

71. $\frac{(x-5)(x-11)}{4}-\frac{(x-12)^2}{5}=\frac{x(x-15)}{5}+\frac{x}{10}$

72. $\frac{(x-12)^2}{6}-\frac{x}{9}+\frac{x(x-9)}{15}=\frac{(x-14)^2}{2}+5.$

73. $\frac{x(2x-10)}{12}-\frac{(x-7)^2}{2}=\frac{(14-x)^2}{3}+(11-x)^2.$

74. $\frac{x-1}{3}+\frac{x-2}{7}-\frac{2x-7}{5}=\frac{x(x-15)}{8}$

75. $\frac{(x+3)^2}{5}-\frac{(2x-20)(x-12)}{3}-\frac{7(x-6)}{2}=-\frac{x}{2}$

76. $\frac{x-1}{x-5}=\frac{3x}{2x-7}$

77. $\frac{3x+2}{x-2}=\frac{4x+5}{x-1}.$

78. $\frac{x-7}{2(x+3)}=\frac{x-6}{x+24}.$

79. $\frac{x+0.5}{x-0.5}=\frac{2x+1}{x+2.5}.$

80. $\frac{4x-1}{x+0.5}=\frac{6x}{x+2.5}$

81. $\frac{3x-1}{4x+\frac{1}{2}}=\frac{2x+\frac{3}{4}}{5x+2\frac{1}{4}}$

82. $\frac{x+1}{x-1}=\frac{4x-3}{x+9}$

83. $\frac{2x-1}{x-4}=\frac{x+1}{x-7}.$

84. $\frac{x}{6}+\frac{35}{x+4}=4\frac{1}{2}.$

85. $\frac{55}{16-x}-\frac{x}{6}=4\frac{1}{2}.$

86. $\frac{6}{x^2-1}+\frac{3}{x+1}=\frac{2}{x-1}+1.$

87. $\frac{2(x+7)}{x+1}+\frac{x-1}{x+1}=\frac{x+11}{x^2-1}+4.$

88. $\frac{x+9}{x-8}+\frac{2(2x+1)}{x+3}=\frac{7}{2}+\frac{7(x+13)}{x^2-9}.$

89. $\frac{2x+1}{x+3}-\frac{x-1}{x^2-9}=\frac{x+3}{3-x}-\frac{4+x}{3+x}$.

90. $\frac{1}{x-3}+\frac{7}{x+3}=\frac{14}{x^2-9}-\frac{x-4}{x+3}$.

91. $\frac{x}{2x-1}+\frac{25}{4x^2-1}=\frac{13}{2x-1}+\frac{1}{27}$.

92. $\frac{15}{3x-1}-\frac{3}{4}=\frac{220}{9x^2-1}-\frac{11}{3x+1}$.

93. $\frac{8-x}{5(x^2-4)}+\frac{x+2}{x-2}+\frac{x-2}{x+2}=\frac{34}{15}$.

94. $\frac{x+4}{3x-7}-\frac{9}{19}-\frac{x}{3x+7}=\frac{52}{9x^2-49}-\frac{15}{52}$.

95. $\frac{1}{x-2}+1-\frac{6-x}{3(x^2-4)}-\frac{1}{2-x}=0$.

96. $\frac{x+1}{x-1}+\frac{x+2}{x-2}-\frac{2x+13}{x+1}=0$.

97. $\frac{x-1}{2x+3}-\frac{x+1}{2x-3}=\frac{x+7}{4x^2-9}-\frac{x-3}{2x-3}$.

98. $\frac{2x-1}{x+1}+\frac{3x-1}{x+2}-\frac{x-7}{x-1}=4$.

99. $\frac{1}{18+x}-\frac{1}{7}=\frac{1}{11}+\frac{1}{x}$.

100. $\frac{1}{24+x}-\frac{1}{13}=\frac{1}{x}+\frac{1}{11}$.

101. $\frac{3x+1}{3(x-5)}-\frac{2x-7}{2x-8}-\frac{5}{2}=0$.

102. $10x-\frac{14x-9}{8x-3}=\frac{18-40x^2}{3-4x}-9$.

103. $\frac{x^2+2x+2}{x+1}+\frac{x^2+8x+20}{x+4}=\frac{x^2+4x+6}{x+2}+\frac{x^2+6x+12}{x+3}$.

104. $\frac{2x-3}{5}+\frac{1}{7}\left(5x-\frac{6x+4}{5x+1}\right)=x+\frac{5x+8}{3x-14}+\frac{1}{3}\left(\frac{x}{7}+\frac{x-9}{5}\right)$.

105. $\frac{x+3}{x+2}+\frac{x-3}{x-2}=\frac{3x+1}{2\left(x-\frac{4}{x}\right)}-\frac{12}{x^2-4}$.

Literal equations with the form $x^2+px+q=0$:

106. $x^2+ax+b=0$.

107. $x^2-ax+b=0$.

108. $x^2+ax-b=0$.

109. $x^2-ax-b=0$.

110. $ax+b=x^2$.

111. $x^2+n=-mx$.

112. $x^2-dx=g$.

113. $bx+c=x^2$.

114. $px+4p^2=x^2-2px$.

115. $bx-2b^2+x^2=2bx$.

116. $x^2+dx-h^4=3h^4-\frac{d^2}{4}$.

117. $x^2-\frac{x}{a}=\frac{3}{4a^2}$.

118. $x^2+mn=-(m+n)x$.

119. $x^2-(n+p)x=-np$.

120. $x^2-\frac{m^2+n^2}{mn}x=\mp 1$.

121. $x^2+\frac{3nx}{2}-n^2=\frac{11n^2}{16}$.

122. $cx-2c-x^2=-2x$.

123. $x^2-\frac{ax}{2}=\frac{a^2}{2}$.

124. $x^2+\frac{mx}{3}=\frac{2m^2}{3}$.

125. $x^2-\frac{3px}{5}=-\frac{p^2}{20}$.

126. $x^2-\frac{5bx}{3}=-\frac{b^2}{4}$.

127. $x^2-\frac{3(a^2+b^2)}{4}-ax=\frac{b}{2}(3a+2x)$.

128. $(m-1)^2x^2-m(x+n)=mx^2(m-2)-nx$.

129. $(x+a)(x-b)+ab=2x^2-ab$.

130. $(m+x)(x-n)=2(x-n)^2+mn$.

131. $\frac{x}{a}\left(1+\frac{x}{b}\right)+1+\frac{x}{b}=0$.

Literal equations with the form $ax^2 + bx + c = 0$:

132. $3x^2 + ax = b$.

133. $2x^2 - 3ax = 2a^2$.

134. $p - ax = -bx^2$.

135. $cx^2 + 5ax = 5a + c$.

136. $ax^2 + 5a^2x = -\frac{9a^3}{4}$.

137. $2x^2 - (a+b)x + \frac{ab}{2} = 0$.

138. $(x-p)^2 = 5px - (x+p)^2$.

139. $2ax^2 - (x + 5a)\,bx = -2b^2(x + a)$.

140. $(2a + x)(2a - x) = 15ab - 9b^2 - 3ax$.

141. $(x + a)(x - 2b) + ab = \frac{3(a+b)^2}{4} - bx$.

142. $(x^2 + px + p^2)(x + p) = (x^2 + p^2)(x - 2p) + 6p^3$.

143. $acx^2 - bcx + adx - bd = 0$.

144. $\frac{bx}{c}(acx + b) + \frac{3a^2x}{c} = \frac{6a^2 + ab - 2b^2}{c^2}$.

145. $(a - b)x^2 + \frac{ab}{a - b} = (a + b)x$.

146. $abx^2 = \frac{1}{ab}\left[x(a + b) - \frac{1}{ab}\right]$.

147. $\frac{3a}{x + a} + \frac{2a}{x + 2a} - \frac{a}{x + 3a} = \frac{4a}{x}$.

Literal equations with the form $ax^2 + 2bx + c = 0$:

148. $ax^2 - 2bx = c$.

149. $ax^2 + 2bx = -c$.

150. $px^2 - 4qx = m$.

151. $-mx^2 - c = 4mx$.

152. $c^2x^2 = 8cx + a^2 + 8a$.

153. $35a^2 + 3x^2 = 22ax$.

154. $21a^2b^2 - 44abx = -15x^2$.

155. $x(x - ab) = ab(x - a)$.

156. $(m - x)(n + x) = x(n - m + 2x) - \frac{mn}{3}$.

157. $(a + x)^2 b = (a + b)^2 - x^2 - \frac{b^2(2a + b)}{b + 1}$.

158. $(x+a)(3x-b)=(3b-a)x-(a-b)^2-ab.$

159. $x^2-4bx=a^2-2ab-3b^2.$

160. $ax\left(\frac{ax}{b^2}-\frac{1}{c}\right)+\frac{1}{c}\left(\frac{b^2}{c}-ax\right)=0.$

161. $\frac{4b}{a}+\frac{a-4b}{x-2b}=\frac{a+4b}{x+2b}.$

162. $\frac{1}{x}+\frac{1}{x-2a^2}-\frac{1}{b^2}=0.$

163. $\frac{1}{x+a}+\frac{1}{x+b}-\frac{1}{x+c}=0.$

164. $\frac{1}{a}+\frac{1}{a+x}+\frac{1}{a+2x}=0.$

165. $\frac{a}{x-a}-\frac{2a}{x-2a}+\frac{3a}{x-3a}=0.$

166. $\frac{x}{a-x}-\frac{a-x}{x}=\frac{a}{x}.$

Find the best solution for the following equations:

167. $abcx^2-a^2b^2x+c^4x-abc^3=0.$

168. $abx^2-(a^2-b^2)x-ab=0.$

169. $x(x-5a)-b(x+6b)=-a(6a+5b).$

170. $4b(b-x)=(a+x)(a-x).$

171. $b(a-x)-c(a+x)=x(a-x)-2c(b-c).$

172. $x(x-2b)+a(b-x)+2c(a-2c)=-b^2.$

173. $x^2-(a^2+b^2)x=2ab(x-a^2-b^2).$

174. $ax(cx-3b)=5d(3b-cx).$

175. $a^5x(c^4x-a^3b)=(a^3b-c^4x)b^2c^3.$

176. $p^3x(m^6p^2x-n^4)+n^2x(m^3-n^2p^3)=n^2\left(\frac{4n^4}{m^3p^2}-m^3x\right).$

177. $x(x+b^2-b)=ax(a+1)-(a+b)^2(a-b).$

178. $x^2+(4m^2-n^2)2mn=(4m^2+n^2)x.$

179. $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}.$

180. $\frac{1}{x}+\frac{1}{x+d}=\frac{1}{c}+\frac{1}{c+d}.$

181. $\frac{p^2+x^2}{p^2-x^2}=\frac{p+2x}{p-2x}.$

182. $\frac{ax^2-bx+c}{px^2-nx+p}=\frac{c}{p}.$

Ex. 28.

Problems Involving Quadratics.

1. Two numbers are in the ratio of $\frac{1}{3}$ to $\frac{1}{5}$, and the sum of their squares is 1666. What are the numbers?

2. The sum of the squares of three integral consecutive numbers is 1202. What are the numbers?

3. The sum of the squares of three consecutive even numbers is 776. What are the numbers?

4. The sum of the squares of three consecutive multiples of 4 is 3104. Find the numbers.

5. In the place of a square field which I have bought, I am offered a rectangular piece of ground of the same perimeter, but the length is 16 rods more than that of the square, and the area is only $\frac{224}{225}$ of that of the square. What is the value of the rectangular garden at \$4 the square rod?

6. The sum of the squares of two numbers is 5690, and the difference of the squares is 1272. What are these numbers?

7. A circular space covered with turf has a surface of 1017.8784 square meters. It is required, without changing the surface, to give to the turf the form of a ring included between two concentric circumferences, the radius of one being double that of the other. What radii must be given to these circumferences? The ratio of the circumference to the diameter is 3.1416.

8. The length of a rectangular field is to its width as 16 is to 11. A neighbor, to make more regular the form of his adjoining property, proposes to diminish the width, but to increase the length; leaving the perimeter the same, so that the new dimensions will be in the ratio of 17 to 10. The rectangular field thus loses 216 square yards. What are its dimensions?

9. Two travellers set out at the same instant from the same point and go one to the south and the other to the east. The first accomplishes 30 miles and the second 40 miles per day. In how many days will they be 250 miles distant from each other?

10. A man buys a certain number of kilograms of coffee, 4 times as much of tea, and 6 times as much of wool. He pays per kilogram for each, as many francs as the number of kilograms he has bought of each. He pays for all 477 francs. How much coffee did he buy?

11. A traveller sets out from A and goes toward the north, accomplishing each day 36 miles. Five days after, a second traveller sets out from A going toward the east, and making each day 28 miles. At the end of how many days will the second traveller be 430 miles from the first?

12. The sum of the squares of three consecutive integral numbers, diminished by the sum of the squares of the two integral numbers preceding, is 288. Find the numbers.

13. A certain even number is such that its square, increased by those of the two even numbers following, is greater by 406 than the sum of the squares of the two odd numbers which follow it. Find the number.

14. Of two numbers, the greater is 48 more than the smaller, and if one adds to their sum the quotient of the greater divided by the smaller, 99 is the result. Find the numbers.

15. A number is the product of three integral consecutive numbers. If it is divided by each of its three factors, the sum of the quotients thus obtained is 767. What is this number?

16. Find two numbers whose sum is double their difference, and whose product diminished by the smaller is equal to 720 times the quotient of the greater divided by the smaller.

17. Find a number which, added to its square and cube, has for the sum 463 times the number in question.

18. A circular piece of water of 864 square yards is changed to another of the same extent but of rectangular form, whose width is 12 yards less than the length. What are the two dimensions?

19. The surface of a rectangular field is 312.8^{a}. If each of the dimensions are increased 1^{m}, the surface will be increased 472^{qm}. What are the two dimensions?

20. If 5 pounds of beef more are sold for \$2, twelve pounds will cost \$0.40 less. How much does one pound cost?

21. A woman buys a certain number of books for \$1.80. If for the same price she had received 4 books more, each would have cost $1\frac{1}{2}$ cents less. How many books did she buy?

22. A merchant bought cattle which cost him in all \$672. If each head had cost him \$4 less, he would have been able to buy 3 more. How many did he buy, and at what price?

23. A man buys 673 young trees which he plants in rows, equally distant, upon two pieces of land forming squares. The side of one contains 6 trees more than the side of the other, and after this he has 7 trees left. How many trees does each square contain?

24. A father is 24 years older than his son. The product of the ages of the father and son is equal to 3 times the square of the age of the son. What is the age of the father and of the son?

25. A tourist, who has accomplished a journey of 630 miles, would have taken 4 days less if he had travelled each day 10 miles more. How many days did it take him to accomplish his journey, and how many miles did he travel each day?

26. A tourist, who has accomplished a journey of 630 miles, would have taken 4 days more if he had travelled each day 10 miles less. How many days did it take him to accomplish his journey, and how many miles did he travel each day?

27. A young merchant buys a certain number of melons which cost him $7.56. After throwing away 5 bad ones, he sold each one which remained for 4 cents more than he had paid for it, and thus gained $0.58. How many melons did he buy?

28. Eight hundred and forty nuts were distributed among some children. If each one had received 2 nuts less he would have had as many as there were sharers in the distribution. How many children were there?

29. I sell a watch for $22.75, and I thus lose as much per cent as it had cost me dollars. How much did I pay for it?

30. The side of a square is 230 yards. Find the dimensions of a rectangle whose perimeter is 12 yards more than that of the square, and the area 460 square yards less.

31. A square field contains a number of square yards equal to the product of 32 times its perimeter, increased by 260. What is the side of the square?

32. A square piece of land, in order to have an area of 260 square yards, must be increased by a number of square yards equal to 32 times the perimeter of the square. What is the side of the square?

33. A woman distributes 144 nuts among 7 children, giving half to the girls, and half to the boys. In this manner each girl receives 6 nuts more than a boy. How many boys and girls are there?

34. Two reservoirs of a cubic form contain together 1853^{1}. The sum of the heights of the reservoirs is 1.7^{m}. What is the side of each?

35. Three workmen, A, B, C, dig a ditch; A in 6 days more, B in 30 days more than the number of days it takes them to do it together; and C in 3 times the number of days it takes them to do it together. How many days would it take them to dig the ditch if they all work together?

36. I buy oranges, each of which costs a number of cents equal to one-third of the number of oranges bought. With the same sum I could buy twice as many melons less 21, paying for each 12 cents more than for an orange. How many oranges did I buy?

37. The denominator of a fraction is greater than the numerator by 3. If 2 is added to each term, the fraction thus obtained is $\frac{2}{21}$ more than the first fraction. What is the first fraction?

38. A cask contains 80^l of wine. A person draws off a certain number of liters, which he replaces by water. From this mixture he again draws off the same number of liters as the first time, and again replaces them by water, after which the 80^l of the cask includes only 45^l of wine. How many liters did he draw off each time?

39. Two messengers, starting from the same point, are sent at the same moment to a city at a distance of 72 miles. The first accomplishes 1 mile per hour more than the second, and arrives at his destination 1 hour sooner. What distance does each messenger travel each hour?

40. A train starts from A for B, a distance of 216 miles, at 2 o'clock in the morning. At 5.30 o'clock there starts from A, for the same destination, a second train which travels 4 miles more per hour than the first, and arrives 1 hour sooner. At what hour does each train arrive in B?

41. A tank can be filled in $26\frac{1}{4}$ hours by two pipes together. If they work separately, the tank will be filled in 18 hours less by the larger than by the smaller. How many hours will it take each pipe to fill the tank?

42. Two numbers are in the ratio m to n; the sum of their squares is s. Find the numbers.

43. Let a, b, c be the three sides of a triangle, and a the greatest. Find the quantity x that must be taken away from each side in order that the triangle whose sides will be $a-x$, $b-x$, $c-x$, will be a right triangle.

Apply the result to the triangle whose sides are 18, 16, and 9.

44. Divide a line a in 2 segments such that twice the sum of the squares made upon the segments equals the square made upon the entire line.

45. Some friends, dining together at a hotel, incur a total expense of \$102. Three of them, being guests, were not allowed to pay; and the others paid \$1$\frac{1}{7}$ more than if each had paid his share. How many friends were there in all?

46. If I take the product of $\frac{1}{60}$, $\frac{1}{20}$, $\frac{1}{15}$ of a certain number, the product is 16 times this number. Find the number.

47. What is the number which is greater than its square root by 182?

48. Two trains set out from two cities, M and N, distant 560 miles, to meet each other. To meet at half the distance, the train from N must start 1$\frac{1}{4}$ hours before the other. If the two trains start at the same moment, at the end of 7 hours the distance which separates them will be $\frac{1}{10}$ of the first distance. How long does it take each train to go from M to N?

49. Two bodies move upon 2 lines which form a right angle, and approach the point of intersection. One body is at a distance of 236 yards, and passes over 7 yards per second; the other is distant 197 yards, and passes over 6 yards per second. At the end of what time will they be 13 yards distant from each other?

Ex. 29.

Roots of Quadratic Equations.

If in the equation $x^2 + px + q = 0$, α and β are the roots; then

$$\alpha + \beta = -p, \quad \alpha\beta = q, \text{ and } (x - \alpha)(x - \beta) = x^2 + px + q.$$

If the equation has the form $ax^2 + bx + c = 0$,

$$\alpha + \beta = -\frac{b}{a}; \text{ and } \alpha\beta = \frac{c}{a}.$$

Form an equation of the second degree whose roots are the following:

1. 3 and 5.
2. 7 and 3.
3. 1 and 2.
4. 8 and 17.
5. 3 and -1.
6. -2 and 7.
7. 5 and -6.
8. -9 and 4.
9. $\frac{1}{2}$ and $\frac{2}{3}$.
10. $\frac{3}{4}$ and $\frac{5}{6}$.
11. 12 and $\frac{7}{8}$.
12. 4 and $-\frac{2}{5}$.
13. -7 and $\frac{3}{5}$.
14. -2 and $-\frac{1}{5}$.
15. 1 and -1.
16. 11 and -11.
17. -6 and -4.
18. -2 and -10.
19. -5 and -11.
20. -3 and $-\frac{1}{4}$.
21. 4.3 and 2.5.
22. 0.4 and 2.5.
23. -0.7 and 0.3.
24. -8.2 and 5.1.
25. -2.4 and -1.3.
26. 2 and 0.5.
27. 4 and 0.25.
28. -7 and $-\frac{1}{7}$.
29. 5 and $-\frac{1}{5}$.
30. -3 and $\frac{1}{3}$.
31. a and b.
32. $a+b$ and $a-b$.
33. a and a^2.
34. $a+3b$; $3a+b$.
35. a and $-a$.
36. a and $-b$.
37. $-m$; $-n$.
38. $3m$; $-2m$.
39. m; 2.
40. $\frac{a}{2}$; $\frac{1}{3}$

If $x^2+px+q=0$; then

$$x=\frac{-p\pm\sqrt{p^2-4q}}{2}.$$

41. In what cases are the roots real?
42. When are they imaginary?
43. Being real, when are they unequal?
44. When are they equal?
45. When are both of the roots positive?
46. When are both negative?
47. In what case is the greatest in absolute value positive, and the least negative?
48. When is the smallest positive and the largest negative?
49. When are the roots commensurable?
50. When are they irrational?

Are the roots of the following equations real or imaginary, positive or negative? Answer without solving:

51. $x^2-6x+8=0$.
52. $x^2-5x+6=0$.
53. $x^2-x-12=0$.
54. $x^2+12x+11=0$.
55. $x^2+9x+8=0$.
56. $x^2+15x+14=0$.
57. $4x^2+52x=87$.
58. $x^2+25x-26=0$.
59. $x^2+11x-12=0$.
60. $8x+20=x^2$.
61. $x^2-21x-46=0$.
62. $16x-63=x^2$.
63. $x^2-16x+64=0$.
64. $x^2=18x-81$.
65. $x^2+10x+25=0$.
66. $2x^2-18x+65=0$.
67. $x^2-31x+246\frac{1}{2}=0$.
68. $2x+5=x^2$.
69. $x^2=2-14x$.
70. $x^2+6x+4=0$.
71. $x^2+8x+25=0$.

72. If the roots of the equation $ax^2+bx+c=0$, are denoted by α and β, what is the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$?

Ex. 30.

Maxima and Minima.

If x may have any value, positive or negative, find the values of x which give to the following expressions a maximum or minimum value:

1. $16-x^2$.
2. $(3+x)(3-x)$.
3. $x(8-x)$.
4. $x(x-10)$.
5. x^2-6x+5.
6. $2x^2+x-3\frac{7}{8}$.
7. $3x^2-4x-\frac{8}{3}$.
8. $(x-1)^2-3$.
9. $(4x-7)^2-8x$.
10. $12x-(3x-1)^2$.
11. $x^3+(4-x)^3-2$.
12. $(x-4)^3-x^3+20$.
13. $\dfrac{x-6}{x^2}$.
14. $\dfrac{x-1}{x^2}$.
15. $\dfrac{(x+12)(x-3)}{x^2}$.
16. $\dfrac{3+x}{3-x}+\dfrac{3-x}{3+x}$.
17. $\dfrac{1}{2+x}+\dfrac{1}{2-x}$.
18. $\dfrac{2x}{1+x^2}$.
19. $\dfrac{x^2+2x-3}{x^2-2x+3}$.
20. $\dfrac{4x}{x^2+4x+4}$.
21. $\dfrac{m^2x^2+n^2}{(m^2-n^2)x}$.
22. $\dfrac{64x^2+36}{28x}$.
23. $\dfrac{(x+1)^2}{x^2-x+1}$.
24. $\dfrac{x^2-1}{x(x+1)}$.
25. $\dfrac{x^2-x-1}{x^2-x+1}$.

26. $\frac{25}{x}+x$.

27. $x+\frac{16}{x}$.

28. $\frac{(20+x)(5+x)}{x}$.

29. $\frac{2x^2+3x-5\frac{7}{8}}{x-3}$.

30. $\frac{2x^2-2x+5}{x^2-2x+3}$.

31. $\frac{4(x+2)}{4x^2+8x+9}$.

32. $\frac{x^2+4x+1}{x^2-4x+1}$.

33. $\frac{x^2+3x+5}{x^2+1}$.

34. $\frac{x-4}{x^2-3x-3}$.

35. $\frac{x}{a^2+x^2}$.

Find the maximum or minimum values in the following examples, if x can have positive values only:

36. $\frac{(x+1)^2}{x}$.

37. $\frac{x^2}{x-1}$.

38. $\frac{x^2-2x+1}{x-3}$.

39. $\frac{x^2-2x+2}{2(x-1)}$.

40. $\frac{x}{(x-3)(x-2)}$.

41. $\frac{x^2-4x+3}{x^2-1}$.

42. $x^3-x^2-16x+10$.

43. x^3-2x^2-x+3.

44. $2x^3-5x^2-4x+19$.

45. $\frac{x^3}{3}-\frac{3x^2}{2}+2x-21$.

Study the variation of the following functions:

46. $y=x^2-6x+5$.

47. $y=x^4-8x^2-9$.

48. $y=x+\frac{1}{x}$.

49. $y=\frac{x^2-3}{2-x}$.

50. $y=\frac{x^2-5x+1}{x^2-x+1}$.

51. $y=\frac{x+4}{7-x^2}$.

52. A given number $2a$ is to be divided into two parts whose product is maximum. Find these parts.

53. Of all the rectangles whose perimeter is $4a$, what is the one whose surface is maximum?

54. Divide a number $2a$ into two such parts that the sum of their square roots is a maximum.

55. Divide a number $2a$ into two such parts that the sum of their squares is a minimum.

56. Of all the triangles of the same perimeter, what is that one whose surface is the greatest?

57. What is the greatest of all the rectangles that can be inscribed in a circle?

58. Inscribe in a triangle a rectangle whose surface is a maximum.

59. Inscribe in a cone, the radius of whose base is r and the height h, a cylinder whose volume is a maximum.

60. Inscribe in a given square whose side is a, another square whose area is a minimum.

61. Circumscribe about a cylinder whose height is h and the radius of the base r, a cone of minimum volume.

62. Of all the cylinders that can be inscribed in a sphere, what is that one whose lateral surface is a maximum?

63. Of all the isosceles triangles that can be inscribed in a circle, what is that one whose surface is a maximum?

64. Of all the cones circumscribed about a sphere, what is that one whose volume is a maximum?

65. Of all the cones circumscribed about a sphere, what is that one whose volume is a minimum?

Ex. 31.

Simultaneous Quadratics.

1. $x^2+y^2=661$,
   $x^2-y^2=589$.

2. $y^2-z^2=-80$,
   $z^2+y^2=82$.

3. $3x^2-y^2=59$,
   $2x^2+3y^2=98$.

4. $\frac{x^2}{2}+3y^2=140$,
   $5x^2-\frac{y^2}{3}=308$.

5. $x+y=12$,
   $xy=35$.

6. $y-z=8$,
   $yz=240$.

7. $x+y=\frac{7}{6}$,
   $xy=\frac{1}{3}$.

8. $x+y=1$,
   $xy=-1$.

9. $x^2+y^2=74$,
   $x+y=12$.

10. $\frac{x+y}{x-y}=12$,
    $x^2-y^2=48$.

11. $\frac{1}{x}-\frac{1}{y}=\frac{1}{36}$,
    $xy^2-x^2y=324$.

12. $5x^2+2y^2=373$,
    $2x+5y=54$.

13. $16x^2=9y^2$,
    $2xy-5x+6y=33$.

14. $60-16xy=-x^2y^2$,
    $x+y=7$.

15. $y^2+z^2=130$,
    $y+z=14$.

16. $x^2+z^2=\frac{241}{400}$,
    $x+z=\frac{19}{20}$.

17. $x^2+y^2=250$,
    $x-y=22$.

18. $x^2-y^2=120$,
    $x+y=20$.

19. $x^2-y^2=-\frac{1}{4}$,
    $x+y=\frac{3}{2}$.

20. $x^2+xy=260$,
    $xy+y^2=140$.

21. $x^2+y^2=218$,
    $xy-y^2=42$.

22. $5x+2y=29$,
    $5xy=-105$.

23. $\frac{9}{x^2}-2y^2=-1$,
    $\frac{2}{x^2}+9y^2=\frac{83}{9}$.

24. $5x^2 - 4y^2 = 109,$
$7x - 5y = 25.$

25. $x + xy + y = 47,$
$x + y = 12.$

26. $x^2 + xy + y^2 = 217,$
$x + y = 17.$

27. $x + y = 23,$
$x + xy = 144.$

28. $xy + x = 20,$
$xy - y = 12.$

29. $xy + x = 104,$
$xy - y = 84.$

30. $2x + 3y = 20,$
$3xy - y^2 = 38.$

31. $\frac{1}{x} + \frac{1}{y} = \frac{9}{20},$
$\frac{1}{x^2} + \frac{1}{y^2} = \frac{41}{400}.$

32. $\frac{1}{x} + \frac{1}{y} = \frac{1}{2},$
$\frac{1}{x^2} + \frac{1}{y^2} = \frac{5}{36}.$

33. $\frac{1}{x} - \frac{1}{y} = \frac{7}{60},$
$\frac{1}{x^2} + \frac{1}{y^2} = \frac{169}{3600}.$

34. $x^2 + xy + y^2 = 79,$
$x^2 - xy + y^2 = 37.$

35. $xy - x^2 - y^2 = -91,$
$x^2 + y^2 + xy = 223.$

36. $42(x^2 + y^2) = 85xy,$
$15xy = 2520.$

37. $x^3 + y^3 + xy = 67,$
$x + y = 9.$

38. $x^3 + y^3 - xy = 141\frac{1}{4},$
$x + y = 4\frac{1}{2}.$

39. $x^3 + y^3 + xy = 541\frac{1}{8},$
$x - y = 6\frac{1}{2}.$

40. $x^2 + y^2 - xy = 84,$
$x - y = 2.$

41. $x + y + x^2 + y^2 = 162,$
$x - y + x^2 - y^2 = -102.$

42. $x + y + x^2 + y^2 = 1\frac{5}{8},$
$y - x + y^2 - x^2 = -1.$

43. $x^2 + y^2 + x + y = 18,$
$2xy = 12.$

44. $x^3 + y^3 = 65,$
$x + y = 5.$

45. $x^3 - y^3 = 1304,$
$x - y = 8.$

46. $x^3 + y^3 = 152,$
$x^2 - xy + y^2 = 19.$

47. $x^3 - y^3 = 279,$
$x^2 + xy + y^2 = 93.$

48. $x^3+y^3=280,$
$x^2-xy+y^2=28.$

49. $x^2-xy+y^2=7,$
$3x^2+13xy+8y^2=162.$

50. $x^3+y^3=152,$
$x^2y+xy^2=120.$

51. $x^3-y^3=335,$
$xy^2-x^2y=-70.$

52. $x^3+y^3=855,$
$xy(x+y)=840.$

53. $x^3-y^3=602,$
$xy(y-x)=-198.$

54. $x^2y^4+y^2=17,$
$xy^2+y=5.$

55. $(x^3+y^3)(x+y)=671,$
$(x^3-y^3)(x-y)=11.$

56. $(x+y)(x^3+y^3)=272,$
$(x^3-y^3)(x-y)=32.$

57. $(x+y)(x^3+y^3)=1695,$
$(x^3-y^3)(x-y)=15.$

58. $(x+y)(x^3+y^3)=175,$
$(x^3-y^3)(x-y)=7.$

59. $(x^2+y^2)(x^4-y^4)=\frac{800}{9}x^2y^2,$
$(x^2+y^2)(x+y)=\frac{40}{3}xy.$

60. $(x^3-y^3)(x+y)=200,$
$x^3-y^3+x+y=30.$

61. $\dfrac{x^3-y^3}{x^2-y^2}=7,$
$\dfrac{x^3+y^3}{x+y}=10.$

62. $xy+z=8,$
$xz+y=18,$
$yz+x=4\frac{1}{2}.$

63. $x(y+z)=90,$
$y(x+z)=84,$
$z(x+y)=48.$

64. $(2+x)(z-3)=30,$
$(5-y)z=6,$
$xy=32.$

65. $xy+3xz-5yz=37,$
$2xy-6xz-7yz=50,$
$5xy-13xz-20yz=121.$

66. $x^2+y^2+z^2=\frac{481}{36},$
$x+y+z=\frac{37}{6},$
$xy=z^2.$

67. $xy=80,$
$\dfrac{x}{y}=5.$

68. $2x - 5y = -18$,
$3xy = 264$.

69. $3x^2 - y^2 = 83$,
$x + y = 15$.

70. $u + x = 5$,
$y + z = 9$,
$u + y^2 = 28$,
$x + z^2 = 18$.

71. $x + y = 6$,
$u + z = 10$,
$x^2 + z^2 = 61$,
$u^2 + y^2 = 17$.

Ex. 32.

PROBLEMS INVOLVING SIMULTANEOUS QUADRATICS.

1. A shepherd, who is paid so much per head for his sheep, receives for the season \$320. If he had had 10 sheep less, and had been paid 50 cents more per head, he would have received \$354. How many sheep were there in his flock, and how much was he paid per head?

2. A shepherd, who is paid so much per head for his sheep, has received for the season \$320. If he had had 10 sheep more, and had been paid 50 cents less per head, he would have received \$354. How many sheep were there in his flock, and how much was he paid per head?

3. A woman carries to market cabbages for which she receives \$4.20. If she had had 12 cabbages more, and had sold each one at 2 cents more, she would have brought back from market \$2.28 more than she really received. How many cabbages did she have, and what was the selling price per dozen?

4. A gardener counts his melons, and thinks he can sell them for \$6. In gathering them he finds 4 more than he had first counted, and he sells them at 15 cents more per dozen than he had expected. He receives \$7.15. How many melons were there, and what was the selling price of each?

5. A woman spends $480 to buy black cloth, and $360 to buy blue cloth, and buys in all 90 yards of cloth. If for the same money she had bought cloth of which the price per yard was equal to the mean price of the two first, she would have received $13\frac{7}{11}$ yards less. How many yards did she buy of black cloth and of blue cloth, and what was the price of each?

6. In accomplishing a distance of 2730^{m}, a fore-wheel of a carriage makes 392 revolutions more than a hind-wheel. If the circumference of each wheel is increased by 30^{cm}, a fore-wheel in passing over the same distance will make only 325 revolutions more than a hind-wheel. What is the circumference of each wheel?

7. A fortress, besieged by the enemy, has still food enough to last 8 days. If 120 men leave, and each one remaining receives $\frac{1}{6}$ of a kilogram of bread less per day, the garrison can hold out 16 days. It will also have food for the same time if 200 men leave, and each soldier remaining receives $\frac{2}{3}$ of a kilogram of bread less than the first ration. How many men does the garrison contain, and what is a man's ration?

8. A garrison has still bread enough to last 11 days. It receives a reinforcement of 200 men; if they wish to make the provisions last as before, each man's ration of bread must be diminished $\frac{1}{4}$ of a pound. If 300 men leave, the provisions will last 12 days, giving to each man $\frac{1}{4}$ of a pound more per day. What is the force of the garrison, and what is the daily ration?

9. A detachment of workmen carry a pile of rocks to a certain distance in 165 hours. If there had been 8 workmen more, and each had carried per hour 2^{kg} less,

the rocks would have been removed in 140 hours. But if, instead, the first detachment had been diminished by 8 men, and each one had carried per hour 4^{kg} more, it would have taken 198 hours to accomplish the work. Of how many men was the detachment composed, and how much did each man carry?

10. The cities A and B are connected by a railroad bordering upon a carriage road. A carriage starts from A at the moment when a train leaves B. After having passed each other it takes the carriage 36 hours to arrive in B, and the train reaches A 9 hours after the meeting. How many hours did it take the carriage and the train to accomplish the distance AB?

11. A sets out from Constance for Berne at the moment when B sets out from Berne for Constance. When they meet A has travelled 30^{km} more than B, and continuing to walk at the same pace he arrives in Berne in 4 days, but it takes B 9 days to reach Constance. What is the distance from Berne to Constance?

12. The diagonal of a rectangle is 89^{m}. If each side of the rectangle were 3^{m} less the diagonal would be 85^{m}. What is the length of each side?

13. The diagonal of a rectangle is 65 feet. If 9 feet are added to the width of the rectangle, and 3 feet are taken away from the length, the diagonal is the same. What are the dimensions of the rectangle?

14. The hypotenuse of a right triangle is 35 feet. If the smaller side of the right triangle is 5 feet less, and the greater side 2 feet more, the hypotenuse will be just 1 foot less. What are the sides of the right triangle?

15. Two square gardens have a total surface of 2137 square yards. A rectangular piece of land whose dimensions are respectively equal to the sides of the two squares, will have 1093 square yards less than the two gardens united. What are the sides of the two squares?

16. The height of a trapezoid is 18 feet. Its surface is equal to that of a rectangle constructed upon the two parallel bases. Three times the smaller base added to the larger base is four times the height of the trapezoid. What are the two bases?

17. A man buys a watch, a cane, an umbrella, and a ring. The two first cost together $24, and the two last $21. If the price of the watch is multiplied by that of the cane, the product will be the same as the product of the cost of the other two articles. The sum of the squares of the prices paid is $585. How much did each article cost?

18. Four travellers, A, B, C, D, meet and reckon up their expenses. A and B together have spent $102, C and D, $96. The product of the sums spent by C and D is $2108. The sum of the squares of the amounts spent by each is $10,300. What is the amount spent by each traveller?

19. If from a number of four figures the number obtained by writing the figures in reverse order is subtracted, the remainder is 4725. The product of the figures is 672; the product of the two middle figures is 28, and the thousands' figure is 5 greater than that of the units'. What is the number?

20. The product of the sum and difference of two numbers is a; the quotient of the sum divided by the difference is b. What are the two numbers?

Ex. 33.

Indeterminate Equations.

1. $x+y=12.$
2. $11x+2y=83.$
3. $6x+31y=147.$
4. $121x+44y=286.$
5. $69x+184y=1173.$
6. $8x+13y=413.$
7. $635x+381y=54{,}737.$
8. $2504x+2817y=157{,}752.$
9. $56x+91y=945.$
10. $4x+9y=53.$
11. $\frac{3}{4}x+5y=92.$
12. $\frac{3x}{4}+\frac{y}{4}=27.$
13. $\frac{5x}{7}+\frac{3y}{7}=15.$
14. $\frac{4x}{5}+\frac{2y}{5}=18.$
15. $\frac{x+15y}{x-21}=-20.$
16. $\frac{13y-62x}{3x-12}=-26.$
17. $\frac{12x^2+3y-70}{5-3x}=-4x.$
18. $\frac{10x^2-18xy-19y^2+96y}{2x-4y}=5x+3y.$
19. $15x-47y=1.$
20. $\frac{14x-8y+1}{5}=\frac{3-2x-5y}{6}.$
21. Find two numbers which, multiplied respectively by 7 and 17, have for their products a sum equal to 1135.
22. If two numbers are multiplied respectively by 8 and 17, the difference of their products is 10. What are the numbers?
23. If two numbers are multiplied respectively by 7 and 15, the first product is 12 greater than the second. Find the numbers.

24. Find two numbers which are respectively multiples of 4 and 5, and the first of which is 6 smaller than the other.

25. Divide 89 in two parts, one of which is divisible by 3, and the other by 8.

26. Divide 314 in two parts, one of which is a multiple of 11, and the other a multiple of 13.

27. Divide 118 in two parts, the first of which is divisible by 7, and the second by 11.

28. What is the smallest number which, divided by 5 and by 7, gives each time 4 for a remainder?

29. A number divided by 7 has 1 for a remainder, and divided by 8, it has 7 for a remainder. Find the number.

30. The difference of two numbers is 151. The first divided by 8 has 5 for a remainder, and 4 must be added to the second to make it divisible by 11. What are the numbers?

31. Divide 352 in two such parts, that the first diminished by 2 is divisible by 7; and the second, increased by 5, is a multiple of 9.

32. Find pairs of fractions whose denominators are 24 and 16, and whose sum is $\frac{19}{24}$.

33. Find pairs of fractions whose denominators are 11 and 13, and whose difference is $\frac{82}{143}$.

34. How can one pay a sum of $87, giving only bills of $5 and $2?

35. A man buys calves at $5 apiece, and pigs for $3 apiece. He spends in all $114. How many did he buy of each?

36. A cook spends $72 to buy lambs at $5.80, and geese at $1.40 each. How many did he buy of each?

37. The diameter of a 5-franc piece of silver is 37^{mm}; that of a piece of 1 franc is 23^{mm}. How can one obtain the length of a meter in pieces of 5 francs and 1 franc?

38. A sovereign is worth 25.21 francs. How can one pay a debt of 317 francs with sovereigns to a person who has only 20-centime pieces to return?

39. How can a person weigh 1^{kg} of goods with weights of 30^{g} and $2\frac{1}{2}^{g}$?

40. A society made an excursion which cost for each grown person $7.15, and for each child $3.10. The total expense was $97.25. How many grown people and how many children were there?

41. A basket contains more than 300 and less than 400 pears. If they are divided among some children, giving 13 to each, 9 are left; but if 15 are given to each, 4 are left. How many pears does the basket contain?

42. A person buys silk at $2.50 a yard, and velvet at $3.50; in all he spends $106. How many yards did he buy of each?

43. Two brothers have worked, the first 18 days, and the second 5 days less; they receive together $142. How much did each earn per day?

44. A woman sells peaches at $0.15 apiece, and apples at $0.59 a dozen, and receives in all $10.87. How many peaches and apples did she sell?

45. A man asked his friend his age on his birthday, Feb. 5, 1880. The friend replied, "I was born in this century, and if you write the year of my birth and then add the four figures, the sum is just the age I am to-day." What is his age?

46. The age of my grandfather in 1808 was equal to the sum of the four figures of the year of his birth. How old was he then, and in what year was he born?

47. A servant receives \$37 to buy potatoes at \$3 a barrel, apples at \$5, and squashes at \$0.80. How many barrels can he buy of each kind?

48. What numbers divided by 2, 3, 7, have for their remainders, respectively, 1, 2, 5?

49. What numbers, divided by 7, 8, 9, have for their remainders, respectively, 6, 7, 8?

50. A farmer sells 15 calves, 14 lambs, and 13 pigs, and receives \$200. Some days after, at the same price, he sells 7 calves, 11 lambs, and 16 pigs, for which he receives \$141. What is the price of each?

51. Divide 111 in three parts, of which the first is divisible by 2, the second by 5, and the third by 7, and three times the first part, two times the second, and four times the third is equal to 400.

Ex. 34.

Inequalities.

Add the following inequalities, member to member:

1. $5 > 3$; $8 > 2$.
2. $a + 1 < x^2$; $a - 5 < 2x$.
3. $x^3 + y > 3 - a$; $y^2 - 2a > b + 4$.
4. $3x + y < 2a - 1$; $3y - 2x < 14 - 2a$.

Subtract the second inequality from the first:

5. $15 > 12$; $3 < 7$.

6. $6\frac{1}{2} < 10$; $2\frac{1}{2} > 1$.

7. $a^2 < 3 - x$; $2x > b^2$.

8. $a(a - x^2) > b^2 - 1$; $a^2 - 2ax^2 < 7$.

Find the products of:

9. $3 > -1$ by 7.

10. $3 > 0.5$ by -2.

11. $4a > -x$ by -2.

12. $(a-b)^2 > (x+y)^2$ by -1.

13. $18 - y^2 < 5$ by a^2.

Find the quotients of:

14. $9 > 6$ by 3.

15. $147 > 91$ by 7.

16. $a^2 - b^2 > a - b$ by $a - b$.

17. $13x^2 + 26b > 91x^2$ by -13.

Perform the operations indicated:

18. $(a - 1 < 7)(a + 1 < 10)$.

19. $(a + b > m)(n < a - b)$.

20. $(a^2 + ab + b^2 > x - y)(x + y < a - b)$.

21. $(8 > 2) \div (1\frac{1}{2} < 7)$.

22. $(x^2 - y^2 > a^2) \div (2a > x + y)$.

23. $(30 > 12)^2$.

24. $(a + b > a - x)^2$.

25. $(x + 1 < y)^4$.

26. $(a - 1 < b - 2)^5$.

27. $(-1 > -2)^5$.

28. $(3 - e > -1)^7$.

29. $[-(1 + a) < -b]^4$.

30. $[-a(1 + m) > x]^3$.

31. $\sqrt[3]{27 > 8}$.

32. $\sqrt[3]{-125 < +64}$.

33. $\sqrt[3]{729 > 343}$.

34. $\sqrt{36 > 25}$.

In exercises like the 23d, each member must be raised to the power indicated, and the proper sign placed between the results. In exercises like the 31st, the root indicated must be extracted from each member, and the two results connected by the proper sign.

Simplify:

35. $4+3ab>2x-(a-b)^2$.

36. $(x-1)^3<x^3-x^2+4x$.

37. $x^3-y^3<(x-y)(x^2+y^2)$.

Solve:

38. $x+4>2-3x$.

39. $(x+1)^2<x^2+3x-5$.

40. $4x-3>\frac{3x}{2}-\frac{3}{5}$.

41. $3x-2<\frac{x}{2}+7\frac{1}{2}$.

42. $2\sqrt{x^2}>\sqrt{4(x-1)^2+12x^2-7}$.

43. $\sqrt{m^2+5mx+4x^2}<2x+3m$.

44. $\sqrt{2}-3\sqrt{x}<\sqrt{4+9x-12}$.

45. $\frac{4x-2}{3}>\frac{3-5x}{7}$.

46. $\sqrt{\frac{2x^2-3}{2}}>x+5$.

47. $2x-3y>4$,
$4x+y>12$.

48. $3x-1>x+3y$,
$x(1-3x)>4x-3x^2-2y$.

49. $2x(2x-3)>(2x+1)^2-3y$,
$4x-y>x+4$.

50. $\sqrt{x^2-3x+y}>x+2$,
$x-y>3$.

51. $\sqrt{(2x-1)(y+x)}>\sqrt{2(xy+x^2)+4}$,
$5x+2y>8$.

52. Prove that $a^2+b^2>2ab$.

53. Prove that a fraction increased by its reciprocal value is greater than 2.

54. Let $\frac{a}{m}, \frac{b}{n}, \frac{c}{p}, \ldots\ldots, \frac{h}{u}$, be fractions whose denominators have the same sign, and arranged in the order of their magnitude, so that $\frac{a}{m}$ may be the smallest. Prove that

$$\frac{a+b+c+\cdots\cdots+h}{m+n+p+\cdots\cdots+u} > \frac{a}{m} \text{ and } < \frac{h}{u}.$$

An inequality of the second degree can always be reduced to the form $ax^2+bx+c>0$, or $ax^2+bx+c<0$.

To *solve* it is to find between what limits the value of x must lie to satisfy the inequality.

Solve:

55. $x^2+x>7x-10$.
56. $x^2+19>1+8x$.
57. $29-11x>-x(1+x)$.
58. $x^2+x-6>0$.
59. $6x^2+7x+2<0$.
60. $x^2+2x-15<0$.
61. $x^2-4x+7<0$.
62. $x^2-10x+32<0$.
63. $x^2+16x-80<0$.
64. $4x+21-x^2>0$.
65. $4x+12-x^2>0$.
66. $15-x-2x^2>0$.
67. $17x-6x^2-5<0$.
68. $38x-7-15x^2<0$.
69. $9x<20x^2+1$.

Ex. 35.

Exponents, Multiplication.

$a^m \times a^n = a^{m+n}$.

$(-a)^n = \begin{cases} -a^n \text{ when } n \text{ is odd.} \\ +a^n \text{ when } n \text{ is even.} \end{cases}$

$a^m \times b^m = (ab)^m$.

If p is even:

$(a-b)^p = (b-a)^p, \quad (a-b)^4 = (b-a)^4$.

$(a-b)^{p+1} = -(b-a)^{p+1}, (a-b)^3 = -(b-a)^3$.

$(a^m)^n = a^{mn} = a^{nm} = (a^n)^m$.

Simplify :

1. $-(+8)^3+(-2)^6$.
2. $(-7)^3-(-3)^7$.
3. $b^{m-3}\times b^3$.
4. $d^{a-5}\times d^6$.
5. $a^{3x-y}a^{2x-y}$.
6. $p^{3a-b}p^{7b-3a}$.
7. $(a+b)^n\times(a+b)^m$.
8. $(x-y)^{m-1}(x-y)$.
9. $(m^x-n^y)^3$.
10. $(p^{m-1}+q^{n-2})^2$.
11. $(a^m+b^n)^3$.
12. $(m^a+n^b+p^c)^2$.
13. $(-y)^{2n+1}\times(-y)^5$.
14. $(-d)^{2n-1}\times(-d)^5$.
15. $(-c)^{2n+1}\times(-c)^{2n-1}$.
16. $(-a)^{2m}\times(-a)^7$.
17. $(-m)^7\times(-m)^{2p}$.
18. $(-x)^{2m+1}\times(-x)^5$.
19. $(-xy)^{2n-1}\times(-xy)^3$.
20. $\left(-\frac{x}{y}\right)^7\times\left(-\frac{x}{y}\right)^8$.
21. $\left(-\frac{a}{b}\right)^3\times\left(\frac{a}{b}\right)^7$.
22. $\left(-\frac{m}{n}\right)^9\times\left(-\frac{n}{m}\right)^{10}$.
23. $\left(-\frac{b}{x}\right)^{12}\times\left(-\frac{x}{c}\right)^9$.
24. $\left(-\frac{x}{y}\right)^{2m}\times\left(-\frac{y}{x}\right)^{2m-1}$.
25. $\left(-\frac{m}{n}\right)^{2m-1}\times\left(-\frac{n}{m}\right)^{2m+1}$.
26. $\left(-\frac{ab}{xy}\right)^8\times\left(-\frac{ax}{by}\right)^{11}$.
27. $\left(\frac{7-c}{m-5}\right)^4\times\left(\frac{5-m}{7-c}\right)^6$.
28. $\left(\frac{m-p}{x-y}\right)^8\times\left(\frac{y-x}{p-m}\right)^{10}$.
29. $\left(\frac{8-a}{x-y}\right)^4\times\left(\frac{x-y}{a-8}\right)^3$.
30. $\left(\frac{x-y}{a-b}\right)^{2m}\times\left(\frac{b-a}{x-y}\right)^{2m+1}$.
31. $\left(\frac{a-b}{x-y}\right)^{2m}\times\left(\frac{y-x}{b-a}\right)^{2m-1}$.
32. $\left(\frac{a^2-b^2}{x^4-y^4}\right)^5\times\left(\frac{x^4-y^4}{b^2-a^2}\right)^7$.
33. $\left(\frac{a^2-b^2}{x^3-y^3}\right)^3\times\left(\frac{x^6-y^6}{b^2-a^2}\right)^4$.
34. $\left(\frac{m^8-n^8}{x^5-y^5}\right)^{2m}\times\left(\frac{y^{10}-x^{10}}{n^4-m^4}\right)^m\times\left(\frac{x^5-y^5}{m^4+n^4}\right)^m$.

35. $(a^4)^3$.

36. $[(b^3)^2]^5$.

37. $(x^{m+1})^2$.

38. $(-a^3)^2$.

39. $(-a^2)^3$.

40. $(-m^3)^7$.

41. $(-y^3)^{2m}$.

42. $(-y^{2m})^3$.

43. $(-y^{3m})^2$.

44. $(-b^2)^{2m-1}$.

45. $(-b^{2n})^{2n-1}$.

46. $(m^n p)^5$.

47. $(5a^{n-1}b^2)^2$.

48. $(3a^3b^{n-2})^3$.

49. $(7a^2x^{n-2}y^{m+1})^3$.

50. $(8a^m b^{n-1}c^{n-2})^3$.

51. $\left(\frac{a^2b^3}{c^4d^5}\right)^2$.

52. $\frac{(a^2x^3)^4}{(ax)^5}$.

53. $\frac{(ab)^6}{(a^2b^3)^3}$.

54. $\frac{(a^6b^3)^2}{(a^5b^2)^4}$.

55. $\left(\frac{a^9b^{28}c^{47}}{d^{10}e^{29}}\right)^{17} \times \left(\frac{d^9e^{36}}{a^8b^{25}c^{42}}\right)^{19}$.

56. $\frac{[(8x-6y)^{2a}]^{5a}}{[(4x-3y)^{5a}]^{2a}}$.

57. $\left[\left(\frac{m^5n^3}{p^2q^2}\right)^3 \times \left(\frac{mq^3}{n}\right)^4\right] \div \left[\left(\frac{m^3n}{p^2q^3}\right)^5 \div \left(\frac{m^2q^3}{n^2p}\right)^6\right]$.

58. $\left[\left(\frac{a^4b^3}{c^2x^3}\right)^5 \div \left(\frac{ax^3}{bc^4}\right)^4\right] \div \left[\left(\frac{ab^3}{c^2x^3}\right)^6 \div \left(\frac{a^2x^3}{b^2c^3}\right)^5\right]$.

Ex. 36.

Exponents, Division.

$$a^m \div a^n = \frac{a^m}{a^n} = \begin{cases} a^{m-n}, \text{ when } m > n. \\ a^0 = 1, \text{ when } m = n. \\ a^{-(n-m)} = \frac{1}{a^{n-m}}, \text{ when } m < n. \end{cases}$$

$a^5 \div a^3 = a^2$. $\quad a^5 \div a^5 = a^0 = 1$.

$a^5 \div a^9 = a^{-4} = \frac{1}{a^4}$.

$a^m \div b^m = (a \div b)^m = \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$.

$1 \div b^n = (1 \div b)^n = \frac{1}{b^n} = \left(\frac{1}{b}\right)^n$.

$a^m \div a^m = a^0 = 1$.

$a^{3m-2n} = \frac{a^{3m}}{a^{2n}}$.

Simplify:

1. $c^x \div c$.
2. $m^{x+3} \div m^3$.
3. $m^{y+5} \div m^6$.
4. $m^{x+y} \div m^y$.
5. $x^a \div x^{2b}$.
6. $x^{a+b} \div x^{a-b}$.
7. $x^{a-b} \div x^{b-a}$.
8. $a^{3x+2y} \div a^{2x-3y}$.
9. $(x+y)^{a-b} \div (x+y)^{b-a}$.
10. $\dfrac{a^{p+2q}\,b^{7p-8q}}{a^{4p-7q}\,b^{3p-11q}} \div \dfrac{a^{2(p-3q)}\,b^{5p+6q}}{a^{6p-17q}\,b^{2(p+2q)}}$.
11. $(x^{4p}-1) \div (x^p-1)$.
12. $(9x^p + 3x^{4p} + 14x^{3p} + 2) \div (1 + 5x^p + x^{2p})$.
13. $(a^n - b^n) \div (a-b)$.
14. $(a^{2n} - b^{2n}) \div (a+b)$.
15. $\dfrac{(a^{n+x} - a^n)(a^n - a^{n-x})}{(a^{n+x} - a^n) - (a^n - a^{n-x})}$.
16. $\left(\dfrac{5a^3}{3c}\right)^n \div \left(\dfrac{15a^2}{6c^3}\right)^n$.
17. $\dfrac{(20a^4 + 8a^2b^2 - 12b^4)^x}{[4(a^2+b^2)]^x}$.
18. $\dfrac{(64m^2 - 49n^2)^b}{(8m-7n)^b}$.
19. $\dfrac{1}{0.25^3}$.
20. $\dfrac{1}{(0.5)^6}$.
21. $\dfrac{1}{(\frac{1}{3})^5}$.

Reduce to the simplest form:

22. $\left(\dfrac{3mn}{5pq}\right)^4 \times \left(\dfrac{5p}{6m}\right)^3 \times \left(\dfrac{4n}{3q}\right)^2$.
23. $\left(\dfrac{m+n}{p-q}\right)^4 \times \left(\dfrac{3}{m+n}\right)^2 \times \left(\dfrac{p-q}{3(m+n)}\right)^5$.

Write the following expressions without using the exponent zero or negative exponents, and simplify the results:

24. x^0.
25. a^{-3}.
26. $\dfrac{1}{x^0}$.
27. $\dfrac{1}{a^{-4}}$.
28. x^{-0}.
29. $\dfrac{1}{x^{-0}}$.
30. $a^3 \times a^0$.
31. $a^3 \times a^{-0}$.
32. $\dfrac{a^5}{q^{-0}}$.

33. $m^{-3} \times x^{-0}$.

34. $m^{-3} \times x^{0}$.

35. $\dfrac{m^{-3}}{x^{-0}}$.

36. $b^{m} \times b^{0}$.

37. $b^{m} d^{-0}$.

38. $b^{m} \times d^{-5}$.

39. $x^{0} \times y^{0}$.

40. $x^{-0} \times y^{-c}$.

41. $(x+y)^{0}$.

42. $x^{0} + y^{0}$.

43. $x^{0} + x^{0}$.

44. $x^{0} x^{0}$.

45. $0^{x} \times 0^{x}$.

46. $x^{-0} + y^{-0}$.

47. $(x+y)^{-0}$.

48. 0^{-x}.

49. 0^{-4}.

50. $(-a)^{-2}$.

51. $(-x)^{-4}$.

52. $(-x)^{-3}$.

53. $(-x^{3})^{-5}$.

54. $\dfrac{a^{2}}{a^{-3}}$.

55. $\dfrac{1}{m^{-2}}$.

56. $(-x^{2})^{0}$.

57. $\dfrac{1}{m^{0}}$.

58. $\dfrac{1}{m^{-0}}$.

59. $\dfrac{a^{0}}{b^{n}}$.

60. $\dfrac{a^{0}}{b^{-n}}$.

61. $\dfrac{x^{-n}}{y^{0}}$.

62. $\dfrac{x^{-n}}{y^{-0}}$.

63. $\dfrac{m^{-5}}{n^{-3}}$.

64. $\dfrac{a^{-x}}{b^{-x}}$.

65. $\dfrac{a^{-m}}{b^{-n}}$.

66. $\dfrac{m^{-6}}{m^{-5}}$.

67. $\dfrac{a^{n-4}}{a^{-5}}$.

68. $\dfrac{p^{-3}}{p^{m-4}}$.

69. $\dfrac{a^{m-2}}{b^{n-3}}$.

70. $\left(\dfrac{1}{n}\right)^{-4}$.

71. $\left(\dfrac{x}{y}\right)^{-1}$.

72. $\left(\dfrac{x}{y}\right)^{0}$.

73. $\left(\dfrac{y}{z}\right)^{-0}$.

74. $\dfrac{1}{m}\left(\dfrac{a}{m}\right)^{-1}$.

75. $p^{2}\left(\dfrac{p}{q}\right)^{-2}$.

76. $m(1-m)^{-1}$.

77. $n(1-n)^{-0}$.

78. $(1+x)(1-x^{2})^{-1}$.

79. $(1+y)^{2}(1-y^{2})^{-2}$.

80. $(m-n)^{3}(m-n)^{-4}$.

81. $(a-1)^{-6} \times (1-a)^{3}$.

82. $\dfrac{3a^{0}b^{-2}c^{-4}}{5a^{-1}b^{-3}c^{-5}}$.

83. $\dfrac{4(a^{0}+b^{0})^{-1} \times x^{-5}}{5x^{-3}y^{-4}}$.

84. $\dfrac{a^0b^{-2}}{9(x^0+y^0+z^0)^{-2}c^3}$.

85. $\dfrac{a^2b^{-4}}{x^{-3}y^{-5}} \times \dfrac{a^{-2}b^4}{x^5y^3}$.

86. $\dfrac{8a^{-5}}{a^2} \times \dfrac{(a^0+y^0)^{-3}}{a^{-10}}$.

87. $\dfrac{m^{2x+4}m^{-(x+5)}}{n^{x-3}}$.

88. $(a^{2x-3})^{-2}$.

89. $(m^{3x-5}-n^{5x-3})^{-x}$.

90. $\left(\dfrac{a^{-2x}+b^{-2y}}{a^{-6x}-b^{-4y}}\right)^{-3}$.

91. $(x^{3-m}+y^{4-n})^{-2}$.

92. $\left(\dfrac{a^{-1}+b^{-1}}{a^{-2}-b^{-2}}\right)^{-2} \div \left(\dfrac{a^{-2}+b^{-2}}{a^{-4}-b^{-4}}\right)^{-3}$.

93. $\left(\dfrac{m^{-2}+n^{-2}}{m^{-4}-n^{-4}}\right)^{-3} \div \left(\dfrac{m^{-1}+n^{-1}}{n^{-2}-m^{-2}}\right)^{-2}$.

Free the following expressions from the form of fractions by using negative exponents:

94. $\dfrac{3}{m}$.

95. $\dfrac{x}{(1-y)^3}$.

96. $\dfrac{x}{(-z^3)}$.

97. $\dfrac{b}{(-p)^4}$.

98. $\dfrac{b}{(-p^4)}$.

99. $a^5 \times \dfrac{1}{a^3}$.

100. $\left(\dfrac{a^m}{xy^2}\right)^3$.

101. $\dfrac{b^n}{by^5}$.

102. $\dfrac{c^4}{c^m z^p}$.

103. $\dfrac{x^m}{x^5}+\dfrac{y^n}{y^3}$.

104. $\dfrac{1}{\left(\dfrac{1}{x^2}+\dfrac{1}{y}\right)^2} \times \dfrac{1}{\left(\dfrac{1}{x^3}-\dfrac{1}{y^4}\right)^3}$.

105. $\dfrac{1}{(x+y)^2} \div \dfrac{1}{x^2+y^2}$.

106. $\dfrac{1}{m^2} \times \dfrac{1}{n^2} + \dfrac{3}{m^4n^4}$.

Calculate the numerical values of the following expressions:

107. $\dfrac{1}{2^0}$.

108. $\dfrac{1}{2^{-1}}$.

109. $\dfrac{2}{3^{-2}}$.

110. $\dfrac{5}{(-4)^3}$.

111. $\dfrac{3}{(-5)^{-3}}$.

112. $\dfrac{2^{-4}}{4^{-2}}$.

113. 2^{-1}.

114. 1^{-1}.

115. 5^{-2}.

116. 100×5^{-3}.

117. $7^0(x+y)^0$.

118. 1^{-n}.

119. 25×5^{-2}.

120. 64×4^{-3}.

121. $(0.5)^{-2} \times 1000$.

122. $(0.3)^{-4} \times 810$.

123. $(-0.8)^{-3} \times 256$.

124. $\dfrac{10}{2^{-3}}$.

125. $\dfrac{5}{3^{-4}}$.

126. $\dfrac{2^c}{(-1)^{-3}}$.

127. $\dfrac{2^0}{(-1)^{-4}}$.

128. $(\frac{1}{5})^{-1}$.

129. $(\frac{1}{4})^{-2}$.

130. $2^{-1} - 2^{-2}$.

131. $9^{-2} - 3^{-4}$.

132. $\dfrac{1}{15^{-3}} - \dfrac{1}{4^{-4}}$.

133. $\dfrac{1}{5^{-4}} - \dfrac{1}{10^{-2}}$.

134. $\dfrac{\dfrac{1}{4^{-3}} - \dfrac{2}{10^{-2}}}{\dfrac{1}{2^{-2}} + \dfrac{1}{4^{-1}}}$.

135. $\left(\dfrac{4^{-5} - \dfrac{1}{3^{-9}}}{\dfrac{1}{5^{-4}} + 8^{-2}}\right)^0$.

136. $64 \times 2^{-4} + 32 \times 2^{-3} + 16 \times 2^{-2} + 8 \times 2^{-1} + 4 \times 2^0$.

Ex. 37.

Negative Exponents, Multiplication.

$a^m \times a^{-n} = a^{m-n}$. $\quad a^{-m} \times a^{-n} = a^{-(m+n)}$.

$(-a)^{-n} = \dfrac{1}{(-a)^n} = \pm\dfrac{1}{a^n}$.

$a^{-m} \times b^{-m} = (ab)^{-m}$. $\quad (a-b)^{-2m} = (b-a)^{-2m}$.

$(a-b)^{-(2m+1)} = -(b-a)^{-(2m+1)}$.

Perform the following operations:

1. $a^4 \times a^{-3}$.
2. $m^7 \times m^{-8}$.
3. $x^{10} \times x^{-6}$.
4. $z^{-6} \times z^4$.
5. $a^5 a^{-6}$.
6. $h^m h^{-n}$.
7. $ab^{-5} \times a^{-1}b^4$.
8. $m^{-2}n^3 \times m^4 n^{-5}$.

9. $a^0b^{-m} \times a^3b^m$.

10. $a^m \times a^{-n} \times a^{-2m}$.

11. $a^{-4} \times a^{-3}$.

12. $b^{-2} \times b^{-5}$.

13. $x^{-m} \times x^{-p}$.

14. $y^{-a} \times y^{-c}$.

15. $x^{-2m} \times x^{-3m}$.

16. $y^{-5x} \times y^{-x}$.

17. $a^{-1} \times a^{-2b}$.

18. $a^{-2x}a^{-3y}$.

19. $(-a)^{-2}$.

20. $(-b)^{-3}$.

21. $(-x)^{-7}$.

22. $(-y)^{-10}$.

23. $(-m)^{-2} \times (-m)^{-3}$.

24. $(-x)^{-1} \times (-x)^{-3}$.

25. $x^{-5}(-x)^{-4}$.

26. $x^{-4}(-x)^{-5}$.

27. $(-y)^{-m} \times (-y)^{-m}$.

28. $(a^{-4} + a^{-3} + a^{-2} + a^{-1}) \times a^{-3}$.

29. $(y^{-5} + y^{-4} + y^{-3} + y^{-2} + y^{-1} + 1) \times (-y)^{-5}$.

30. $\dfrac{a^{-1}}{b^{-2}} \times \dfrac{a^{-2}}{b^{-1}}$.

31. $\left(\dfrac{1}{x}\right)^{-1} \times \left(\dfrac{y}{x}\right)^{-2}$.

32. $\dfrac{a^{-1}b^{-2}}{m^{-3}n^{-4}} \times \dfrac{a^{-2}b^{-3}}{m^{-2}n^{-1}} \times \dfrac{m^{-6}n^{-7}}{a^{-3}b^{-8}}$.

33. $(x^{-1} - 2x^{-2} + 3x^{-3}) \times (x^0 + x^{-1} - x^{-2})$.

34. $4^{-2} \times 3^{-2}$.

35. $5^{-1} \times 6^{-1}$.

36. $8^{-2} \times 5^{-2}$.

37. $1^{-3} \times 2^{-3} \times 3^{-3}$.

38. $5^{-4} \times 2^{-4}$.

39. $(4-1)^{-2}$.

40. $(1-4)^{-2}$.

41. $(5-2)^{-3}$.

42. $-(2-5)^{-3}$.

43. $(\frac{2}{3})^{-2} \times (\frac{3}{4})^{-2}$.

44. $0.2^{-4} \times 0.5^{-4}$.

45. $(\frac{1}{2}a)^{-1} \times (2c)^{-1}$.

46. $(\frac{3}{5}x)^{-4} \times (\frac{5}{9}y)^{-4}$.

47. $(a-b)^{-4}(b-a)^{-4}$.

48. $(x^2 - y^2)^{-3}(y^2 - x^2)^{-3}$.

49. $(3m^5 - 2n)^{-2}(2n - 3m^5)^{-2}$.

50. $(1-x^2)^{-7}(x^2-1)^{-7}$.

51. $\left(\frac{a^2}{b}+\frac{m^{-1}}{n^{-2}}\right)^{-3}\left(\frac{a^2}{b}-\frac{m^{-1}}{n^{-2}}\right)^{-3}$.

52. $\left(\frac{x}{y^3}-\frac{z^{-1}}{y^{-4}}\right)^{-7}\left(\frac{x}{y^3}+\frac{z^{-1}}{y^{-4}}\right)^{-7}$.

53. $\left(\frac{3x}{a}+\frac{2m^{-1}}{b^{-2}}\right)^{-2}\left(\frac{2m^{-1}}{b^{-2}}-\frac{3x}{a}\right)^{-2}$.

54. $\left(\frac{x^{-3}}{y^{-4}}+\frac{a^{-1}}{z^2}\right)^{-3}\left(\frac{a^{-1}}{z^2}-\frac{x^{-3}}{y^{-4}}\right)^{-3}$.

55. $\left(\frac{2}{y^{-3}}+\frac{1}{z^{-1}}\right)^{-6}\left(\frac{1}{z^{-1}}-\frac{2}{y^{-3}}\right)^{-6}$.

Ex. 38.

Negative Exponents, Powers, Division.

$(a^{-m})^n = a^{-mn} = (a^{-n})^m$.

$(a^m)^{-n} = a^{-mn} = (a^{-n})^m = (a^n)^{-m}$.

$(a^{-m})^{-n} = a^{mn}$.

$a^{-m} \div a^{-n} = a^{-m+n}$.

$a^{-m} \div a^n = a^{-(m+n)}$.

$a^m \div a^{-n} = a^{m+n}$.

$a^{-m} \div b^{-m} = \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$.

$a^{-5} \div a^{-3} = a^{-2} = \frac{1}{a^2}$.

$a^{-5} \div a^{-5} = a^0 = 1$.

$a^{-5} \div a^{-8} = a^3$.

$1 \div b^{-m} = \left(\frac{1}{b}\right)^{-m} = b^m$.

Find the values of:

1. $(2^{-3})^2$.
2. $(2^{-2})^3$.
3. $(5^{-3})^{-4}$.
4. $(10^{-2})^4$.
5. $(3^{-2})^{-3}$.
6. $(5^{-3})^{-1}$.
7. $[(2^{-3})^{-2}]^2$.
8. $(100^{-1})^{-3}$.
9. $(\frac{2}{3})^{-3}$.
10. $(\frac{1}{5})^{-4}$.
11. $2^{-3} \times (\frac{3}{4})^{-2}$.
12. $(\frac{3}{5})^{-3} \div (\frac{9}{10})^{-2}$.
13. $[(2a)^{-2}]^{-3}$.
14. $[(4a)^{-2}]^2$.
15. $(x^{-3})^{-2}$.
16. $(y^2)^{-2}$.
17. $(-1^{-1})^{-2}$.
18. $(-a)^{-2}$.
19. $[(-b)^2]^{-2}$.
20. $(-m)^{-3}$.
21. $(a^2b^{-1})^{-2}$.

22. $(m^{-1}n^{-3})^{-1}$.
23. $(a^{-2}b^{-5})^{2}$.
24. $(-a^{-1}b^{-2})^{-3}$.
25. $(-x^{-2}y^{-4})^{-5}$.
26. $(-x^{-1}y^{-2})^{-2}$.
27. $(a^{-m})^{-n}$.
28. $(bn)^{-3}$.
29. $(d^{-5})^{-p}$.
30. $(c^{-x})^{2}$.
31. $[(-m)^{-2x}]^{y}$.
32. $[(-a)^{2y}]^{-1}$.
33. $(a^{-m}b^{n})^{-2}$.
34. $(b^{-1}c^{-y})^{-3}$.
35. $(-x^{-a}y^{-m})^{2n}$.
36. $(-3a^{-2})^{-2x}$.
37. $[(-a^{-1})^{-2}]^{-3}$.
38. $[(-a^{-2})^{-2n}]^{-(2n+1)}$.
39. $[(a^{-1})^{-3}]^{-2n}$.
41. $\left(\frac{a^{-1}b^{2}}{c^{-m}d^{-n}}\right)^{-2}$.
43. $\left(\frac{x^{-1}y}{az^{0}}\right)^{-1}$.
40. $[(-m^{-p})^{-2q}]^{-x}$.
42. $\left(\frac{a^{0}bc^{-5}}{x^{-2}y^{4}}\right)^{-m}$.
44. $\left(\frac{mn^{0}}{x^{-p}y^{-q}}\right)^{-1}$.
45. $\left(\frac{m+n}{m-n}\right)^{-3}\times\left(\frac{m-n}{m+n}\right)^{-2}$.
46. $\left(\frac{m+n}{m-n}\right)^{-2}\times\left(\frac{p+q}{m-n}\right)^{2}$.
47. $\left(\frac{x-y}{a-b}\right)^{2}\times\left(\frac{x-y}{a+b}\right)^{-2}$.
48. $\left(\frac{x-y}{p+q}\right)^{-4}\times\left(\frac{x+y}{p+q}\right)^{4}\times\left(\frac{x+y}{x-y}\right)^{-5}$.
49. $\left(\frac{m+n}{m-n}\right)^{-4}\times(m-n)^{-5}\times\left(\frac{1}{m+n}\right)^{-5}$.
50. $\left(\frac{1}{1+x}\right)^{5}\times\left(\frac{1}{1-x}\right)^{-7}\times\left(\frac{x-1}{x+1}\right)^{-6}$.
51. $\left[\left(\frac{a-b}{x+y}\right)^{-3}\right]^{2}\times\left[\left(\frac{x+y}{a-b}\right)^{-2}\right]^{3}$.
52. $\left[\left(\frac{a-b}{x+y}\right)^{-2}\right]^{-5}\times\left[\left(\frac{x+y}{a-b}\right)^{-5}\right]^{-3}$.
53. $\left[\left(\frac{a+b}{c-d}\right)^{-2}\right]^{7}\times\left[\left(\frac{c-d}{a+b}\right)^{-6}\right]^{2}$.

54. $\left[\left(\frac{a+b}{x-y}\right)^{-4}\right]^{-5} \times \left[\left(\frac{a+b}{y-x}\right)^{-7}\right]^{3}$.

55. $\left[\left(\frac{c-d}{m-n}\right)^{-3}\right]^{-5} \times \left[\left(\frac{d-c}{n-m}\right)^{-7}\right]^{2}$.

56. $2^{-4} \div 2^{-5}$.

57. $3^{-4} \div 3^{-7}$.

58. $5^{-1} \div 5^{-3}$.

59. $7^{-4} \div 7^{-2}$.

60. $6^{-7} \div 6^{-3}$.

61. $10^{-8} \div 10^{-6}$.

62. $7^{-9} \div 7^{-5}$.

63. $3^{2} \div 3^{-2}$.

64. $3^{-2} \div 3^{2}$.

65. $7^{-3} \div 7^{2}$.

66. $25^{-8} \div 25^{-8}$.

67. $(\frac{3}{4})^{-5} \div (\frac{3}{4})^{-6}$.

68. $(\frac{2}{3})^{-4} \div (\frac{2}{3})^{-7}$.

69. $(\frac{1}{2})^{2} \div (\frac{1}{2})^{-3}$.

70. $(\frac{2}{5})^{-5} \div (\frac{2}{5})^{-2}$.

71. $(0.3)^{-1} \div (0.3)^{-3}$.

72. $(0.7)^{-6} \div (0.7)^{-8}$.

73. $(0.4)^{-7} \div (0.4)^{-4}$.

74. $(0.1)^{-5} \div (0.1)^{-2}$.

75. $(0.2)^{-5} \div (0.2)^{-2}$.

Perform the following operations, and give the results freed from negative exponents:

76. $a^{-3} \div a^{-5}$.

77. $a^{-4} \div a^{-3}$.

78. $a^{-7} \div a^{-5}$.

79. $m^{-x} \div m^{-2x}$.

80. $m^{-(x+2)} \div m^{-2x}$.

81. $x^{-2a} \div x^{-3a}$.

82. $y^{-a} \div y^{-m}$.

83. $b^{-m+1} \div b^{-(m+1)}$.

84. $c^{-(2m+3x)} \div c^{-(2m-3x)}$.

85. $(a+b)^{-(x+y)} \div (a+b)^{-(x-y)}$.

86. $\frac{a^{-5}b^{-x}}{m^{-3}n^{-4}} \div \frac{a^{-6}b^{-(x+2)}}{m^{-7}n^{-3}}$.

87. $(a^{-6} - b^{-4}) \div (a^{-3} - b^{-2})$.

88. $(x^{-m} - 1) \div (x - 1)$.

89. $(x^{m} - 1) \div (x^{-1} - 1)$.

Find the values of:

90. $5^{-8} \div 10^{-8}$.

91. $3^{-5} \div 2^{-5}$.

92. $14^{-3} \div 7^{-3}$.

93. $8^{-5} \div 16^{-5}$.

94. $9^{-4} \div 3^{-4}$.

95. $1^{-7} \div 2^{-7}$.

96. $3^{-4} \div 1^{-4}$.

97. $(0.2)^{-3} \div (0.5)^{-3}$.

98. $(0.6)^{-2} \div (0.7)^{-2}$.

99. $3^{-5} \div (0.3)^{-5}$.

100. $(\frac{2}{5})^{-3} \div (\frac{4}{15})^{-3}$.

101. $(\frac{3}{4})^{-4} \div (\frac{2}{3})^{-4}$.

102. $(\frac{3}{7})^{-2} \div (\frac{9}{28})^{-2}$.

103. $\left(\frac{3\frac{2}{5}}{4\frac{1}{5}}\right)^{-3} \div \left(\frac{8\frac{1}{2}}{10\frac{1}{2}}\right)^{-3}$.

104. $\left(\frac{13\frac{1}{2}}{11\frac{1}{2}}\right)^{-5} \div \left(\frac{16\frac{1}{5}}{18\frac{2}{5}}\right)^{-5}$.

105. $(1 \div a^{-m})^2 \div \left(\frac{1}{a}\right)^{-2m}$.

106. $\left(\frac{a^2}{b^{-3}}\right)^{-2} \div \left(\frac{a^{-3}}{b^4}\right)^{-2}$.

107. $\left[x^{-6} - \left(\frac{1}{y^{-1}}\right)^{-2}\right] \div \left[x^{-3} + \left(\frac{1}{y^{-1}}\right)^{-1}\right]$.

Ex. 39.

Fractional Exponents.

$$\sqrt[p]{a^{mp}} = a^m = a^{\frac{mp}{p}}.$$

$$\sqrt[m]{a^{-2}} = \sqrt[m]{\frac{1}{a^2}} = \frac{1}{a^{2m}} = \frac{1}{\sqrt[m]{a^2}}.$$

$$\sqrt[m]{\frac{a^x}{b^y}} = \frac{\sqrt[m]{a^x}}{\sqrt[m]{b^y}} = \frac{a^{\frac{x}{m}}}{b^{\frac{y}{m}}}.$$

Express with fractional exponents:

1. $\sqrt{a^3}$.
2. $\sqrt{a^5}$.
3. $(\sqrt{a})^6$.
4. $\sqrt[3]{n^5}$.
5. $(\sqrt[4]{a})^5$.
6. $\sqrt{a^3b}$.
7. $\sqrt[3]{a^3b^2}$.
8. $\sqrt[3]{ab^5}$.
9. $\sqrt[3]{a^3 - 3a^2b + 3ab^2 - b^3}$.
10. $\sqrt{m^2 - 2mp + p^2}$.
11. $\sqrt[3]{27a^{-2}b^4}$.
12. $\sqrt[3]{\frac{m^5}{n^4}}$.
13. $\sqrt[4]{\frac{a^6}{b^8}}$.
14. $\sqrt[3]{x^6}$.
15. $\sqrt{m^3}$.
16. $\sqrt[3]{\sqrt{a^5}}$.
17. $\sqrt[4]{\sqrt[3]{a^7}}$.

Write the following expressions, using only integral and positive exponents:

18. $a^{\frac{1}{2}}$.
19. $1^{\frac{2}{3}}$.
20. $m^{\frac{3}{4}}$.
21. $x^{\frac{p}{q}}$.
22. $n^{\frac{3}{5}}$.
23. $b^{\frac{2}{3}}$.
24. $c^{\frac{4}{5}}$.
25. $h^{\frac{1}{2}}$.
26. $x^{\frac{a}{b}}$.
27. $y^{\frac{1}{m}}$.
28. $a^{-\frac{1}{2}}$.
29. $c^{-\frac{2}{3}}$.
30. $e^{-\frac{m}{n}}$.
31. $x^{-\frac{2m}{3}}$.
32. $y^{-\frac{a+b}{m+1}}$.
33. $z^{p-\frac{1}{2}}$.
34. $(x^3+y^3)^{\frac{1}{3}}$.
35. $x^{\frac{1}{2}}y^{\frac{2}{3}}$.
36. $(m^3-n^3)^{-\frac{1}{2}}$.
37. $\left(\frac{a^{-1}}{b^{-2}}\right)^{-\frac{1}{2}}$.
38. $\left(\frac{m^5}{n^4}\right)^{\frac{2}{3}}$.
39. $\left(\frac{x^{-3}}{y^{-4}}\right)^{-\frac{1}{2}}$.
40. $\left(\frac{a^2}{b^{-2}}\right)^{-1\frac{1}{2}}$.

Find the numerical values of the following:

41. $4^{\frac{1}{2}}$.
42. $27^{\frac{1}{3}}$.
43. $625^{\frac{1}{4}}$.
44. $16^{\frac{1}{4}}$.
45. $9^{\frac{1}{2}}$.
46. $64^{\frac{1}{6}}$.
47. $3125^{\frac{1}{5}}$.
48. $216^{\frac{1}{3}}$.
49. $49^{\frac{1}{2}}$.
50. $729^{\frac{1}{6}}$.
51. $16^{\frac{1}{4}}$.
52. $81^{\frac{1}{4}}$.
53. $125^{\frac{1}{3}}$.
54. $32^{\frac{1}{5}}$.
55. $81^{\frac{1}{4}}$.
56. $256^{\frac{1}{4}}$.
57. $512^{\frac{1}{9}}$.
58. $128^{\frac{1}{7}}$.
59. $16^{-\frac{1}{4}}$.
60. $64^{-\frac{1}{3}}$.
61. $1024^{-\frac{1}{2}}$.
62. $64^{-\frac{1}{6}}$.
63. $512^{-\frac{1}{9}}$.
64. $81^{-\frac{1}{4}}$.
65. $625^{-\frac{1}{4}}$.
66. $(\frac{25}{36})^{\frac{1}{2}}$.
67. $(\frac{8}{27})^{\frac{1}{3}}$.
68. $(\frac{625}{1296})^{\frac{1}{4}}$.
69. $(\frac{32}{243})^{\frac{1}{5}}$.
70. $(\frac{1}{64})^{\frac{1}{6}}$.
71. $(\frac{25}{36})^{-\frac{1}{2}}$.
72. $(\frac{8}{27})^{-\frac{1}{3}}$.
73. $(\frac{625}{1296})^{-\frac{1}{4}}$.
74. $(\frac{32}{243})^{-\frac{1}{5}}$.
75. $(\frac{1}{64})^{-\frac{1}{6}}$.
76. $(3\frac{3}{8})^{-\frac{1}{3}}$.
77. $(6\frac{1}{4})^{\frac{1}{2}}$.
78. $(7\frac{19}{32})^{\frac{2}{5}}$.
79. $(7\frac{19}{32})^{-\frac{2}{5}}$.
80. $(0.25)^{\frac{1}{2}}$.
81. $(0.027)^{\frac{1}{3}}$.
82. $(0.125)^{-\frac{1}{3}}$.
83. $(-0.027)^{\frac{1}{3}}$.
84. $(-0.00032)^{\frac{1}{5}}$.

Ex. 40.

Fractional and Negative Exponents.

$a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a^{\frac{2}{2}}$.

$a^{-\frac{m}{n}} \times a^{\frac{p}{n}} = a^{\frac{p-m}{n}}$.

$\left(a^{\frac{m}{n}}\right)^{\frac{x}{b}} = a^{\frac{mx}{bn}}$.

$a^{-\frac{m}{n}} + a^{-\frac{x}{y}} = a^{\frac{x}{y}-\frac{m}{n}}$ $\qquad a^{-\frac{m}{n}} + a^{\frac{x}{y}} = a^{-\left(\frac{m}{n}+\frac{x}{y}\right)}$.

$a^{\frac{m}{n}} + a^{-\frac{x}{y}} = a^{\frac{m}{n}+\frac{x}{y}}$.

Add:

1. $a^{\frac{1}{2}}$; $3a^{\frac{1}{2}}$.
2. $\frac{1}{2}b^{\frac{2}{3}}$; $4b^{-\frac{2}{3}}$.
3. $5m^{\frac{3}{4}}$; $-10m^{-\frac{3}{4}}$.
4. $5x^{-3}$; $4x^{-3}$.
5. $0.5m^{-\frac{1}{2}}$; $0.31m^{-\frac{1}{2}}$.
6. $4c^{-\frac{2}{3}}$; $-0.6c^{-\frac{2}{3}}$; $-3c^{-\frac{2}{3}}$.
7. d^{-4}; $3d^{-4}$; $-5.3d^{-4}$.
8. $x^{-2} - x^{-3}y^{\frac{1}{2}} - 3x^{-4}y^{\frac{1}{2}}$; $2x^{-3}y^{\frac{1}{2}} - 2x^{-2} + 5x^{-4}y^{\frac{1}{2}}$; $-x^{-4}y^{\frac{1}{2}} + x^{-2} - 3x^{-3}y^{\frac{1}{2}}$.
9. $x^{-\frac{1}{2}} - 2ax^{-\frac{1}{2}} + 2bx^{-\frac{1}{2}}$; $3ax^{-\frac{1}{2}} - mx^{-\frac{1}{2}} - cx^{-\frac{1}{2}}$; $bx^{-\frac{1}{2}} + dx^{-\frac{1}{2}} - 2x^{-\frac{1}{2}}$.
10. $y^{-\frac{1}{2}} + by^{-\frac{1}{2}} - cy^{\frac{1}{2}}$; $4y^{-\frac{1}{2}} - cy^{-\frac{1}{2}} + 2y^{\frac{1}{2}}$; $my^{-\frac{1}{2}} - 2y^{-\frac{1}{2}}$; $4by^{-\frac{1}{2}} - my^{-\frac{1}{2}}$; $-y^{-\frac{1}{2}} + y^{\frac{1}{2}}$.

From the first expression subtract the following expressions in succession:

11. $b^{-\frac{1}{2}}$; $2b^{-\frac{1}{2}}$; $ab^{-\frac{1}{2}}$.
12. $d^{-\frac{2}{3}}$; $4d^{-\frac{2}{3}}$; $0.7d^{-\frac{2}{3}}$.
13. e^{-x}; $-2e^{-x}$; $2.5e^{-x}$.
14. $0.8h^{-4}$; $-0.05h^{-4}$; $2h^{-4}$.
15. $-2.3m^{-5}$; $0.7m^{-5}$; $-7m^{-5}$.
16. $\frac{1}{2}p^{-3}$; $0.7p^{-3}$; $-1\frac{1}{2}p^{-3}$.
17. $3x^{-1} - 2x^{-2} + 0.5x^{-3}$; $4x^{-2} + 2x^{-1} - \frac{3}{5}x^{-3}$; $0.7x^{-2} - 0.4x^{-1} + 2x^{-3}$; $-x^{-3} - x^{-1} + x^{-2}$.

Perform the following multiplications:

18. $a^{\frac{1}{2}} \times a^{\frac{1}{3}}$.

19. $b^{-\frac{1}{2}} \times b^{\frac{1}{3}}$.

20. $a^{-\frac{1}{4}} \times a^{-\frac{1}{2}}$.

21. $c^{-\frac{1}{5}} \times c^{-\frac{1}{10}}$.

22. $d^{-\frac{1}{3}} \times d^{-1}$.

23. $m^{-\frac{1}{2}} \times m^{-\frac{1}{3}}$.

24. $m^{-\frac{1}{4}} \times m^{\frac{1}{2}}$.

25. $x^{-\frac{1}{m}} \times x^{\frac{1}{n}}$.

26. $x^{\frac{m}{n}} \times x^{\frac{p}{q}}$.

27. $y^{-\frac{a}{c}} \times y^{-\frac{m}{n}}$.

28. $1^{-\frac{1}{m}} \times a^{-\frac{x}{y}}$.

29. $z^{-\frac{c}{m}} \times z^{0}$.

30. $u^{-1} \times u^{\frac{1}{2}}$.

31. $d^{-\frac{2}{m}} \times d^{-\frac{n}{2m}}$.

32. $e^{-\frac{2x}{y}} \times e^{-\frac{x}{y}}$.

33. $(a^{\frac{2}{3}})^{\frac{5}{12}}$.

34. $(b^{\frac{1}{2}})^{\frac{1}{3}}$.

35. $(h^{\frac{2}{3}})^{\frac{3}{5}}$.

36. $(m^{\frac{1}{x}})^{\frac{a}{b}}$.

37. $(n^{-\frac{a}{b}})^{\frac{c}{d}}$.

38. $(x^{\frac{r}{2}} + 2x^{\frac{r}{4}}y^{p} + 3y^{2p}) \times (x^{\frac{r}{2}} - 2x^{\frac{r}{4}}y^{p} + 3y^{2p})$.

39. $(x^{\frac{p}{2}} - x^{\frac{p}{4}}y^{\frac{q}{4}} + y^{\frac{q}{2}}) \times (x^{\frac{p}{2}} + x^{\frac{p}{4}}y^{\frac{q}{4}} + y^{\frac{q}{2}})$.

40. $(x^{-1} + x^{-\frac{1}{2}}y^{-\frac{1}{2}} + y^{-1}) \times (x^{-1} - x^{-\frac{1}{2}}y^{-\frac{1}{2}} + y^{-1})$.

41. $(c^{-\frac{4}{m}} - 3c^{-\frac{3}{m}} + 5c^{-\frac{2}{m}} - c^{-\frac{1}{m}}) \times (c^{-\frac{3}{m}} - 2c^{-\frac{2}{m}} - c^{-\frac{1}{m}})$.

42. $(\frac{1}{2}d^{-\frac{1}{x}} + \frac{1}{3}d^{-\frac{2}{x}} - \frac{1}{4}d^{-\frac{3}{x}} + \frac{1}{5}d^{-\frac{4}{x}}) \times (2d^{-\frac{1}{x}} - 3d^{-\frac{2}{x}} + 4d^{-\frac{3}{x}})$.

43. $\left(\frac{5x^{-\frac{3}{4}}}{4y^{-\frac{3}{4}}} + \frac{3a^{-\frac{1}{2}}}{2b^{-\frac{1}{2}}}\right) \times \left(\frac{5x^{-\frac{3}{4}}}{4y^{-\frac{3}{4}}} - \frac{3a^{-\frac{1}{2}}}{2b^{-\frac{1}{2}}}\right)$.

44. $(2a^{-1}b^{-\frac{1}{3}} - 3a^{-\frac{1}{2}}b^{-\frac{1}{6}})^{2}$.

Perform the following divisions:

45. $e^{-\frac{1}{3}} \div e^{-\frac{1}{4}}$.

46. $h^{-\frac{2}{3}} \div h^{-\frac{1}{4}}$.

47. $k^{-\frac{2}{3}} \div k^{-\frac{1}{4}}$.

48. $l^{-\frac{2}{3}} \div l^{-\frac{1}{4}}$.

49. $g^{-\frac{3}{5}} \div g^{-\frac{2}{3}}$.

50. $a^{\frac{2}{3}} \div a^{-\frac{1}{4}}$.

51. $b^{-\frac{1}{2}} \div b^{\frac{1}{3}}$.

52. $m^{-\frac{1}{2}} \div m^{-\frac{1}{3}}$.

53. $p^{-\frac{1}{m}} \div p^{\frac{1}{n}}$.

54. $q^{\frac{1}{a}} \div q^{-\frac{1}{b}}$.

55. $t^{-\frac{m}{n}} \div t^{-\frac{1}{a}}$.

56. $v^{-\frac{1}{p}} \div v^{\frac{a}{b}}$.

57. $x^{-\frac{m}{n}} \div x^{-\frac{b}{2m}}$.

58. $z^{-a} \div z^{\frac{p}{q}}$.

59. $y^{-1} \div y^{-\frac{m}{n}}$.

60. $c^{-\frac{2}{m}} \div c^{-\frac{a}{m}}$.

61. $d^{-\frac{a}{m}} \div d^{-\frac{b}{2m}}$.

62. $e^{\frac{m}{n}} \div e^{-\frac{1}{a}}$.

63. $ab^{-\frac{p}{q}} \div a^{-\frac{1}{2}}b^{\frac{2p}{q}}$.

64. $\dfrac{1}{a^{-\frac{1}{2}}} \div \dfrac{3b^{-\frac{2}{3}}}{a^{-\frac{2}{3}}}$.

65. $(x^{\frac{3}{4}} - xy^{\frac{1}{4}} + x^{\frac{1}{4}}y - y^{\frac{3}{4}}) \div (x^{\frac{1}{2}} - y^{\frac{1}{2}})$.

66. $(x^{\frac{3}{2}} - x^2 - 4x^{\frac{1}{2}} + 6x - 2x^{\frac{1}{2}}) \div (x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 2)$.

67. $(\frac{2}{5}x^{\frac{5}{3}} + \frac{1}{2}x^{\frac{1}{3}} - \frac{2}{5}x^{\frac{1}{3}} - \frac{3}{5}x^{\frac{1}{6}} + x^{\frac{2}{3}} - \frac{3}{10}x^{\frac{1}{6}}) \div (x^{\frac{1}{3}} - \frac{3}{5}x^{\frac{1}{6}})$.

68. $(x + y + z - 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}) \div (x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}})$.

Ex. 41.

Powers of Binomials.

Expand:

1. $(a+b)^3$.
2. $(m+n)^5$.
3. $(c+d)^8$.
4. $(h+l)^{10}$.
5. $(x+y)^{11}$.
6. $(y-z)^7$.
7. $(x-a)^9$.
8. $(y-b)^3$.
9. $(c-d)^5$.
10. $(m-a)^8$.
11. $(1+a)^4$.
12. $(a-1)^4$.
13. $(1+d)^6$.
14. $(1-h)^8$.
15. $(1+x)^{10}$.
16. $(2+a)^5$.
17. $(3-a)^6$.
18. $(m-2)^9$.
19. $(x-5)^3$.
20. $(y-3)^5$.
21. $(a^3+b^2)^7$.
22. $(m^5-n^2)^8$.
23. $(x^3+y^4)^5$.
24. $(a^4-y^3)^8$.
25. $(a-b^2)^7$.
26. $\left(\frac{a}{b}+\frac{x}{y}\right)^2$.
27. $\left(\frac{a}{c}-\frac{d}{x}\right)^3$.
28. $\left(\frac{m}{n}+\frac{p}{q}\right)^5$.
29. $\left(\frac{m}{p}-\frac{n}{q}\right)^6$.
30. $\left(\frac{1}{a}-\frac{b}{c}\right)^4$.
31. $(m^{\frac{1}{2}}-n^{\frac{1}{2}})^5$.
32. $(a^{\frac{2}{3}}+b^{\frac{2}{3}})^6$.
33. $(x^{\frac{1}{2}}+y^{\frac{1}{4}})^7$.
34. $(x^{\frac{1}{2}}-z^{\frac{1}{3}})^4$.
35. $(a^{\frac{1}{2}}+b^{\frac{2}{3}})^5$.
36. $(a^{-1}+b^{-2})^5$.
37. $(a^{-3}+x^{-1})^8$.
38. $(m^{-4}-n^{-1})^7$.
39. $(d^{-2}-x^2)^6$.
40. $(c^{-5}+z)^4$.
41. $(m^{-\frac{1}{2}}+n^{-\frac{1}{3}})^5$.
42. $(n^{-\frac{2}{3}}+p^{-\frac{1}{3}})^5$.
43. $(n^{-\frac{2}{3}}-p^{-\frac{1}{3}})^4$.
44. $(a^{-\frac{1}{4}}+b^{-\frac{1}{3}})^5$.
45. $(x^{-\frac{1}{2}}+y)^3$.
46. $(y^{-2}-z^{-\frac{2}{3}})^6$.
47. $(u^{-\frac{2}{3}}+v^3)^4$.
48. $(1^{-2}-z^{-\frac{1}{3}})^5$.
49. $(2x+y)^4$.
50. $(3y-z)^5$.
51. $(4z-u)^6$.
52. $(2a-b^2)^7$.
53. $(m^2+3b)^6$.

54. $(a-2c)^5$.

55. $(x^3-3d)^4$.

56. $(u^{-\frac{1}{2}}-4e)^3$.

57. $(2a^2-3b)^7$.

58. $(3x^{-1}+2y^{\frac{1}{2}})^6$.

59. $(2x+3z^{-\frac{1}{2}})^8$.

60. $\left(\frac{a}{2}-1\frac{1}{2}d^{-1}\right)^5$.

61. $(5m^{-2}+\frac{1}{5}n^{-1})^5$.

62. $\left(\frac{2a}{b}-\frac{3c}{d}\right)^6$.

63. $\left(\frac{1}{2a}+\frac{3}{b}\right)^5$.

64. $\left(\frac{2}{a^{-1}}+\frac{d^{-2}}{c^{-3}}\right)^4$.

65. $\left(\frac{x}{2}-\frac{y^{\frac{1}{2}}}{3}\right)^7$.

66. $\left(\frac{a}{3b}+\frac{2m^{\frac{1}{2}}}{x}\right)^5$.

67. $\left(1-\frac{3y}{4}\right)^6$.

68. $\left(\frac{3a}{b}-\frac{2a^2}{x}\right)^7$.

69. $(5m^x-\frac{1}{5}n^{2y})^4$.

70. $(3b^{m+1}+2d^{n-1})^5$.

71. $(4x^{a-3}-3y^{2-b})^4$.

72. $(5m^{2x-1}+\frac{3}{4}n^{y-2})^3$.

73. $(2a^3b-\frac{5}{3}mb^3)^2$.

74. $(\frac{1}{4}a^mb^x-\frac{1}{3}a^{m+1}b^{x-1})^4$.

75. $(2a^2b^m+\frac{1}{2}a^mb^2)^5$.

Ex. 42.

Transformation of Radicals.

$a=\sqrt[m]{a^m}$. $\qquad a\sqrt[m]{b}=\sqrt[m]{a^m}\times\sqrt[m]{b}=\sqrt[m]{a^mb}$.

$\sqrt[m]{ab}=\sqrt[m]{a}\times\sqrt[m]{b}$. $\qquad \sqrt[r]{a^rb}=a\sqrt[r]{b}$.

Express as entire surds:

1. $2\sqrt{2}$.
2. $5\sqrt{7}$.
3. $6\sqrt{5}$.
4. $2\sqrt{21}$.
5. $3\sqrt[3]{2}$.
6. $7\sqrt[3]{3}$.
7. $8\sqrt[3]{5}$.
8. $4\sqrt[4]{4}$.
9. $2\sqrt[5]{3}$.
10. $3\sqrt[7]{2}$.
11. $10\sqrt[3]{4}$.
12. $9\sqrt[3]{10}$.
13. $8\sqrt{41}$.
14. $a\sqrt{5}$.
15. $x\sqrt[4]{2}$.
16. $y\sqrt[5]{6}$.
17. $z\sqrt[3]{7}$.
18. $(-b)\sqrt[4]{6}$.
19. $(-d)\sqrt[5]{7}$.
20. $-m\sqrt[6]{10}$.
21. $-n\sqrt[3]{15}$.
22. $-3\sqrt{a}$.
23. $-5\sqrt[3]{b}$.
24. $-2\sqrt[5]{x}$.
25. $-4\sqrt[4]{y}$.
26. $3\sqrt{bx}$.
27. $a\sqrt{b}$.
28. $m^2\sqrt{yz^3}$.
29. $n^3\sqrt[5]{xy^3}$.
30. $-a^3\sqrt[4]{m^3n}$.
31. $-x^5\sqrt[3]{y^4z}$.
32. $\frac{1}{2}\sqrt{a}$.
33. $\frac{2}{3}\sqrt[3]{m^2}$.
34. $\sqrt{8}$.

Express as mixed surds:

35. $\sqrt{12}$.

36. $\sqrt{28}$.

37. $\sqrt{50}$.

38. $\sqrt{72}$.

39. $\sqrt[3]{72}$.

40. $\sqrt[3]{500}$.

41. $\sqrt[3]{108}$.

42. $\sqrt[3]{192}$.

43. $\sqrt[3]{56}$.

44. $\sqrt[5]{160}$.

45. $\sqrt[5]{2048}$.

46. $\sqrt[6]{3645}$.

47. $\sqrt[4]{567}$.

48. $\sqrt[4]{112}$.

49. $2\sqrt{405}$.

50. $3\sqrt[3]{864}$.

51. $5\sqrt[4]{144}$.

52. $\sqrt{a^3b}$.

53. $\sqrt[3]{m^3n^2}$.

54. $\sqrt[4]{b^8a^3}$.

55. $\sqrt[4]{b^4c}$.

56. $\sqrt[5]{c^{15}x^3}$.

57. $\sqrt[3]{x^6y^4}$.

58. $\sqrt{y^4z^3}$.

59. $\sqrt[3]{m^9n^7}$.

60. $\sqrt[4]{x^5y^6}$.

61. $\sqrt[3]{u^4v^{10}}$.

62. $\sqrt[4]{y^7z^8}$.

63. $\sqrt[10]{a^{12}b^{15}}$.

64. $\sqrt{4a^3b}$.

65. $\sqrt{25m^4x}$.

66. $\sqrt{36a^3b^2}$.

67. $\sqrt[3]{64x^6y}$.

68. $\sqrt[4]{81m^6n^3}$.

69. $\sqrt[6]{64a^8b^6}$.

70. $2\sqrt{80c^4d}$.

71. $\sqrt{\dfrac{4a^3b}{9}}$.

72. $\sqrt[3]{\dfrac{125m^2}{216n^5}}$.

73. $\sqrt{\dfrac{a^5}{4}}$.

74. $\sqrt[5]{\dfrac{m^6n}{243}}$.

75. $\sqrt[4]{\dfrac{x^5y^6}{1296}}$.

76. $\sqrt{\dfrac{(m^2-2mn+n^2)y}{49}}$.

77. $\sqrt[3]{\dfrac{(x^3-3x^2+3x-1)z^3}{512}}$.

78. $\sqrt{\dfrac{(x^2-y^2)^3}{4(x+y)}}$.

Ex. 43.

Reduction of Radicals.

$$\sqrt[m]{a^p} = \sqrt[mn]{a^{pn}};\quad \sqrt[m]{a^n} = \sqrt[\frac{m}{p}]{a^{\frac{n}{p}}}.$$

$$\sqrt{a}+\sqrt{b} = \sqrt{a+b+2\sqrt{ab}}.$$

$$\sqrt{a}-\sqrt{b} = \sqrt{a+b-2\sqrt{ab}}.$$

$$\sqrt{a}+b\sqrt{-1} = \sqrt{a-b^2+2b\sqrt{a}\sqrt{-1}}.$$

$$\sqrt{a}-b\sqrt{-1} = \sqrt{a-b^2-2b\sqrt{a}\sqrt{-1}}.$$

Reduce the following radicals to the least common index possible:

1. $\sqrt{3}$; $\sqrt[6]{4}$.
2. $\sqrt[4]{6}$; $\sqrt{7}$.
3. $\sqrt{5}$; $\sqrt[3]{4}$.
4. $\sqrt[3]{2}$; $\sqrt[4]{3}$.
5. $\sqrt[5]{8}$; $\sqrt{2}$.
6. $\sqrt[3]{4}$; $\sqrt[4]{8}$.
7. $\sqrt{2}$; $\sqrt[3]{5}$; $\sqrt[4]{6}$.
8. $\sqrt[4]{7}$; $\sqrt[5]{5}$; $\sqrt[10]{120}$.

Change to similar surds:

9. $\sqrt{3}$; $\sqrt{12}$.
10. $\sqrt{5}$; $\sqrt{20}$.
11. $\sqrt{63}$; $\sqrt{7}$.
12. $\sqrt{72}$; $\sqrt{8}$.
13. $3\sqrt{11}$; $\sqrt{176}$.
14. $\sqrt[3]{54}$; $2\sqrt[3]{2}$.
15. $\sqrt[3]{72}$; $\sqrt[3]{243}$.
16. $\sqrt[4]{5}$; $\sqrt[4]{405}$.
17. $\sqrt{18}$; $\sqrt{128}$; $\sqrt{32}$.
18. $7\sqrt[3]{54}$; $3\sqrt[3]{16}$; $\sqrt[3]{432}$.
19. $\sqrt{27}$; $2\sqrt{48}$; $3\sqrt{108}$.
20. $\sqrt[3]{128}$; $\sqrt[3]{686}$; $\sqrt[3]{16}$.
21. $\sqrt[4]{162}$; $3\sqrt[4]{32}$; $\sqrt[4]{2592}$.
22. $\sqrt{\frac{4}{5}}$; $2\sqrt{\frac{25}{3}}$.
23. $\frac{1}{2}\sqrt{\frac{2}{5}}$; $\sqrt{\frac{2}{45}}$.
24. $3\sqrt{\frac{50}{147}}$; $\sqrt{\frac{2}{363}}$.
25. $\frac{1}{4}\sqrt{0.2}$; $\frac{1}{5}\sqrt{5}$.
26. $\sqrt{20}$; $3\sqrt{\frac{1}{5}}$; $4\sqrt{125}$; $7\sqrt{\frac{9}{5}}$; $\sqrt[4]{\frac{25}{16}}$; $3\sqrt{\frac{9}{80}}$.
27. $\sqrt[3]{\frac{8}{5}}$; $\sqrt[3]{\frac{9}{8}}$; $\sqrt[3]{\frac{72}{343}}$; $\sqrt[3]{\frac{5}{8}}$; $2\sqrt[3]{\frac{2}{4}}$.
28. $\sqrt[5]{\frac{3}{16}}$; $\sqrt[5]{192}$; $\sqrt[5]{\frac{3}{512}}$; $\sqrt[5]{\frac{18}{3125}}$; $\sqrt[5]{\frac{2}{81}}$.

29. $\sqrt{\frac{1}{2}+\frac{3}{4}}$; $\sqrt{\frac{3}{9}-\frac{1}{8}}$.

30. $\sqrt{\frac{1}{4}-\frac{1}{5}}$; $\sqrt{\frac{4}{5}-\frac{7}{9}}$.

31. $\sqrt{\frac{13}{18}+\frac{1}{36}}$; $\sqrt{\frac{1}{27}+\frac{5}{9}}$.

32. $\sqrt[3]{\frac{12}{125}-\frac{1}{25}}$; $\sqrt[3]{\frac{47}{8}-\frac{1}{2}}$.

33. $\sqrt[6]{a^7b}$; $\sqrt[6]{a^{13}b^7}$.

34. $\sqrt[3]{27m^4}$; $\sqrt[3]{8m^7}$.

35. $\sqrt[3]{x^5-3x^4}$; $\sqrt[3]{\frac{x-3}{x^2}}$.

36. $\sqrt{\frac{a^4c}{b^3}}$; $\sqrt{\frac{a^2c^3}{bx^2}}$; $\sqrt{\frac{a^2cx^2}{by^2}}$.

37. $\sqrt{\frac{a^3x}{b^3}}$; $\sqrt{\frac{ax^5}{b^5}}$; $\sqrt{\frac{x^3}{ab}}$.

38. $\sqrt[4]{\frac{1}{a^3}}$; $\sqrt[4]{\frac{b^4}{a^3}}$; $\sqrt[4]{a}$; $\sqrt[4]{\frac{a}{c^4x^8}}$.

39. $\sqrt{1-\frac{1}{a^2}}$; $\sqrt{\frac{a-1}{a+1}}$.

40. $\sqrt{\left(\frac{1}{b}-a\right)}$; $\sqrt{\frac{bd^2-ab^2d^2}{c^2}}$.

41. $\sqrt{\left(\frac{a^2-b^2}{(a+b)}\right)^3}$; $\sqrt{\frac{(a^2+b^2)^2}{a-b}}$.

42. $\sqrt[3]{(x-b^2)^2}$; $\sqrt[3]{\frac{(y-z)^3}{x-b^2}}$; $\sqrt[3]{\frac{5y^5}{x-b^2}}$.

43. $\sqrt{\frac{m^3}{1-2x+x^2}}$; $\sqrt{\frac{4m-8mx+4mx^2}{c^2n^2}}$.

Change to a single surd:

44. $\sqrt{2}+\sqrt{18}$. 46. $\sqrt{50}+\sqrt{18}$. 48. $\sqrt{6}+\sqrt{24}$.

45. $\sqrt{12}+\sqrt{75}$. 47. $\sqrt{45}+\sqrt{245}$. 49. $\sqrt{5}+\sqrt{45}$.

50. $\sqrt{7}+\sqrt{28}$. 51. $\sqrt{11}+\sqrt{44}$.

Which is the greater:

52. $\sqrt{7}+\sqrt{10}$ or $\sqrt{3}+\sqrt{19}$?

53. $\sqrt{8}+\sqrt{12}$ or $\sqrt{2}+\sqrt{20}$?

54. $\sqrt{10}+\sqrt{8}+\sqrt{6}$ or $\sqrt{2}+\sqrt{3}+\sqrt{24}$?

Ex. 44.

Addition and Subtraction of Radicals.

Perform the following operations:

1. $2\sqrt{3}+3\sqrt{3}$.
2. $5\sqrt{7}+3\sqrt{7}+2\sqrt{7}$.
3. $8\sqrt{10}+3\sqrt{10}+2\sqrt{10}$.
4. $7\sqrt{2}-3\sqrt{2}$.
5. $4\sqrt{11}+3\sqrt{11}-5\sqrt{11}$.
6. $2\sqrt{3}-5\sqrt{3}+9\sqrt{3}$.
7. $4\sqrt[3]{3}-3\sqrt[3]{3}+2\sqrt[3]{3}$.
8. $11\sqrt{13}-4\sqrt{13}$.
9. $7\sqrt{2}-\sqrt{18}$.
10. $5\sqrt[3]{4}+2\sqrt[3]{32}-\sqrt[3]{108}$.
11. $5\sqrt{2}+2\sqrt{2}-4\sqrt{2}$.
12. $3\sqrt[5]{2}+4\sqrt[5]{2}-\sqrt[5]{64}$.
13. $\frac{1}{2}\sqrt[3]{5}+2\frac{1}{2}\sqrt[3]{5}+\frac{1}{4}\sqrt[3]{40}$.
14. $3\sqrt[4]{3}-5\sqrt[4]{48}+\sqrt[4]{243}$.
15. $2\sqrt{175}-3\sqrt{63}+5\sqrt{28}$.
16. $\sqrt{2}+3\sqrt{32}+\frac{1}{2}\sqrt{128}-6\sqrt{18}$.
17. $\sqrt{75}+\sqrt{48}-\sqrt{147}+\sqrt{300}$.
18. $20\sqrt{245}-\sqrt{5}+\sqrt{125}-2\frac{1}{2}\sqrt{180}$.
19. $2\sqrt{20}+\frac{1}{2}\sqrt{12}-2\sqrt{27}+5\sqrt{45}-9\sqrt{12}$.
20. $3\sqrt{7}+2\sqrt{54}-5\sqrt{63}+3\sqrt{150}+4\sqrt{63}+7\sqrt{24}$.
21. $7\sqrt{25}+4\sqrt{45}-\sqrt{9}-2\sqrt{80}+\sqrt{20}-4\sqrt{64}$.
22. $\sqrt{275}+\sqrt{1300}-\sqrt{44}+\sqrt{52}-2\sqrt{99}-2\sqrt{208}$.
23. $\sqrt{\frac{45}{4}}+\frac{3}{\sqrt{8}}-\sqrt{20}-\frac{\sqrt{245}}{6}-5\sqrt{\frac{1}{18}}-\sqrt{\frac{49}{2}}$.
24. $\sqrt[3]{\frac{4}{27}}-\sqrt[3]{\frac{10}{2}}-3\sqrt[3]{256}+\sqrt[3]{625}+\sqrt[3]{\frac{32}{27}}-\sqrt[3]{\frac{125}{54}}$.
25. $\sqrt{9}+\sqrt{20}-\sqrt{\frac{1}{16}}-\sqrt{\frac{9}{5}}+\sqrt{\frac{9}{16}}+\sqrt{\frac{25}{81}}+\sqrt{\frac{20}{9}}$.
26. $\sqrt[3]{54}+\sqrt{\frac{1}{2}}-\sqrt[3]{250}-\frac{3}{4}\sqrt{\frac{2}{9}}+0.5\sqrt{\frac{128}{9}}+\sqrt[3]{6\frac{3}{4}}$.
27. $2\sqrt{\frac{5}{3}}+\sqrt{60}-\sqrt{15}+\sqrt{\frac{3}{5}}+\sqrt{\frac{4}{15}}$.

28. $3\frac{1}{2}\sqrt{24} - 5\frac{3}{4}\sqrt{54} + 13\frac{1}{8}\sqrt{99} + 2\frac{17}{24}\sqrt{216} - 21\sqrt{44}$.

29. $\frac{8}{2}\sqrt[3]{\frac{2}{49}} + 0.8\sqrt{\frac{8}{8}} - \frac{1}{15}\sqrt{96} + 1.5\sqrt[3]{\frac{7}{4}} - \frac{11}{28}\sqrt[3]{1750} + 8\sqrt{\frac{3}{2}}$.

30. $\sqrt{a} - \frac{1}{2}\sqrt{a} + \frac{3}{2}\sqrt{a}$.

32. $\sqrt{a^3} + b\sqrt{a} - 3\sqrt{a}$.

31. $\sqrt[3]{a^2} + \frac{1}{2}\sqrt[3]{a^2} - 3\sqrt[3]{27a^2}$.

33. $\sqrt{25b} + 2\sqrt{9b} - 3\sqrt{4b}$.

34. $\sqrt[3]{27c^4} - \sqrt[3]{8c^4} + \sqrt[3]{125c}$.

35. $\sqrt[5]{a^5b} - \sqrt[5]{b^6} + \sqrt[5]{32b}$.

36. $\sqrt{a^4x} + \sqrt{b^4x} - \sqrt{4a^2b^2x}$.

37. $\sqrt{4x^3y^2z} + \sqrt{y^4z} + \sqrt{x^4z}$.

38. $\sqrt{a^2b^2c} - a\sqrt{4c} + b\sqrt{a^2c}$.

39. $\sqrt[4]{81a^5} - \sqrt[4]{16a} + \sqrt[4]{256a^5}$.

40. $\sqrt[3]{27m^4} - \sqrt[3]{125m} + \sqrt[3]{216m}$.

41. $\sqrt{8a} - \sqrt{50a^3} - 3\sqrt{18a}$.

42. $6a\sqrt{63ab^3} - 3\sqrt{112a^3b^3} + 2ab\sqrt{343ab} - 5b\sqrt{28a^3b}$.

43. $3\sqrt{125m^3n^2} + n\sqrt{20m^3} - \sqrt{500m^3n^2} - m\sqrt{45mn^2}$.

44. $\sqrt{32a^4b^5} + 6\sqrt{72b} + 3\sqrt{128a^2b^3} - 4\sqrt{288a^2b^3}$.

45. $2\sqrt[3]{a^6b} - 3a^2\sqrt[3]{64b} + 5a\sqrt[3]{a^3b} + 2a^2\sqrt[3]{125b^4} + 3\sqrt[3]{27a^6b}$.

Ex. 45.

Multiplication of Radicals.

$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$. $\sqrt[n]{a^y} = \sqrt[nz]{a^{ny}}$.

$\sqrt[m]{a^p} \times \sqrt[n]{c^q} = \sqrt[mn]{a^{pn}} \times \sqrt[mn]{c^{mq}} = \sqrt[mn]{a^{pn}c^{mq}}$.

$(\sqrt[n]{a})^n = a$.

Perform the following multiplications:

1. $\sqrt{3} \times \sqrt{27}$.

2. $\sqrt{5} \times \sqrt{20}$.

3. $\sqrt{2} \times \sqrt{18}$.

4. $\sqrt[3]{3} \times \sqrt[3]{9}$.
5. $\sqrt[3]{2} \times \sqrt[3]{32}$.
6. $\sqrt[3]{16} \times \sqrt[3]{32}$.
7. $\sqrt[4]{27} \times \sqrt[4]{3}$.
8. $\sqrt[4]{108} \times \sqrt[4]{12}$.
9. $\sqrt[5]{4} \times \sqrt[5]{8}$.
10. $\sqrt[5]{27} \times \sqrt[5]{9}$.
11. $\sqrt{2} \times \sqrt{32}$.
12. $\sqrt[3]{7} \times \sqrt[3]{-49}$.
13. $\sqrt[6]{1331} \times \sqrt[6]{11} \times \sqrt[6]{121}$.
14. $\sqrt[5]{8} \times \sqrt[5]{-4}$.
15. $\sqrt[4]{9} \times \sqrt[4]{81} \times \sqrt[4]{729}$.
16. $\sqrt{6} \times \sqrt{12} \times \sqrt{72}$.
17. $\sqrt[8]{5^3} \times \sqrt[8]{5^7} \times \sqrt[8]{5^6}$.
18. $\sqrt[3]{-9^4} \times \sqrt[3]{9} \times \sqrt[3]{9^4}$.
19. $\sqrt[3]{-108} \times \sqrt[3]{50} \times \sqrt[3]{40}$.
20. $\sqrt[5]{7^3} \times \sqrt[5]{-112} \times \sqrt[5]{14}$.
21. $\sqrt{3} \times \sqrt{6}$.
22. $\sqrt[3]{3} \times \sqrt[3]{18}$.
23. $\sqrt[4]{6} \times \sqrt[4]{8}$.
24. $\sqrt[5]{54} \times \sqrt[5]{9}$.
25. $\sqrt[3]{81} \times \sqrt[3]{-45}$.
26. $2\sqrt{8} \times \sqrt{2}$.
27. $3\sqrt[3]{16} \times 5\sqrt[3]{4}$.
28. $\frac{1}{2}\sqrt[4]{8} \times \frac{3}{4}\sqrt[4]{2}$.
29. $\frac{2}{3}\sqrt[3]{18} \times \frac{3}{4}\sqrt[3]{-3}$.
30. $5\sqrt{2} \times 0.4\sqrt{72}$.
31. $(\sqrt{18} + 2\sqrt{72} - 3\sqrt{8} - \frac{1}{2}\sqrt{128})\sqrt{2}$.
32. $(\sqrt[3]{32} - \frac{1}{2}\sqrt[3]{864} + 3\sqrt[3]{4})\sqrt[3]{2}$.
33. $(\frac{1}{3}\sqrt{27} - \frac{1}{4}\sqrt{2187} + \frac{1}{8}\sqrt{432} - \sqrt{16{,}875})\sqrt{3}$.
34. $(3\sqrt[3]{24} + 4\sqrt[3]{81} - 5\sqrt[3]{375} + \frac{1}{2}\sqrt[3]{192})\sqrt[3]{2}$.
35. $(\sqrt{5 - \sqrt{4}})(\sqrt{5 + \sqrt{4}})$.
36. $(\sqrt{3 - \sqrt{2}})(\sqrt{3 + \sqrt{2}})$.
37. $(\sqrt{9 - \sqrt{17}})(\sqrt{9 + \sqrt{17}})$.
38. $(\sqrt{6 + \sqrt{11}})(\sqrt{6 - \sqrt{11}})$.
39. $(\sqrt{8 + \sqrt{15}})(\sqrt{8 - \sqrt{15}})$.
40. $(\sqrt[3]{10 - \sqrt{36}})(\sqrt[3]{10 + \sqrt{36}})$.

41. $(\sqrt[3]{9+\sqrt{17}})(\sqrt[3]{9-\sqrt{17}})$.

42. $(\sqrt[3]{12+\sqrt{19}})(\sqrt[3]{12-\sqrt{19}})$.

43. $(3+2\sqrt{5})(2-\sqrt{5})$.

44. $(8+3\sqrt{2})(2-\sqrt{2})$.

45. $(5+2\sqrt{3})(3-5\sqrt{3})$.

46. $(3-\sqrt{6})(6-3\sqrt{6})$.

47. $(2\sqrt{6}-3\sqrt{5})(\sqrt{3}+2\sqrt{2})$.

48. $(\sqrt{7}-\sqrt{3})(\sqrt{2}+\sqrt{5})$.

49. $(\sqrt[3]{9}-2\sqrt[3]{4})(4\sqrt[3]{3}+\sqrt[3]{2})$.

50. $(2\sqrt{30}-3\sqrt{5}+5\sqrt{3})(\sqrt{8}+\sqrt{3}-\sqrt{5})$.

51. $(3\sqrt{5}-2\sqrt{3}+4\sqrt{7})(3\sqrt{7}-4\sqrt{5}-5\sqrt{3})$.

52. $(2\sqrt{\frac{1}{2}}-3\sqrt{3}+4\sqrt{8})(3\sqrt{8}+\sqrt{18}-\sqrt{3})$.

53. $(3\sqrt{3}-4\sqrt{8}+5\sqrt{18})(5\sqrt{2}+\sqrt{3}+\sqrt{\frac{1}{2}})$.

54. $(4\sqrt{8}+\frac{1}{2}\sqrt{12}-\frac{1}{4}\sqrt{32})(8\sqrt{32}-4\sqrt{50}-2\sqrt{2})$.

55. $(\sqrt[3]{6}-\sqrt[3]{3}+\sqrt[3]{16})(\sqrt[3]{36}+\sqrt[3]{9}-\sqrt[3]{4})$.

56. $(\sqrt[3]{16}-2\sqrt[3]{2}+4\sqrt[3]{54})(5\sqrt[3]{4}+3\sqrt[3]{\frac{1}{2}}-2\sqrt[3]{2})$.

57. $(\sqrt[3]{9}-7\sqrt[3]{72}+6\sqrt[3]{1125})(3\sqrt[3]{2}+8\sqrt[3]{3}-4\sqrt[3]{\frac{1}{9}})$.

58. $(0.4\sqrt[3]{25}+0.2\sqrt[3]{200}-\frac{1}{2}\sqrt[3]{75})(2\sqrt[3]{135}-5\sqrt[3]{5}-10\sqrt[3]{15})$

59. $(2\sqrt{\frac{2}{3}}-8\sqrt{\frac{3}{8}}+3\sqrt{\frac{3}{2}})(3\sqrt{\frac{2}{3}}-\sqrt{12}-\sqrt{6})$.

60. $(2\sqrt{\frac{5}{6}}-4\sqrt{\frac{3}{2}}-7\sqrt{\frac{6}{5}})(3\sqrt{\frac{5}{6}}-5\sqrt{30}-2\sqrt{1\frac{5}{2}})$.

61. $(3\sqrt{2}+4\sqrt{5}-2\sqrt{3})^2$.

62. $(5\sqrt{6}-3\sqrt{3}-2\sqrt{24})^2$.

63. $(3\sqrt{5}-4\sqrt{45}+2\sqrt{20})^2$.

64. $(2\sqrt{12}+3\sqrt{3}+6\sqrt{\frac{1}{3}})^2$.

65. $(2\sqrt{7}-8\sqrt{28}-5\sqrt{63})^2$.

66. $(\frac{1}{2}\sqrt{24}-\frac{3}{4}\sqrt{6}+5\sqrt{3})^2$.

67. $(3\sqrt{x^5}-2\sqrt{x^3})^2$.

68. $(a\sqrt{y}+b\sqrt{y^3})^2$.

69. $(m^2\sqrt{ab^3}-m\sqrt{a^3b})^2$.

70. $(2p\sqrt{12z^5}-\frac{1}{2}q\sqrt{18z})^2$.

71. $\sqrt{5} \times \sqrt[3]{4}$.

72. $\sqrt{125} \times \sqrt[4]{36}$.

73. $\sqrt[3]{16} \times \sqrt[6]{250}$.

74. $\sqrt[4]{36} \times \sqrt[8]{1296}$.

75. $\sqrt[4]{64} \times \sqrt[3]{16}$.

76. $\sqrt[3]{20} \times \sqrt[9]{1000}$.

77. $\sqrt[3]{\frac{1}{2}} \times \sqrt[12]{256^3}$.

78. $\sqrt{15} \times \sqrt[6]{27 \times 125}$.

79. $\sqrt[5]{12} \times \sqrt[10]{96}$.

80. $3 \times \sqrt[3]{72}$.

81. $\sqrt[15]{27} \times \sqrt[4]{42}$.

82. $\sqrt[24]{256} \times \sqrt[5]{2}$.

83. $\sqrt[7]{7^5} \times \sqrt{5^6}$.

84. $\sqrt[6]{6^2} \times \sqrt[10]{42^7}$.

85. $\sqrt[5]{\frac{3}{8}} \times \sqrt[3]{\frac{2}{4}}$.

86. $\sqrt[4]{\frac{7}{9}} \times \sqrt[5]{\frac{15}{7}}$.

87. $\sqrt[8]{\frac{4}{81}} \times \sqrt[5]{\frac{27}{32}}$.

88. $\sqrt[9]{\frac{9}{28}} \times \sqrt[4]{\frac{2}{8}}$.

89. $\sqrt[10]{\frac{125}{16}} \times \sqrt[3]{\frac{4}{5}}$.

Ex. 46.

Division of Radicals.

Perform the following divisions:

1. $\sqrt{243} \div \sqrt{3}$.

2. $\sqrt[3]{81} \div \sqrt[3]{3}$.

3. $\sqrt[3]{56} \div \sqrt[3]{7}$.

4. $\sqrt[3]{500} \div \sqrt[3]{4}$.

5. $\sqrt[3]{1080} \div \sqrt[3]{5}$.

6. $\frac{\sqrt[3]{72}}{\sqrt[3]{9}}$.

7. $\sqrt[4]{80} \div \sqrt[4]{5}$.

8. $\sqrt[4]{243} \div \sqrt[4]{3}$.

9. $\sqrt[4]{4375} \div \sqrt[4]{7}$.

10. $\sqrt[4]{2048} \div \sqrt[4]{8}$.

11. $\sqrt{\frac{1}{2}} \div \sqrt{\frac{2}{9}}$.

12. $\sqrt{\frac{5}{6}} \div \sqrt{\frac{3}{5}}$.

13. $\sqrt{\frac{7}{12}} \div \sqrt{\frac{8}{7}}$.

14. $\sqrt{\frac{2}{9}} \div \sqrt{\frac{1}{2}}$.

15. $\sqrt{\frac{12}{35}} \div \sqrt{\frac{21}{5}}$.

16. $\sqrt{\frac{14}{15}} \div \sqrt{\frac{24}{35}}$.

17. $\sqrt{\frac{22}{27}} \div \sqrt{\frac{6}{11}}$.

18. $\sqrt{\frac{52}{35}} \div \sqrt{\frac{17}{65}}$.

19. $(3\sqrt{6} + 45\sqrt{2}) \div 3\sqrt{3}$.

20. $(42\sqrt{5} - 30\sqrt{3}) \div 2\sqrt{15}$.

21. $(84\sqrt{15} + 168\sqrt{6}) \div 3\sqrt{21}$.

22. $(15\sqrt{105} - 36\sqrt{10} + 30\sqrt{3}) \div 3\sqrt{15}$.

23. $(56\sqrt{30} - 84\sqrt{10} + 100\sqrt{14}) \div 4\sqrt{35}$.

24. $(30\sqrt[3]{4} - 36\sqrt[3]{10} + 30\sqrt[3]{90}) \div 3\sqrt[3]{20}$.

25. $(50\sqrt[3]{18} + 18\sqrt[3]{20} - 48\sqrt[3]{5}) \div 2\sqrt[3]{30}$.

26. $(2 - \sqrt{2}) \div (2 + \sqrt{2})$.

27. $\sqrt{3a} \div \sqrt{a}$.

28. $\sqrt{2x} \div \sqrt{2}$.

29. $\sqrt{10y} \div \sqrt{5y}$.

30. $\sqrt{12m} \div \sqrt{6}$.

31. $\sqrt{5b} \div \sqrt{2b}$.

32. $\sqrt{a^5} \div \sqrt{a^3}$.

33. $\sqrt{3a^7} \div \sqrt{a^3}$.

34. $\sqrt{8c^9} \div \sqrt{2c^5}$.

35. $\sqrt{\frac{a^3}{b^5}} \div \sqrt{\frac{a}{b^3}}$.

36. $\sqrt{\frac{3}{a^3}} \div \sqrt{\frac{6}{a}}$.

37. $\sqrt[3]{54a^7} \div \sqrt[3]{2a}$.

38. $\sqrt[3]{96n^8} \div \sqrt[3]{6n^5}$.

39. $\sqrt[3]{49x^6} \div \sqrt[3]{\frac{1}{7}x}$.

40. $\sqrt[3]{2y} \div \sqrt[3]{\frac{1}{4y^2}}$.

41. $\sqrt[3]{a^2x} \div \sqrt[3]{ax^5}$.

42. $\sqrt[4]{125a^7} \div \sqrt[4]{\frac{a^3}{5}}$.

43. $\sqrt[4]{\frac{27}{2}y} \div \sqrt[4]{\frac{8y^5}{3}}$.

44. $\sqrt[4]{\frac{54a^3}{y}} \div \sqrt[4]{\frac{8y^5}{9a}}$.

45. $\sqrt[4]{\frac{2}{5}m^3} \div \sqrt[4]{\frac{5m}{8}}$.

46. $(a - b) \div (\sqrt{a} - \sqrt{b})$.

47. $(1 - a) \div (1 - \sqrt{a})$.

48. $(b - 1) \div (\sqrt{b} - 1)$.

49. $1 \div \sqrt{\frac{x + 2y}{x^3 - 3xy^2 + 2y^3}}$.

50. $(acx^2\sqrt{y} - bcy\sqrt{x}) \div c\sqrt{xy}$.

51. $(acy^2\sqrt[3]{z^2} - adz\sqrt[3]{y}) \div a\sqrt[3]{yz^2}$.

52. $\sqrt{54} \div \sqrt[4]{36}$.

53. $\sqrt[3]{9} \div \sqrt{3}$.

54. $\sqrt[3]{49} \div \sqrt{7}$.

55. $\sqrt[3]{72} \div \sqrt{6}$.

56. $\sqrt[3]{12} \div \sqrt{6}$.

57. $\sqrt[4]{0.0324} \div \sqrt{0.5}$.

58. $\sqrt[6]{0.064} \div \sqrt{10}$.

59. $\sqrt[8]{10.4976} \div 2\sqrt{5}$.

60. $\sqrt{\frac{8}{45}} \div \sqrt[4]{6\frac{1}{4}}$.

61. $\sqrt{\frac{8}{5}} \div \sqrt[6]{3\frac{3}{8}}$.

62. $\sqrt[5]{\frac{4}{5}} \div 2\sqrt[10]{\frac{1}{400}}$.

63. $\sqrt{27} \div 3$.

64. $\sqrt{196} \div 2$.

65. $\sqrt{45} \div 3$.

66. $\sqrt{1452} \div 11$.

67. $6 \div \sqrt{24}$.

68. $8 \div \sqrt{72}$.

69. $7 \div \sqrt{63}$.

70. $30 \div \sqrt{210}$.

71. $6 \div (3\sqrt{3} - 5)$.

72. $4 \div (3\sqrt{2} - 4)$.

73. $\sqrt{a} \div \sqrt[3]{a^2}$.

74. $\sqrt[3]{b^2} \div \sqrt[5]{b^3}$.

75. $\sqrt[3]{m^5} \div \sqrt[4]{m^7}$.

76. $\sqrt{x^3} \div \sqrt[9]{x^8}$.

77. $\sqrt[3]{2x} \div \sqrt{x^3}$.

78. $\sqrt[5]{y^4} \div \sqrt[7]{2y^3}$.

79. $\sqrt[6]{\frac{z^9}{27}} \div \sqrt{6z^5}$.

80. $\sqrt[11]{a^5b^9} \div \sqrt[7]{a^4b^3}$.

81. $\sqrt[7]{\frac{a^2}{c^3}} \div \sqrt[13]{\frac{a^3}{c^8}}$.

82. $a^2 \div \sqrt{a}$.

83. $b^3 \div \sqrt{b}$.

84. $az^5 \div \sqrt{a^2z}$.

85. $3ny^4 \div \sqrt{6n^2y}$.

86. $\sqrt[3]{25m^4x} \div 5mx^2$.

87. $\sqrt[4]{8x^3y^3} \div 2x^2y^3$.

88. $\sqrt[6]{1600a^{22}x^{16}} \div 5a^2x^4$.

89. $(x+y) \div \frac{1}{3}\sqrt{x^2 - y^2}$.

90. $(3mn\sqrt[3]{n} + 4n^2\sqrt{m}) \div 6\sqrt[4]{mn^6}$.

Perform the following operations:

91. $(\sqrt[-2]{25})$.

92. $\sqrt[-3]{\frac{1}{64}}$.

93. $\sqrt[-6]{\frac{64}{15625}}$.

94. $\sqrt[-4]{36}$.

95. $\sqrt[1]{17}$.

96. $\sqrt[-1]{6}$.

97. $\sqrt[-1]{\frac{1}{4}}$.

98. $\sqrt[-5]{243}$.

99. $\sqrt[-2]{a}$.

100. $\sqrt[-3]{m^6}$.

101. $\sqrt[-1]{a}$.

102. $\sqrt[-6]{\frac{m^{14}}{n^{18}}}$.

103. $\sqrt[-4]{\frac{a^6}{b^{10}}}$.

104. $\sqrt[-2]{a^3} \times \sqrt{a^5}$.

105. $\sqrt[3]{m^5} \times \sqrt[-5]{m^3}$.

106. $\sqrt[-3]{x^5} \times \sqrt[-2]{x^7}$.

107. $\sqrt[-5]{\frac{1}{a^2}} \times \sqrt[-2]{a^3}$.

108. $\frac{\sqrt{a^3}}{\sqrt[-2]{a^5}}$.

109. $\frac{\sqrt[-3]{m^2}}{\sqrt[3]{m^5}}$.

110. $\frac{\sqrt{a}}{\sqrt[-3]{a}}$.

111. $\frac{\sqrt{x^5}}{\sqrt[-5]{x}}$.

112. $\frac{\sqrt[-3]{a^2}}{\sqrt[-5]{a^3}}$.

113. $\sqrt[\frac{1}{2}]{4}$.

114. $\sqrt[\frac{1}{3}]{12}$.

115. $\sqrt[\frac{1}{3}]{27}$.

Ex. 47.

Simplification of Radicals.

Perform the following operations:

1. $(\sqrt{a^3})^2$.
2. $(\sqrt[3]{m^2})^3$.
3. $(\sqrt[5]{n^3})^5$.
4. $(\sqrt[4]{x^3})^4$.
5. $(\sqrt[7]{y^3})^7$.
6. $(\sqrt[9]{ab^3})^9$.
7. $(\sqrt{m^3})^5$.
8. $(\sqrt[3]{n^2})^4$.
9. $(\sqrt[4]{y})^5$.
10. $(\sqrt[3]{b^2c})^4$.
11. $(\sqrt[8]{x^3z^5})^7$.
12. $(\sqrt[3]{2a^4})^2$.
13. $(\sqrt{\sqrt[3]{2\tfrac{1}{4}}})^9$.
14. $(\sqrt[3]{\sqrt{27a^6}})^7$.
15. $(\sqrt[m]{\sqrt[n]{a^{2mn}c^{mn}}})^n$.
16. $(\sqrt[5]{\sqrt[4]{16a^{17}}})^5$.
17. $(\sqrt[3]{ax^4})^6$.
18. $[\sqrt{(x+y)^3}]^4$.
19. $[\sqrt[5]{(x-y)^2}]^4$.
20. $[\sqrt[6]{(a^2-b^2)^4}]^3$.
21. $[\sqrt[8]{(1+a^2)^6}]^3$.
22. $[\sqrt[m]{(x^2-y^2)^{2m}}]^n$.
23. $(\sqrt[3]{a^2b}-\sqrt{m^3}+\sqrt[4]{ax^3})^2$.
24. $(\sqrt[x]{a^2}-\sqrt[y]{b})^2$.
25. $(\sqrt[x]{m}-\sqrt[y]{n}+\sqrt[z]{p^2})^2$.
26. $(-\sqrt{2a^3b})^2$.
27. $(-\sqrt[3]{xy^2})^3$.
28. $(-\sqrt[3]{x^2-y^2})^6$.
29. $(-\sqrt[3]{a^2-b^2})^9$.
30. $(-\sqrt{5})^2$.
31. $(-\sqrt[3]{4})^6$.
32. $(-\sqrt[4]{127})^4$.
33. $(-\sqrt[5]{131})^5$.
34. $(-\sqrt[3]{9})^4$.
35. $(-\sqrt[4]{8})^3$.
36. $(-\sqrt[7]{13})^{14}$.
37. $(-\sqrt{-\sqrt{6}})^4$.
38. $(\sqrt[3]{n})^4$.
39. $(\sqrt[3]{v^2})^3$.
40. $(\sqrt[5]{x^4})^{15}$.
41. $(\sqrt[4]{x^3})^4$.
42. $(\sqrt[3]{y^5})^{12}$.
43. $\sqrt{\sqrt[3]{a^3}}$.
44. $\sqrt[3]{\sqrt{b}}$.
45. $\sqrt[4]{\sqrt[3]{c}}$.
46. $\sqrt[5]{\sqrt[6]{2m}}$.
47. $\sqrt[4]{\sqrt{3a^3}}$.
48. $\sqrt[6]{\sqrt[3]{a^6}}$.
49. $\sqrt[5]{\sqrt[3]{m^5}}$.
50. $\sqrt[6]{\sqrt[7]{y^{14}}}$.
51. $\sqrt[5]{\sqrt[3]{a^{10}}}$.
52. $\sqrt[4]{\sqrt[3]{m^3}}$.
53. $\sqrt[3]{\sqrt{125}}$.
54. $\sqrt[4]{\sqrt[3]{729}}$.
55. $\sqrt[4]{\sqrt{(x-y)^8}}$.

56. $\sqrt[3]{\sqrt[6]{(a-b)^{36}}}$. 59. $\sqrt{\sqrt[5]{(1-x^2)^{60}}}$. 62. $\sqrt[11]{\sqrt[3]{m^{33}n^{22}}}$.

57. $\sqrt[5]{\sqrt[3]{(m+n)^{30}}}$. 60. $\sqrt[9]{\sqrt[3]{512x^{18}}}$. 63. $\sqrt[13]{6\sqrt[5]{m^{65}}}$.

58. $\sqrt[7]{\sqrt[3]{(3a-2b)^{14}}}$. 61. $\sqrt[3]{\sqrt[5]{32a^{45}}}$. 64. $\sqrt[7]{128\sqrt[5]{243\,a^{70}}}$.

Ex. 48.

RATIONALIZATION OF THE DENOMINATORS OF RADICALS.

In the following expressions make the denominators rational:

1. $\dfrac{2\sqrt{2}}{3\sqrt{2}-2\sqrt{3}}$. 7. $\dfrac{\sqrt{11}+\sqrt{5}}{\sqrt{11}-\sqrt{5}}$. 13. $\dfrac{7-3\sqrt{10}}{5+4\sqrt{5}}$.

2. $\dfrac{21\sqrt{3}}{4\sqrt{3}-3\sqrt{2}}$. 8. $\dfrac{\sqrt{13}-\sqrt{10}}{\sqrt{13}+\sqrt{10}}$. 14. $\dfrac{2+2\sqrt{2}}{6+5\sqrt{6}}$.

3. $\dfrac{120\sqrt{2}}{5\sqrt{6}+3\sqrt{10}}$. 9. $\dfrac{\sqrt{8}+\sqrt{7}}{\sqrt{7}-\sqrt{2}}$. 15. $\dfrac{7+3\sqrt{7}}{12-6\sqrt{11}}$.

4. $\dfrac{75\sqrt{14}}{8\sqrt{2}+2\sqrt{7}}$. 10. $\dfrac{\sqrt{6}+\sqrt{10}}{\sqrt{10}-\sqrt{6}}$. 16. $\dfrac{18}{4+3\sqrt{2}+5\sqrt{3}}$.

5. $\dfrac{1200\sqrt{10}}{12\sqrt{5}-4\sqrt{7\frac{1}{2}}}$. 11. $\dfrac{3+5\sqrt{3}}{4-3\sqrt{2}}$. 17. $\dfrac{1-\sqrt{2}+\sqrt{3}}{1+\sqrt{2}-\sqrt{3}}$.

6. $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$. 12. $\dfrac{1-4\sqrt{5}}{6+2\sqrt{3}}$. 18. $\dfrac{2+\sqrt{6}-\sqrt{2}}{2-\sqrt{6}+\sqrt{2}}$.

19. $\dfrac{3-\sqrt{5}-\sqrt{2}}{3+\sqrt{5}+\sqrt{2}}$.

20. $\dfrac{7-2\sqrt{3}+3\sqrt{2}}{3+3\sqrt{3}-2\sqrt{2}}$. 21. $\dfrac{2\sqrt{3}-3\sqrt{6}}{\sqrt{2}-\sqrt{3}+\sqrt{5}-\sqrt{6}}$.

Ex. 49.

Imaginary Expressions.

$\sqrt{-1} = i.$ $\quad i^{4n+1} = \sqrt{-1}.$

$(\sqrt{-1})^2 = i^2 = -1.$ $\quad i^{4n+2} = -1.$

$(\sqrt{-1})^3 = i^3 = -\sqrt{-1}.$ $\quad i^{4n+3} = -\sqrt{-1}.$

$(\sqrt{-1})^4 = i^4 = +1.$ $\quad i^{4n+4} = +1.$

Place the following quantities in the form $a\sqrt{-1}$.

1. $\sqrt{-4}$.
2. $\sqrt{-25}$.
3. $\sqrt{-81}$.
4. $\sqrt{-144}$.
5. $\sqrt{-a^2}$.
6. $\sqrt{-b^4}$.
7. $\sqrt[4]{-16}$.
8. $\sqrt[4]{-81}$.
9. $\sqrt[4]{-625}$.
10. $\sqrt[6]{-64}$.
11. $\sqrt[6]{-729}$.
12. $\sqrt[8]{-256}$.
13. $\sqrt[4]{-x^8}$.
14. $\sqrt[4]{-y^9}$.
15. $\sqrt[6]{-z^{18}}$.
16. $\sqrt[10]{-u^{20}}$.
17. $\sqrt{-\frac{1}{4}}$.
18. $\sqrt{-\frac{a^4}{b^2}}$.
19. $\sqrt{-x^2-y^2}$.
20. $\sqrt[4]{-(x^3+y^3)^2}$.
21. $\sqrt[6]{-(3x-2y^2)^{12}}$.
22. $\sqrt{-9x^6}$.
23. $\sqrt{-81m^6}$.

What is the value of:

24. $(\sqrt{-1})^6$; $(\sqrt{-1})^8$; $(\sqrt{-1})^{36}$; $(\sqrt{-1})^{16}$; $(\sqrt{-1})^{21}$; $(\sqrt{-1})^{27}$; $(\sqrt{-1})^{51}$?
25. i^7; i^{11}; i^{28}; i^{32}; i^{14}; i^{13}; i^{38}?

Add:

26. $\sqrt{-25}+\sqrt{-49}+\sqrt{-121}-\sqrt{-64}+\sqrt{-1}-\sqrt{-36}$.
27. $\sqrt{-a^4}+\sqrt{-a^2}-\sqrt{-4a^4}+\sqrt{-16a^4}-\sqrt{-81a^2}+\sqrt{-a^2}$.
28. $\sqrt{-(x-y)^2}$; $\sqrt{-(x^2-2xy+y^2)}$; $+\sqrt{-64x^2y^2}$.
29. $\sqrt{-25a^6}$; $\sqrt{-16a^6}$; $-\sqrt{-(a-1)^2a^6}$.
30. $\sqrt{-(m-n)^2}$; $-\sqrt{-(n-2m)^2}$; $\sqrt{-4m^2n^2}$; $-\sqrt{-4(m-n)^2}$.

31. $5+\sqrt{-16}$; $3-\sqrt{-4}$; $8+\sqrt{-4}$; $-(7+\sqrt{-36})$; $2-\sqrt{-1}$.

32. $a+bi$; $2a+3bi$; $\frac{1}{2}a-4bi$; $5a-\frac{1}{2}bi$; $4a-16bi$.

33. $3+2i$; $4-2i$; $7+3i$; $8-i$; $4+2bi$.

Multiply:

34. $\sqrt{-a}\times\sqrt{-a}$.

35. $\sqrt{-16}\times\sqrt{-9}$.

36. $\sqrt{-m}\times\sqrt{-n}$.

37. $\sqrt{-y}\times\sqrt{-1}$.

38. $\sqrt{25}\times\sqrt{-49}$.

39. $\sqrt{a^6}\times\sqrt{-a^4}$.

40. $\sqrt{-9y^2}\times\sqrt{-16z^6}$.

41. $\sqrt{-m^2}\times(-\sqrt{-9m^4})$.

42. $i\sqrt{-z^2}$.

43. $i\sqrt{-36}\times\sqrt{25}$.

44. $i\sqrt{-28}\times i\sqrt{-32}$.

45. $i^2\sqrt{-49}\times\sqrt{-121}$.

46. $\sqrt{-(a+b)}\times\sqrt{-(a-b)}$.

47. $\sqrt{x-y}\times\sqrt{y-x}$.

48. $\sqrt{y-5}\times\sqrt{5-y}$.

49. $i\sqrt{-9}\times\sqrt{2-y}$.

50. $(a+\sqrt{-1})(a-\sqrt{-1})$.

51. $(m+bi)(m-bi)$.

52. $(3+5i)(4-7i)$.

53. $(x+2i)(y-bi)$.

54. $(3x-8i)(2y+3i)$.

55. $(2a+3bi)(c-di)$.

56. $(4-i\sqrt{8})(3+i\sqrt{2})$.

57. $(m-3i\sqrt{b})(n+4i\sqrt{c})$.

58. $(\sqrt{5}+i\sqrt{6})(\sqrt{6}-i\sqrt{8})$.

Perform the following divisions:

59. $\frac{a}{\sqrt{-1}}$.

60. $\frac{b}{\sqrt{-b^2}}$.

61. $\frac{d}{\sqrt{-4}}$.

62. $\frac{\sqrt{-9}}{\sqrt{-3}}$.

63. $\frac{\sqrt{-m}}{\sqrt{-n}}$.

64. $\frac{\sqrt{-ax}}{\sqrt{-x}}$.

65. $\frac{x}{\sqrt{-x}}$.

66. $\frac{z}{\sqrt{-z^2}}$.

67. $\frac{m}{i\sqrt{m}}$.

68. $\frac{\sqrt{-x^3}}{\sqrt{-x}}$.

69. $\frac{\sqrt{-8}}{\sqrt{-2}}$.

70. $\frac{\sqrt{-10m^3}}{\sqrt{-5m}}$.

Ex. 50.

Radical Equations.

Solve:

1. $2\sqrt{x+5}=\sqrt{28}$.
2. $3\sqrt{4x-8}=\sqrt{13x-3}$.
3. $\sqrt{x+9}=5\sqrt{x-3}$.
4. $4=2\sqrt{x}-3$.
5. $5-\sqrt{3y}=4$.
6. $9+4\sqrt{x}=11$.
7. $6-3\sqrt{x}=4$.
8. $7+2\sqrt[3]{3x}=5$.
9. $11-4\sqrt{5x}=9$.
10. $6+\sqrt{8x}=2$.
11. $\sqrt[3]{2x-3}=-3$.
12. $\sqrt{8y+17}=4$.
13. $\sqrt[3]{3x+7}=3$.
14. $\sqrt[5]{80x-43}=-3$.
15. $14+\sqrt[3]{4x-40}=10$.
16. $\sqrt{2x+8}=\sqrt{5x-2}$.
17. $\sqrt[3]{10y-4}=\sqrt[3]{7y+11}$.
18. $\sqrt[5]{2x-3}=\sqrt[5]{3x+2}$.
19. $2x-8=\sqrt{4x^2-12x+32}$.
20. $\sqrt[3]{3y+2}=\sqrt[6]{9y^2+10y-6}$.
21. $\sqrt{x-1}=\sqrt[8]{(x-1)^2(x+3)^3}$.
22. $\sqrt{(2x-5)}=\sqrt[6]{\frac{1}{8}(2x-5)^4}$.
23. $\sqrt{(3x-4)}=\sqrt[6]{(3x-4)^3(9x-6)}$.
24. $2\sqrt{y-2}=\sqrt[4]{32(y-2)^3}$.
25. $\sqrt[4]{9x^4+6x^3+5x^2-2x}=\sqrt{3x^2+x}$.
26. $\sqrt[6]{8x^3-60x^2+145x-115}=\sqrt{2x-5}$.
27. $\sqrt[5]{(3x^3-5x+2)(2x+4)}=\sqrt[6]{(3x^3-5x+2)^2(4x^2-5x+1)}$.
28. $\sqrt{(5x^5-x^4+2)(5x+3)}=\sqrt[4]{(5x^5-x^4+2)^2(25x^2+25x-6)}$.
29. $3\sqrt{16x+9}=12\sqrt{4x}-9$.
30. $\sqrt{\frac{15}{4}+x}=\frac{3}{2}+\sqrt{x}$.
31. $\sqrt{32+x}=16-\sqrt{x}$.

32. $\sqrt{x}-\sqrt{x-5}=\sqrt{5}$.

33. $\sqrt{x-7}=\sqrt{x-14}+1$.

34. $\sqrt{x-7}=\sqrt{x+1}-2$.

35. $\sqrt{22-x}-2=\sqrt{10-x}$.

36. $\sqrt{x+20}-\sqrt{x-1}-3=0$.

37. $\sqrt{x+15}-7=7-\sqrt{x-13}$.

38. $x=7-\sqrt{x^2-7}$.

39. $\dfrac{2x^2-x+2}{\sqrt{x(2x-1)+2}}=\dfrac{\sqrt{2(x^2+1)-x}}{\sqrt{5x-19}}$.

40. $\dfrac{2(5x^3-22x^2+11x-12)}{3\sqrt{5x^2-2x+3}}=3\sqrt{5x^2-2x+3}$.

41. $\dfrac{24\sqrt[3]{2x+136}+13}{15+4\sqrt[3]{2x+136}}=-5$.

42. $\dfrac{1}{2}-\dfrac{3}{x}=\sqrt{\dfrac{1}{4}-\dfrac{1}{x}\sqrt{9-\dfrac{36}{x}}}$.

43. $\dfrac{\sqrt{x}-3}{\sqrt{x}+3}=\dfrac{\sqrt{x}+1}{\sqrt{x}-2}$.

44. $(2-x)^{\frac{1}{2}}=(4-5x-3x^2)^{\frac{1}{2}}$.

45. $y^{\frac{1}{2}}-y^{-\frac{1}{2}}=(y+1)^{\frac{1}{2}}$.

46. $x^{\frac{1}{2}}+(x-9)^{\frac{1}{2}}=36(x-9)^{-\frac{1}{2}}$.

47. $\dfrac{1+(1-x)^{\frac{1}{2}}}{1-(1-x)^{\frac{1}{2}}}=3$.

48. $[(3+x^2)x^{-2}]^{\frac{1}{2}}=[\frac{1}{3}(9x^{-3}+x^{-2})]^{\frac{1}{2}}$.

49. $\sqrt{2x}+(2x)^{-\frac{1}{2}}(1+x)=2(2x)^{-\frac{1}{2}}$.

50. $\left(\dfrac{y}{5}+4\right)^{\frac{1}{2}}-\left(\dfrac{y}{5}-4\right)^{\frac{1}{2}}=\left(\dfrac{4y-2}{5}\right)^{\frac{1}{2}}$.

51. $(12+x)^{\frac{1}{2}}=x^{\frac{1}{2}}+6$.

52. $(x-3)^{\frac{1}{2}}+x^{\frac{1}{2}}=\dfrac{3}{(x-3)^{\frac{1}{2}}}$.

53. $x^{\frac{1}{2}}-\left(\dfrac{a}{x}\right)^{\frac{1}{2}}=(a+x)^{\frac{1}{2}}$.

54. $\dfrac{2}{x+\sqrt{2-x^2}}+\dfrac{2}{x-\sqrt{2-x^2}}=x.$

55. $\dfrac{\sqrt{1-z}}{2-\sqrt{1+z}}=\dfrac{\sqrt{1+z}}{2+\sqrt{1-z}}.$

56. $\dfrac{1}{1+\sqrt{1-x}}+\dfrac{1}{1-\sqrt{1-x}}=\dfrac{2x}{9}.$

57. $\sqrt{2x-3}=x-3.$

58. $\sqrt{2x-5}+6=x+2.$

59. $\sqrt{2x+1}+2\sqrt{x}=\dfrac{21}{\sqrt{2x+1}}.$

60. $\dfrac{\sqrt{x}}{21-\sqrt{x}}+\dfrac{21-\sqrt{x}}{\sqrt{x}}=2\tfrac{1}{2}.$

61. $\sqrt{3x^2-6x+\tfrac{8}{3}}=\sqrt{5x-2x^2-2\tfrac{2}{3}}.$

62. $\sqrt{(x-1)(3x-6)}=x-2.$

63. $\dfrac{\sqrt{2x-1}}{\sqrt{x^2-9}}=\dfrac{3}{\sqrt{x+11}}.$

64. $\dfrac{\sqrt{2x-3}}{3}+\dfrac{6}{\sqrt{2x-3}}=\dfrac{\sqrt{x-6}}{\sqrt{2x-3}}+\sqrt{2x-3}.$

65. $\sqrt{2(x-1)}=\sqrt[4]{3x^2-x+4}.$

66. $\sqrt[6]{8x^3+150x^2+31x+3}=\sqrt{2x+5}.$

67. $\dfrac{\sqrt{10+x}}{\sqrt{10-x}}=\dfrac{\sqrt{x-2}}{\sqrt{10-x}}+1.$

68. $\sqrt{2(x-3)}=\sqrt[4]{x^2-4x+11}.$

69. $\sqrt[6]{(3x+2)(5x-2)}=\sqrt[3]{2(x+2)}.$

70. $\sqrt{2x+7}=\sqrt{x}+2.$

71. $\sqrt{5x-4}=\sqrt{2x+1}+1.$

72. $(3x-1)^{\frac{1}{2}}-\dfrac{5}{(x+1)^{\frac{1}{2}}}=\dfrac{4\sqrt{2}-5}{(x+1)^{\frac{1}{2}}}$

73. $(x+4)^{\frac{1}{2}}-(5x)^{\frac{1}{2}}=-(x-1)^{\frac{1}{2}}.$

74. $2(3x-\frac{1}{2})^{\frac{1}{2}}+4(3x-\frac{1}{2})^{-\frac{1}{2}}=3(3x+\frac{5}{2})^{\frac{1}{2}}.$

75. $\sqrt{x+17}+\sqrt{x-4}=\frac{7}{4}\sqrt{2x}.$

76. $\sqrt{2x+1}+\sqrt{x-3}=2\sqrt{x}.$

77. $\sqrt{x+4}-\sqrt{x}=\sqrt{x+\frac{8}{2}}.$

78. $\sqrt{2x-\frac{1}{4}}+\sqrt{x+1}=3.$

79. $\sqrt{12+x}=\sqrt{7x+8}-2.$

80. $(x^3+8x^2+16x-1)^{\frac{1}{3}}-x=3.$

81. $(x^4-4x^3+6x^2+15x+9)^{\frac{1}{4}}=(x^2-2x+6)^{\frac{1}{2}}.$

82. $\dfrac{\sqrt{x^2+x+3}}{\sqrt{2x^2+5x-2}}=\dfrac{3}{4}.$

83. $\dfrac{\sqrt{3x^2+x+5}}{\sqrt{4x^2-x+1}}=\dfrac{3}{2}.$

84. $\dfrac{(2-3x-x^2)^{\frac{1}{2}}}{(8-5x-2x^2)^{\frac{1}{2}}}=\dfrac{4}{5}.$

85. $\dfrac{\sqrt{x^2-6-x}}{7(43+x-x^2)^{\frac{1}{2}}}=\dfrac{6}{7}.$

86. $\left(\dfrac{8x^2+12x+1}{4x^2+2x-1}\right)^{-\frac{1}{2}}=\dfrac{1}{3}.$

87. $\dfrac{(z^2+2z+1)^{\frac{1}{2}}}{(3z^2+4z+2)^{\frac{1}{2}}}=\dfrac{2}{3}.$

88. $\left(\dfrac{\sqrt{x^2-3x+1}}{\sqrt{3x^2+x-50}}\right)^{-1}=\dfrac{[3(x+5)]^{\frac{1}{2}}}{(x+6)^{\frac{1}{2}}}.$

89. $\dfrac{4}{x+\sqrt{4-x^2}}+\dfrac{4}{x-\sqrt{4-x^2}}=\dfrac{12}{7}.$

90. $\dfrac{5}{x+\sqrt{5+x^2}}-\dfrac{5}{x-\sqrt{5+x^2}}=6.$

91. $\dfrac{1}{1-\sqrt{1-x^2}}-\dfrac{1}{1+\sqrt{1-x^2}}=\dfrac{\sqrt{3}}{x^2}.$

92. $\dfrac{5x-1}{\sqrt{5x}+1}=1+\dfrac{\sqrt{5x}-1}{2}$

93. $\dfrac{4x-1}{\sqrt{2x+1}}+3\sqrt{2x+1}=7\sqrt{x}.$

94. $\dfrac{\sqrt{3x^2+4}-\sqrt{2x^2+1}}{\sqrt{3x^2+4}+\sqrt{2x^2+1}}=\dfrac{1}{7}$

95. $\dfrac{\sqrt{27x^2+4}+\sqrt{9x^2+5}}{\sqrt{27x^2+4}-\sqrt{9x^2+5}}=7.$

96. $\dfrac{\sqrt{5x-4}+\sqrt{5-x}}{\sqrt{5x-4}-\sqrt{5-x}}=\dfrac{\sqrt{4x}+1}{\sqrt{4x}-1}$

97. $\sqrt{ax^2-bx-b}=\sqrt{-ax}.$

98. $\dfrac{\sqrt{x+a}}{\sqrt{3x^2-5ax+\dfrac{4a^2}{3}+1}}=\dfrac{\sqrt{x^2+3ax+2a^2}}{\sqrt{x+2a}}.$

99. $\dfrac{\sqrt{ax+b}+\sqrt{ax}}{\sqrt{ax+b}-\sqrt{ax}}=\dfrac{1+\sqrt{ax-b}}{1-\sqrt{ax-b}}.$

100. $\dfrac{4x}{a-b}=\dfrac{1}{\sqrt{a}-x}.$

101. $\sqrt{a+x}-\sqrt{a-x}=\sqrt{a}.$

102. $\sqrt{6x^2+5a^2+2ax}=a+3x.$

103. $\sqrt{10ax-x^2+25a^2}=x+3a.$

104. $\sqrt{\dfrac{(3x+2b)x+\frac{2}{3}(\frac{1}{2}b^2-2ab)}{3}}=\dfrac{a-b}{3}.$

119. $\frac{a-x}{\sqrt{a-x}}+\frac{x-b}{\sqrt{x-b}}=\sqrt{a-b}.$

120. $\sqrt{\frac{a-x}{b+x}}-\sqrt{\frac{b+x}{a-x}}=c.$

121. $\sqrt{\frac{a-x}{b-x}}+\sqrt{\frac{b-x}{a-x}}=c.$

122. $\sqrt{\frac{a-x}{b-x}}-\sqrt{\frac{b-x}{a-x}}=c.$

123. $\frac{9a}{2}+\frac{2x}{3}=\sqrt{72a^2+\frac{x^2}{4}}-\frac{a}{2}.$

Ex. 51.

SOLUTION OF PROBLEMS BY LOGARITHMS.

Find by logarithms the amounts at compound interest of:

1. \$2640 at 5 %, for 8 years.
2. \$7800 at 4½ %, for 13 years.
3. \$2315 at 2½ %, for 30 years.
4. \$5240 at 3¼ %, for 60 years.
5. \$745.20 at 5¼ %, for 12 years.
6. \$500 at 5 %, for 200 years.
7. \$1 at 6 %, for 150 years.
8. \$.20 at 4¼ %, for 70 years.
9. \$.05 at 3¾ %, for 400 years.
10. \$3000 at 3 %, for 21 years and 134 days.
11. \$800 at 1½ %, for 91 years, 219 days.
12. \$415 at 4½ %, for 8 years, 7½ months.

13. What will $600 become at 5%, for 18 years, the interest being compounded every six months?

14. What is $1600 worth at the end of 10 years at 4%, the interest being compounded: (i.) every year? (ii.) every six months? (iii.) every three months? (iv.) every month?

Find what capital must be put at compound interest to produce:

15. $13,823.13 in 13 years, at 4½%.

16. $1927.65 in 10 years, at 3½%.

17. $6411.86 in 20 years, at 2%.

18. $1600 in 15 years, at 3½%.

19. $2000 in 18 years, at 5%.

20. What is the actual value of a sum of $2400, payable in 9 years, and discounted at compound interest at 4½%?

Find during what time it is necessary to place, at compound interest:

21. $620 at 2½%, to produce $755.41.

22. $2412 at 4½%, to produce $3914.32.

23. $1200 at 4%, to produce $1600.

24. $1400 at 5%, to produce $20,000.

25. In what time will a sum, placed at compound interest, double itself: (i.) at 3%? (ii.) at 3½%? (iii.) at 4%? (iv.) at 5%?

Find at what rate it is necessary to lend, at compound interest:

26. $1800, to produce $12,791.68 in 50 years.

27. $918, to produce $1757.92 in 14 years.

28. $2500, to produce $3715.24 in 9 years.

29. $856.40, to produce $1546.75 in 20 years.

30. $1, to produce $18.42 in 50 years.

31. At the birth of a child, there is placed in a bank, which pays compound interest at $4\frac{1}{2}\%$, a sum of $500. How much money will he receive at the age of 21 years?

32. A man, who has to pay a sum of $4520 on June 30, 1880, pays his debt March 31, 1877, getting a compound discount of $4\frac{1}{2}\%$. How much does he pay?

33. A usurer pays $1000 for a note of $1800, payable in three years. What rate of compound interest does the usurer receive?

34. If a bank pays to its depositors 4%, and lends the money at $4\frac{3}{4}\%$, how much will the bank gain in 10 years upon $1000, reckoning compound interest in both cases?

35. The population of a certain city has increased in 24 years from 32,500 to 66,066 people. How much per cent has it increased each year?

36. A cask contains 140 gallons of wine. A person draws out 4 gallons, which he replaces by water. The same operation is repeated 24 times, taking care to mix each time the water equally with the wine. How much wine remains in the cask?

37. A forest contains to-day 120,000^{cbm} of wood. How much will it contain in 15 years, if the annual increase is $3\frac{1}{4}\%$?

38. When Jacob arrived in Egypt, his family was composed of 70 persons; 430 years after, when the Israelites went out, they numbered 660,000. How much per cent was the increase each year, if from 1000 persons 25 died annually?

39. What is the principal which, at the end of 8 years, at 4 %, is worth as much as $2700, at 3 %, in 12 years?

40. Two sums of money, of which the second is worth $1420 more than the first, are placed at compound interest, the first at 4 %, the second at 5 %. At the end of 16 years they are worth, together, $211,084. What are these sums of money?

41. Two sums of money, one being $393 greater than the other, are placed at compound interest, the smaller at 5¼ %, the greater at 3¼ %. What are these sums of money, knowing that, at the end of 40 years, the smaller is double the other?

42. A man lends at the same time at compound interest $38,000 at 4½ %, and $99,398 at 3½ %. In what time will the two sums be equal?

43. A house is for sale. A offers $30,000 cash; B, $35,000, payable in 3 years; C, $33,000, in three equal sums, payable at the commencement of each year, from to-day. Which is the most advantageous offer, and how much?

If, at the end of each year, commencing in a year, there is placed during n years a sum s, at compound interest, the value S of all these annuities and of their interests is given by the formula

$$S = \frac{s}{r}[(1+r)^n - 1].$$

If to-day there is placed a capital C, and each year, to commence in a year, s dollars are added to this capital, at the end of n years the value of S is given by the formula

$$S = C(1+r)^n + \frac{s}{r}[(1+r)^n - 1].$$

It follows that, to pay in n years, by an annuity s, commencing in one year, a capital C borrowed to-day, we have

$$C(1+r)^n = \frac{s}{r}[(1+r)^n - 1].$$

The actual value A, which must be given to-day to buy up an annuity of s dollars, is given by the following formulas. If the annuity commences:

1. In one year to be perpetual,

$$A = \frac{s}{r}.$$

2. In $(m+1)$ years to be perpetual,

$$A = \frac{s}{r(1+r)^m}.$$

3. In one year to cease in m years,

$$A = \frac{s}{r} - \frac{s}{r(1+r)^m} = \frac{s[(1+r)^m - 1]}{r(1+r)^m}.$$

4. In $(m+1)$ years and ceases p years later,

$$A = \frac{s}{r(1+r)^m} - \frac{s}{r(1+r)^{m+p}} = \frac{s[(1+r)^p - 1]}{r(1+r)^{m+p}}.$$

44. A smoker, who commenced to smoke at the age of 17, spends \$1.80 per week. If, renouncing this habit, he had, at the end of his eighteenth year, placed in a bank at 4 % the sum that he had spent for tobacco, and, if he had done the same at the end of each year, how much would he have possessed at the age of 60 years?

45. During how many years must a sum of \$250 be placed at 4½ %, in order that the principal and interest may amount to \$8195.76?

46. An engineer who, each year, has taken $500 from his salary in order to place it at compound interest at 5 % a year, finds himself, at the age of 54 years, a possessor of $34,880.37. At what age did he commence his deposits?

47. What is the total value, after the twenty-sixth deposit, of an annuity of $1400, deposited half June 30, and half December 31, the interest at 4 % being compounded every six months?

48. A man has placed in a bank, December 31, 1874, a sum of $2800, at compound interest at 4 %. At the end of December, in each of the following years, he adds to the capital $450. How much will he possess at the end of December, 1887?

49. In a city of 800,000 people, where the immigration brings each year 2000 persons, the population, both the old and the new, increases each year 2 %. What will be the population of the city in 15 years?

50. A father died, leaving a fortune of $80,000, which produces 4½ %. In order to bring up the children, an annual sum of $5000 must be spent for 8 years. What will remain of the property at the end of this time?

51. A stockholder, who possesses a capital of $120,000, spends each year $6800 for his support. The interest being at 4½ %, how much will he have left in 20 years?

52. A forest contains 80,000 cords of wood, and increases 3 % per year. How many cords of wood can be cut each year, so that in 30 years the forest will contain 100,000 cords?

53. I borrow \$18,000 at 4½ %, and pay each year 10 % of this sum. In how many years will my debt be paid?

54. A man inherits a fortune of \$120,000, producing interest at 5½ %. He spends, besides what he earns, \$9500 per year. In how many years will he have spent his fortune?

55. A man borrows, at 5 %, a sum of \$50,000, which he places in business, producing \$6000 per year. Using all the proceeds for the payment of the interest and the principal, in how many years will the debt be paid?

56. How much must be given to-day to redeem a perpetual annuity of \$500, which is to commence in a year, if the interest is at 4 %?

57. What is the actual value of a perpetual annuity of \$1200, which is to commence in 12 years, if the interest is at 4½ %?

58. A sum of \$18,324.46, due to-day, is to be converted into a perpetual annuity of \$2000. In how many years ought the first payment to be made, if the interest is at 5 %?

59. An annual rent of \$2800, which must be paid 14 years more, is changed to another annual rent payable during 20 years. To how much will the last be raised, the interest being reckoned at 4½ %?

60. How much must be given to-day to redeem, at 5 %, an annuity of \$9500, which commences in 10 years, and lasts 40 years?

61. In order to redeem, at 6 %, an annuity which commences in 25 years, and lasts 50 years, \$19,464.20 are given. How much is the annuity?

62. A man pays \$10,200 to redeem a perpetual annuity of \$459, which commences in 1 year. At what rate is the interest reckoned?

Ex. 52.

Ratio and Proportion.

1. An express-train goes from A to B in $2\frac{1}{4}$ hours; a coach accomplishes the same distance in 9 hours. What is the ratio of the two velocities?

2. A hall is 39^m long and 26^m wide. Express the ratio of the length to the width.

3. The diameter of a column of the Tuscan order is 1.17^m and its height is 8.19^m. Express the ratio of the diameter to the height.

4. The diameter of a column of the Corinthian order is $2\frac{7}{12}$ feet and its height is $25\frac{5}{6}$ feet. What is the ratio of the diameter to the height?

5. The height of a column of the Ionian order is $24\frac{3}{4}$ feet and its diameter is $2\frac{3}{4}$ feet. Find the ratio of the height to the diameter.

6. The diameter of a column being $1\frac{5}{6}$ feet, the height in the Tuscan order will be $12\frac{5}{6}$ feet, in the Dorian order $14\frac{7}{8}$ feet, in the Ionian order $16\frac{1}{2}$ feet, and in the Corinthian order $18\frac{1}{3}$ feet. What are the ratios of these heights, the diameters being the same?

7. If two workmen, A and B, are paid in proportion to their activity, in what ratio will be their salaries if to do the same work A employs 18 days and B 22 days?

8. A garrison has food for 260 days; it receives orders to hold out 60 days more with the same provisions. What is the ratio of the new ration to the old?

9. Two pedestrians walk 75 yards and 90 yards per minute. In what ratio will be the time that they each use to accomplish the same distance?

10. The lengths of two rectangular fields are to each other as 2 : 3; the widths are in the ratio of 5 : 6. What is the ratio of their surfaces?

11. The volumes of two stones are to each other as 3 : 4; the weights of equal volumes are as 2 : 5. What is the ratio of the weight of the first stone to that of the second?

12. The surfaces of two circles are as the squares of their radii. Find the ratio of the surfaces of two circles, the first having a radius of 18 feet, and the other a radius of 24 feet.

13. The volumes of two spheres are to each other as the cubes of their radii. In what ratio are the volumes of two spheres having radii of 8 feet and 12 feet?

14. Of two moving bodies one accomplishes m feet in t seconds, the other n feet in p seconds. What is the ratio of their velocities? What is the ratio if $m = 35$, $t = 9$, $n = 25$, $p = 6$?

Form a proportion with the numbers:

15. 75, 18, 27, 50.

16. 969, 663, 285, and 195.

Find x in:

17. $27 : 90 = 45 : x$.

18. $10 : 56 = x : 21$.

19. $3.15 : x = 6.75 : 20$.

20. $x : 3\frac{3}{4} = 4\frac{1}{2} : 11\frac{1}{4}$.

21. $3\frac{3}{4} : 5\frac{2}{3} = 7\frac{1}{2} : x.$

22. $5\frac{2}{5} : 3\frac{3}{4} = x : 7\frac{2}{3}.$

23. $\frac{4}{7} = \frac{8}{x}.$

24. $\frac{3}{7} = \frac{x}{42}.$

25. $\frac{24\frac{2}{3}}{15\frac{1}{2}} = \frac{2x}{7\frac{3}{4}}.$

26. $\frac{x}{1.5} = \frac{1\frac{5}{7}}{1.8}.$

27. $\frac{3}{4}mn : \frac{5}{6}mp = x : 3\frac{1}{5}p.$

28. $\frac{10}{3}h^2q : \frac{5}{6}ht = \frac{4}{3}hq^2 : x.$

29. $\frac{221\,a^2b}{51\,pq} = \frac{26\,ab^2}{x}.$

30. $\frac{17(m+n)}{x} = \frac{51(m^2-n^2)}{13(m-n)}.$

31. $\frac{3a}{5b} : \frac{12a}{7c} = \frac{14c}{15b} : x.$

32. $\frac{15(a+b)}{14(a-b)} : \frac{10(a+b)^2}{21(a-b)^2} = \frac{a-b}{a+b} : x.$

33. $\left(\frac{m^3-n^3}{m-n} - mn\right) : \left(\frac{m^3+n^3}{m+n} + mn\right) = 1 : x.$

34. $\left(\frac{p^3-q^3}{p-q} - pq\right) : (p+q)^2 = \left(\frac{p^3+q^3}{p+q} + pq\right) : x.$

35. $\frac{1}{m-n} : \frac{m^2-mn+n^2}{m^3-3mn(m-n)-n^3} = \frac{m^3-n^3}{m^3+n^3} : x.$

36. $\frac{a^2-b^2}{ac+c^2} : \frac{a+b}{a^2-c^2} = x : a + \frac{ac}{a-c}.$

37. $\frac{b(1+b)}{1-a^2} : \frac{1-b^2}{a(1+a)} = \left(b + \frac{ab}{1-a}\right) : x.$

38. $\frac{m^2-4m+4}{m^2-4m+3} : x = \frac{m^2-5m+6}{m^2-2m+1} : \frac{m^2-6m+9}{m^2-3m+2}.$

Find a fourth proportional to:

39. 5, 6, 10.

40. 8, 7, 5.

41. $3\frac{3}{4}$, $7\frac{1}{2}$, $22\frac{2}{3}$.

42. m, n, p.

43. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$.

44. $\frac{m-n}{m+n}, \frac{(m-n)^2}{(m+n)^2}, \frac{m}{n}$.

45. $\frac{1+n}{1-m^2}$, $\frac{1-n^2}{m+m^2}$, $1+\frac{m}{1-m}$. 46. $\frac{a^2-b^2}{a^2+b^2}$, $1+\frac{a}{b}$, $1-\frac{b}{a}$.

Find a third proportional to:

47. 9, 6.

48. 16, 12.

49. 9, 15.

50. 8, 9.

51. m, n.

52. $\frac{p^2-q^2}{r}$, $\frac{p-q}{r}$.

53. $\frac{p}{(p+m)^2}$, $\frac{2p^2}{m^3+p^3}$.

54. $\frac{m^2}{l^2-m^2}$, $\frac{lm-m^2}{(l+m)^2}$.

55. $\frac{36a^2b^2}{(a^2-b^2)^2}$, $\frac{4(a^2-ab)}{b(a+b)^2}$.

Find a mean proportional between:

56. 27 and 3.

57. 3 and 12.

58. 5 and 20.

59. 2 and $12\frac{1}{2}$.

60. 3 and $16\frac{1}{3}$.

61. $\frac{mn}{p}$ and $\frac{mp}{n}$.

62. $\frac{2p}{q}$ and $\frac{q^3}{8p^3}$.

63. $\frac{m-5}{m^2+10m+25}$ and $\frac{(m+5)^2}{m^2-25}$.

64. $\frac{c+d}{f-g}$ and $\frac{(f^2-g^2)(c+d)}{f+g}$.

Ex. 53.

Arithmetical Progression.

$$l=a+(n-1)d;\ s=\frac{n}{2}(a+l)=\frac{n}{2}[2a+(n-1)d].$$

Find l and s in:

1. 7, 11, 15 ..... (13 terms).
2. 5, 8, 11 ..... (12 t.).
3. $\frac{1}{2}$, $6\frac{1}{2}$, $12\frac{1}{2}$ ..... (30 t.).
4. 1, 1.1, 1.2 ..... (200 t.).
5. $\frac{1}{7}$, $\frac{2}{7}$, $\frac{1}{7}$ ..... (15 t.).
6. 63, 58, 53 ..... (8 t.).

7. $3n$, $5n$, $7n$ ..... (36 t.).

8. $2m$, $2m+4n$, $2m+8n$ ..... (14 t.).

9. 1, $1+3m$, $1+6m$ ..... (11 t.).

Find d and s in:

10. 2 ..... 134, (13 t.).

11. 0 ..... 200, (51 t.).

12. $3\frac{1}{4}$ ..... 64, (82 t.).

13. 169 ..... 8, (24 t.).

14. 7 ..... -3.5, (36 t.).

15. $5a^3$ ..... $145a^3$, (21 t.).

16. $2b^3$ ..... $(2b^2+17a^3)$, (35 t.)

Insert between:

17. 13 and 76, eight means.

18. 1 and 2, fifteen means.

19. $\frac{1}{2}$ and $\frac{3}{4}$, eight means.

20. 47 and 2, eight means.

Find a and s in:

21. ..... 142, 149; $n=22$.

22. ..... 221, 242; $n=12$.

23. ..... $26\frac{1}{3}$, $27\frac{5}{6}$; $n=19$.

24. ..... -20, -22; $n=40$.

25. ..... 5, 0; $n=14$.

Find n and s in:

26. 17 ..... 350; $d=9$.

27. 34 ..... 10; $d=-2$.

28. $116\frac{1}{4}$ ..... 0; $d=-\frac{5}{4}$.

Find d and n in:

29. $1\frac{1}{2}$ ..... 54; $s=999$.

30. 5 ..... 161; $s=3320$.

31. 2 ..... 87; $s=801$.

32. -13 ..... 27; $s=77$.

Find d and l when:

33. $a=10$; $n=14$; $s=1050$.

34. $a=1$; $n=20$; $s=305$.

35. $a=0$; $n=30$; $s=290x$.

36. $a=16$; $n=9$; $s=0$.

Find a and d when :

37. $l = 21$; $n = 7$; $s = 105$.

38. $l = 148$; $n = 27$; $s = 2241$.

39. $l = 11\frac{2}{3}$; $n = 37$; $s = 209\frac{2}{3}$.

40. $l = 105$; $n = 16$; $s = 840$.

Find a and l when :

41. $n = 21$; $s = 1197$; $d = 4$.

42. $n = 12$; $s = 594$; $d = 7$.

43. $n = 25$; $s = -75$; $d = \frac{1}{2}$.

44. $n = 33$; $s = -33$; $d = -\frac{3}{4}$.

Find n and l when :

45. $s = 1095$; $a = 38$; $d = 5$.

46. $s = 624$; $a = 9$; $d = 4$.

47. $s = 2551.3$; $a = 32$; $d = 0.3$.

48. $s = 1782$; $a = 18$; $d = 6$.

Find a and n when :

49. $s = 623$; $d = 5$; $l = 77$.

50. $s = 1008$; $d = 4$; $l = 88$.

51. $s = 281\frac{1}{7}$; $d = \frac{3}{7}$; $l = 15\frac{3}{7}$.

52. $s = 2640$; $d = -20$; $l = 15$.

53. When a train arrives at the top of a long slope, the last car is detached and begins to descend, passing over 50^{cm} in the first second, three times 50^{cm} in the second, five times 50^{cm} in the third, etc. At the end of 12 minutes it reaches the bottom of the slope. How many meters does it pass over in the last second?

54. A man wishes to have his horse shod. The blacksmith asks him \$2 a shoe, or 1 cent for the first nail, 3 for the second, 5 for the third, etc. Each shoe has 8 nails. Ought the man to accept the second proposition?

55. There are placed in a straight line upon a lawn 50 eggs 3 feet distant from each other. A person is required to pick them up one by one and carry them to a basket 3 feet from the first egg, while a runner, starting from the basket, touches a goal and returns. At what distance ought the goal to be placed that both men may have the same distance to pass over?

56. Starting from a box, there are placed upon a straight line 40 stones, at the distances 1 foot, 3 feet, 5 feet, etc. A man placed at the box is required to take them and carry them back one by one. What is the total distance that he has to accomplish?

57. A stone which falls freely passes over respectively in the first three seconds of its fall 4.9^m, 3×4.9^m, and 5×4.9^m. In the famous balloon ascension which was made in 1875, the balloon rose to 8643.6^m. What space would a stone have passed over in the last second falling from the balloon to the ground?

58. A body, which moves vertically, moves with a velocity which decreases in the same manner as the velocity of a falling body increases; in the last second the rising body passes over 4.9^m; in the second before the last, 3×4.9^m; in the preceding second, 5×4.9^m. Supposing that a ball is shot vertically from a field-piece with a velocity of 445.9^m per second, find (i.) during what time it will rise; (ii.) what height it will reach?

59. How much time will it take a stone, falling from the top of the dome of the Pantheon at Paris, to reach the ground? The highest point of the cupola is 83^m.

60. The product of five numbers in arithmetical progression is 379,440, and their sum is 85. Find them.

61. The sum of three numbers in arithmetical progression is 33, and the sum of their squares is 461. What are the numbers?

62. The sum of five terms in arithmetical progression is 50, and the sum of their squares 590. Find the terms.

63. The sum of 11 numbers in arithmetical progression is 77, and the sum of their squares 1529. What are these numbers?

Ex. 54.

Arithmetical Progression of a Higher Order.

If we represent by S_2 the sum of the squares, and by S_3 the sum of cubes of the natural numbers from 1 to n inclusive, we have:

$$S_2 = \frac{n(n+1)(2n+1)}{1\times 2\times 3}. \qquad S_3 = \left(\frac{n(n+1)}{1\times 2}\right)^2.$$

If we represent by N the number of balls contained in a pile, and by n the number of balls upon each side of the lower tier, we have:

Pile with a square base,

$$N = \frac{n(n+1)(2n+1)}{1\times 2\times 3}.$$

Pile with a triangular base,

$$N = \frac{n(n+1)(n+2)}{1\times 2\times 3}.$$

Pile with a rectangular base,

$$N = \frac{n(n+1)(3m-n+1)}{1\times 2\times 3}.$$

In this last formula, m is the number of balls upon the large side of the base, and n the number upon the small side.

What is the sum of the squares of the natural numbers between the following limits, inclusive:

1. From 1 to 13?
2. From 1 to 40?
3. From 1 to 53?
4. From 1 to 100?
5. From 1 to 125?
6. From 1 to 315?
7. From 1 to 1000?
8. From 50 to 200?

What is the sum of the cubes of the natural numbers between the following limits, inclusive:

9. From 1 to 15?
10. From 1 to 39?
11. From 15 to 29?
12. From 20 to 50?
13. From 32 to 90?
14. From 7 to 35?

What is the number of balls contained in a complete pile with a square base, having upon each side of its base:

15. 6 balls?
16. 9 balls?
17. 7 balls?
18. 17 balls?
19. 21 balls?
20. 27 balls?

What is the number of balls contained in a truncated pile with a square base, having:

21. Lower base, 11 balls; upper base, 5 balls?
22. Lower base, 9 balls; upper base, 4 balls?
23. Lower base, 14 balls; upper base, 6 balls?
24. Lower base, 17 balls; upper base, 9 balls?
25. Lower base, 18 balls; upper base, 5 balls?
26. Lower base, 16 balls; upper base, 6 balls?

How many balls does a complete triangular pile contain, having upon each side:

27. 6 balls?
28. 12 balls?
29. 17 balls?
30. 10 balls?
31. 19 balls?
32. 25 balls?

How many balls does a truncated triangular pile contain, having:

33. Lower base, 14 balls; upper base, 6 balls?
34. Lower base, 17 balls; upper base, 13 balls?
35. Lower base, 24 balls; upper base, 17 balls?
36. Lower base, 20 balls; upper base, 14 balls?

How many balls does a rectangular pile contain, having upon the two sides of the lower base:

37. 11 and 5 balls?
38. 9 and 4 balls?
39. 21 and 13 balls?
40. 24 and 17 balls?
41. 31 and 19 balls?
42. 27 and 23 balls?

Find the number of balls which form a rectangular truncated pile, having:

43. Lower base, 19 and 14 balls; upper base, 15 and 10 balls.
44. Lower base, 22 and 16 balls; upper base, 16 and 10 balls.
45. Lower base, 18 and 11 balls; upper base, 11 and 4 balls.
46. Lower base, 28 and 19 balls; upper base, 19 and 10 balls.

Ex. 55.

Geometrical Progression.

$$l = ar^{n-1}, \quad S = \frac{lr - a}{r - 1} = \frac{a(r^n - 1)}{r - 1},$$

or, for decreasing progressions,

$$S = \frac{a(1 - r^n)}{1 - r}.$$

Find l and s:

1. 4, 8, 16 ..... (7 terms).
2. 2, 6, 18 ..... (9 t.).
3. 1, 4, 16 ..... (7 t.).
4. 9, 3, 1 ..... (11 t.).

5. 13, 1.3, 0.13 ..... (6 t.).

6. 56, 28, 14 ..... (12 t.).

7. 1, $\frac{1}{2}$, $\frac{1}{4}$ ..... (10 t.).

8. 1, 1.5, 2.25 ..... (6 t.).

9. 1, $\frac{1}{2}$, $\frac{1}{4}$ .....

10. 8, 2, $\frac{1}{2}$ .....

11. 2, $\frac{2}{7}$, $\frac{2}{49}$ .....

12. 5, 3, $\frac{9}{5}$ .....

13. 1, $\frac{1}{4}$, $\frac{1}{16}$ .....

14. m^3, m^3n, m^3n^2, (7 t.).

15. a, $a(1+x)$, $a(1+x)^2$ ..... (8 t.).

16. a, $\frac{a}{a^2-1}$, $\frac{a}{(a^2-1)^2}$ ..... (10 t.).

17. b, $\frac{b}{1-m}$, $\frac{b}{(1-m)^2}$ ..... (9 t.).

18. $b(1+x)^{n-1}$, $b(1+x)^{n-2}$, $b(1+x)^{n-3}$ ..... (n t.).

Find r and s when :

19. 8 ..... 10,368, (5 t.).

20. 2 ..... 31,250, (7 t.).

21. 36 ..... $\frac{4}{81}$, (7 t.).

22. 3 ..... 49,152, (8 t.).

23. 48 ..... $\frac{3}{64}$, (6 t.).

24. 7 ..... 3584, (10 t.).

Between the following terms insert the number of means indicated :

25. 14 and 686, (1 mean).

26. 47 and 1269, (2 m.).

27. 31 and 496, (3 m.).

28. 13 and 101,088, (4 m.).

29. 81 and $\frac{256}{81}$, (7 m.).

30. 128 and 31,250, (5 m.).

31. $\frac{5}{64}$ and 640, (12 m.).

32. $\frac{2}{243}$ and 1458, (10 m.).

33. 6250 and $\frac{2}{3125}$, (9 m.).

Find a and s when :

34. $l = 128$; $r = 2$; $n = 7$.

35. $l = 78{,}125$; $r = 5$; $n = 8$.

36. $l = 29{,}296{,}875$; $r = 5$; $n = 11$.

37. $l = 14\frac{2150}{2187}$;
$r = \frac{4}{3}$;
$n = 8$.

38. $l = \frac{15309}{262144}$;
$r = \frac{2}{8}$;
$n = 8$.

39. $l = \frac{2}{27}$;
$r = \frac{1}{3}$;
$n = 5$.

Find s and n when:

40. $a = 9$;
$l = 2304$;
$r = 2$.

41. $a = 2$;
$l = 64$;
$r = 2$.

42. $a = 3$;
$l = 192\sqrt{2}$;
$r = \sqrt{2}$.

43. $a = 54$;
$l = \frac{2}{2187}$;
$r = \frac{1}{3}$.

44. $a = 9$;
$l = \frac{32}{27}$;
$r = \frac{2}{3}$.

Find r and n when:

45. $a = 2$;
$l = 1458$;
$S = 2186$.

47. $a = 10$;
$l = \frac{5}{16}$;
$S = 19\frac{11}{16}$.

46. $a = 1$;
$l = 2401$;
$S = 2801$.

48. $a = 3125$;
$l = 5$;
$S = 3905$.

Find the two unknown quantities in the following examples:

49. $a = 3$;
$r = 3$;
$S = 29{,}523$.

51. $r = 2$;
$n = 7$;
$S = 635$.

53. $l = 1296$;
$r = 6$;
$S = 1555$.

50. $a = 8$;
$r = 2$;
$S = 4088$.

52. $r = 8$;
$n = 6$;
$S = 149{,}796$.

54. $l = 32{,}768$;
$r = 4$;
$S = 43{,}690$.

55. The first term of a geometrical progression is 18, the number of terms is 3, and the sum of the terms is 1026. Find the last term and the ratio.

56. A cask contains 240 gallons of wine. The first day of January, half is drawn out; the following day, half of the rest; the third day, half of what then remains; and so on. How much will be drawn off on January 31?

57. What is the value of the decimal fraction 0.7272 .....?

58. The sum of the two extreme terms of a geometrical progression of four terms is 195; the sum of the two mean terms is 60. What is the first term, and what is the ratio?

59. What is the sum of the terms of the progression 1, -3, 9, -27, to the twentieth term, inclusive?

60. What is the sum of the first twelve terms of the progression, $-2, +4, -8, +16$ .....?

61. What is the sum of the terms to infinity of the progression $9, -3, +1, -\frac{1}{3}$ .....?

Ex. 56.

Permutations and Combinations.

The permutations of m letters are the results obtained by writing, in every possible order, these m letters, one after another, so that each letter may occur in each permutation once, and once only.

If P_m is the number of permutations of m letters, we have

$$P_m = 1 \times 2 \times 3 \times 4 \ldots\ldots m = \underline{|m}.$$

If in m objects there are p objects of one kind, q of a second kind, r of a third kind, the number of possible permutations is given by

$$P_m = \frac{1 \times 2 \times 3 \ldots\ldots m}{(1 \times 2 \times 3 \ldots\ldots p)(1 \times 2 \times 3 \ldots\ldots q)(1 \times 2 \times 3 \ldots\ldots r)} = \frac{\underline{|m}}{\underline{|p}\,\underline{|q}\,\underline{|r}}.$$

If P_n is the number of permutations of m things, taken n at a time ($n < m$), then

$$P_n = m(m-1)(m-2)\ldots\ldots(m-n+1).$$

1. In how many different ways can the 26 letters of the alphabet be placed, one after another?

2. In how many ways can 14 persons be placed at table?

3. In how many ways can the 30 pupils of a class be placed?

4. In how many ways can the 8 bells of a chime be struck successively?

5. In how many ways can 25 men be placed in a rank?

6. It is required to place 20 vases of flowers in a half circle. In how many different ways can they be arranged?

7. How many numbers can be formed with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, each figure occurring in each number once, and once only?

8. In the same case, how many different numbers can be formed with the figures 1, 2 ..... 9, 0? The numbers beginning with zero not to be counted.

9. In how many different ways can be placed, one after another, 24 colored eggs, of which 4 are yellow, 8 blue, 7 brown, and the others red?

10. In a library are 30 volumes, of which 10 are bound in white, 9 in red, 6 in green, and the 5 others in brown. In how many ways can they be placed as to their colors?

How many permutations can be made with the following products:

11. a^2bc. 12. $a^3b^7c^2$. 13. $a^7x^5cd^4$. 14. $a^5x^5y^8z^2$.

15. How many words can be made with 9 letters, 3 letters at the beginning remaining inseparable, and keeping the same order?

16. What will be the answer to the preceding problem if the three inseparable letters can be arranged in any order?

17. With 7 letters, among which are several a's, 210 different words can be formed. How many a's are there?

18. How many arrangements can be formed with 12 objects taken 2 at a time?

19. How many arrangements can be formed with 30 objects taken 8 at a time?

20. A girl has two rings of different form. In how many ways can she place them upon her fingers, either on the left hand or on the right, putting one ring on a finger? (The thumb does not count.)

21. How many arrangements can be formed with 15 objects taken 6 at a time?

22. How many arrangements can be formed with 26 letters, if 4 are taken at a time? if 6 are taken at a time?

23. How many numbers of three figures can be formed with the numbers 1 to 9 inclusive?

24. With the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, how many numbers can be formed: (i.) of 3 figures? (ii.) of 4 figures? The numbers beginning with zero not to be counted.

25. How many words can be formed with 20 consonants and 6 vowels, each word containing 3 consonants and 2 vowels, and the latter occupying only the second and fourth places?

26. What would be the answer to the preceding problem if the vowels could occupy any place whatever?

27. How many words of 3 letters can be made with 20 consonants and 6 vowels, taking two consonants and a vowel for each word, if the vowels can represent in all, by means of accents, 10 different sounds?

28. In how many ways can the 10 bells of a chime be struck: (i.) 4 at a time? (ii.) 5 at a time?

29. The number of arrangements of n objects taken 3 at a time is to the number of arrangements of the same objects, taken 4 at a time, as 1 is to 20. Find n.

30. A number of m objects is to the number of its arrangements, taken 3 at a time, as 1 is to 240. Find m.

31. How many numbers of 4 figures can be formed: (i.) with the figures 1, 2, 3, 4; (ii.) with the figures from 1 to 9 inclusive, if in both cases the figures can be repeated?

32. What is the number of the arrangements, with repetitions, of twelve objects taken 5 at a time?

33. What is the total number of the arrangements, without repetitions, that can be made with 16 letters, taking 1, 2, 3 ..... 16 at a time?

34. What will be the answer to the preceding problem if the arrangements are made with repetitions?

35. What is the total number of the arrangements, with repetitions, of 26 letters taken 2, 3, 4 ..... 10 at a time?

36. The number of arrangements, with repetitions, of n objects taken 8 at a time is to the number of arrangements, with repetitions, of the same objects taken 3 at a time, as 537,824 is to 1. Find n.

37. The total number of the arrangements, 1, 2, 3 ..... 8 at a time, of p objects, is to the total number of the arrangements 1, 2, 3, 4 at a time, as 1297 is to 1. In the two cases there are repetitions. Find p.

COMBINATIONS.

Suppose m letters are taken 2, 3, 4 ..... n at a time, and from them groups are formed in which the *order* of the elements is indifferent, but such that any two groups whatever differ from each other at least by one letter, the results that are obtained are *combinations* or *different products*.

If C_n is the number of combinations of m letters taken n at a time, we have

$$C_n = \frac{m(m-1)(m-2)\ldots\ldots(m-n+1)}{1\times 2\times 3\ldots\ldots n}.$$

If $\Sigma C_{1,2,3\ldots\ldots m}$ is the sum of all the combinations of m objects taken 1, 2, 3 ..... m at a time, we have

$$\Sigma C_{1,2,3\ldots\ldots m} = 2^m - 1.$$

If we have m collections of objects, containing respectively p, q, r, s ..... different objects, and if we make combinations, taking each time *only one* object in each collection, we have for the total number C of the combinations,

$$C = p\times q\times r\times s\ldots\ldots$$

If two collections contain respectively m and n objects, the number C of the combinations that can be obtained by taking p objects of the first and q objects of the second collection is given by the formula,

$$C = \frac{m(m-1)(m-2)\ldots\ldots(m-p+1)}{1\times 2\ldots\ldots p}\times\frac{n(n-1)(n-2)\ldots\ldots(n-q+1)}{1\times 2\ldots\ldots q}.$$

38. How many different products can be formed with the natural numbers, from 2 to 13 inclusive, multiplying 3 and 3?

39. A captain, having under his command 60 men, wishes to form a guard of 8 men. In how many different ways can it be formed?

40. A merchant who has 8 kinds of coffee wishes to make mixtures of equal parts, taking for each mixture 3 different qualities. How many different mixtures can he make?

41. To form a floor, a man has hexagonal bricks of 12 different colors. Arranging the colors 4 and 4, how many combinations of colors can be obtained?

42. An assembly containing 50 members wishes to elect a commission of 12 members taken from its midst. Among how many combinations can the assembly choose?

43. A number has for prime factors 2, 3, 7, 13, 17. How many divisors has this number, including the prime factors? The number itself and unity are not counted.

44. The actual number of 4 companies is reduced respectively to 40, 62, 70, and 73 men. In how many ways can an officer form a picket of 4 men, taking 1 soldier from each company?

45. In an election there are 6 candidates for 4 offices. If each elector can have upon his vote any number of names whatever, without exceeding 4, in how many ways can he vote?

46. It is wished to divide 15 objects in lots, each containing 3 objects. In how many ways can the lots be made?

47. A man has 5 pairs of pants, 8 waistcoats, and 7 coats. In how many different costumes can he appear?

48. How many different compounds will 66 simple bodies form, uniting 2 and 2, 3 and 3, 4 and 4, in the proportions of an atom of one element for an atom of the other?

49. How many combinations will these same elements form combining 2 and 2, 3 and 3, 4 and 4, if each element enters in the compounds for 1, 2, 3, or 4 atoms?

50. A detachment of 30 men must furnish each night a guard of 4 men. For how many nights can a different guard be formed, and how many times will each soldier serve?

51. How many combinations can be made with 10 letters, a, b, c, d ....., taken 5 at a time? In how many combinations will a enter? In how many will a, b, c enter at a time?

52. We have m different objects. Find the value of n, so that, combining these m objects n at a time, the number of the combinations is a maximum.

53. The number of combinations of $n+2$ objects, taken 4 at a time, is to the number of combinations of n objects, taken 2 at a time, as 11 is to 1. Find n.

54. The total number of the combinations of $2n$ objects, taken 1, 2, 3 together, is to the number of n objects as 387 is to 1. Find n.

55. Find how many divisors a number has which is equal to $2^5 \times 3 \times 5^4 \times 7 \times 13$.

56. How many divisors has the number
$$2^3 \times 5^2 \times 7 \times 11 \times 13?$$

57. How many divisors has the number
$$2^4 \times 3^5 \times 5^2 \times 7 \times 11 \times 13?$$

58. Prove that the number of combinations of m objects, taken n at a time, is the same as that of the combinations taken $(m-n)$ at a time.

59. It is required to select 13 cards from a pack of 52 cards. How many years will it take to form all the combinations possible, forming 4 combinations per minute, and working every day, 15 hours per day? The year being counted as 365 days and 6 hours.

60. An editor wishes to publish for youth a volume embellished with 12 chromos and 18 engravings. It is necessary to choose among 30 chromos and 37 engravings. How many different combinations can he make to illustrate the volume?

61. An assembly of stockholders, composed of 40 merchants, 20 lawyers, 30 manufacturers, and 10 physicians, wishes to elect a commission of 4 merchants, 3 manufacturers, 1 physician, 2 lawyers. In how many ways can the commission be formed?

Ex. 57.

Binomial Theorem.

If m is an entire positive quantity, we have (A):

$$(a+b)^m = a^m + \frac{m}{1}a^{m-1}b + \frac{m(m-1)}{1\times 2}a^{m-2}b^2$$
$$+ \frac{m(m-1)(m-2)}{1\times 2\times 3}a^{m-3}b^3 + \ldots\ldots \frac{m(m-1)\ldots\ldots 2}{1\times 2\times 3\times (m-1)}ab^{m-1}$$
$$+ \frac{m(m-1)\ldots\ldots 2\times 1}{1\times 2\times 3\ldots\ldots m}b^m.$$

The nth term $= \dfrac{m(m-1)(m-2)\ldots\ldots(m-n+2)}{1\times 2\times 3\ldots\ldots(n-1)}a^{m-n+1}b^{n-1}$.

Apply the formula to the development of the following expressions:

1. $(a+x)^8$.
2. $(b+x)^4$.
3. $(d+y)^5$.
4. $(c+z)^6$.
5. $(a+p)^7$.
6. $(m+n)^{11}$.

7. $(p+v)^{13}$.
8. $(a-b)^{4}$.
9. $(a-b)^{5}$.
10. $(a-x)^{7}$.
11. $(h+z)^{13}$.
12. $(h-z)^{11}$.
13. $(a-c)^{8}$.
14. $(b-p)^{9}$.
15. $(z-d)^{13}$.
16. $(1+x)^{12}$.
17. $(1-x)^{9}$.
18. $(1-a)^{8}$.
19. $(1+a)^{15}$.
20. $(1-z)^{14}$.
21. $(x+1)^{9}$.
22. $(x-1)^{11}$.
23. $(y-1)^{8}$.
24. $(z+1)^{7}$.
25. $(z-2)^{8}$.
26. $(2+x)^{8}$.
27. $(2-x)^{4}$.
28. $(3+x)^{8}$.
29. $(x-3)^{9}$.
30. $(z+5)^{6}$.
31. $(a-5)^{8}$.
32. $(4-b)^{7}$.
33. $(h-2)^{10}$.
34. $(a-10)^{6}$.
35. $(b+4)^{5}$.
36. $(\frac{1}{2}+x)^{5}$.
37. $(y+\frac{1}{3})^{6}$.
38. $(\frac{1}{4}-v)^{7}$.
39. $(a-\frac{1}{5})^{8}$.
40. $(0.3+x)^{6}$.
41. $(0.4-z)^{7}$.
42. $(3.2+b)^{5}$.
43. $(c+1.5)^{4}$.
44. $(a-2.1)^{6}$.
45. $(2a+x)^{5}$.
46. $(3a-y)^{7}$.
47. $(\frac{1}{2}a+z)^{6}$.
48. $(\frac{1}{3}b-y)^{8}$.
49. $(\frac{2}{3}x+y)^{9}$.
50. $(\frac{3}{4}a-b)^{11}$.
51. $(x-\frac{3}{5})^{4}$.
52. $(-x+2a)^{7}$.
53. $(-3x+b)^{8}$.
54. $(-y+2c)^{9}$.
55. $(-z+3a)^{5}$.
56. $(2x+3a)^{6}$.
57. $(5d-3y)^{5}$.
58. $(4d-3z)^{7}$.
59. $(0.2a+0.3b)^{5}$.
60. $(1.2b-0.2y)^{7}$.
61. $(\frac{1}{3}a+\frac{1}{2}b)^{5}$.
62. $(\frac{1}{5}c-\frac{1}{4}d)^{8}$.
63. $(\frac{2}{5}m+\frac{3}{4}p)^{9}$.
64. $(\frac{2}{5}z-\frac{3}{7}b)^{6}$.
65. $(\frac{3}{5}h-\frac{1}{2}v)^{7}$.
66. $(a^{2}+b^{3})^{5}$.
67. $(x^{3}-a^{5})^{6}$.
68. $(y^{2}+b)^{7}$.
69. $(d^{3}-b^{2})^{8}$.
70. $(a^{3}-y^{2})^{9}$.
71. $(d^{3}-3x^{2})^{8}$.
72. $(2h^{2}-3x^{3})^{5}$.
73. $(3a^{2}+5b^{3})^{7}$.
74. $(1-2x^{3})^{5}$.
75. $(3x^{2}-1)^{8}$.
76. $(1+\frac{2}{3}a^{5})^{8}$.
77. $(\frac{1}{2}p^{5}+3y^{4})^{5}$.
78. $(\frac{2}{3}a^{3}+\frac{3}{4}b^{2})^{5}$.
79. $(0.2a^{3}-\frac{2}{3}x^{4})^{4}$.
80. $(\frac{5}{3}l^{2}+0.3p^{3})^{7}$.
81. $(2a-3cx)^{7}$.

82. $(3a^2+2c^2x^3)^6$.

83. $(bc^2-2ay^3)^5$.

84. $(4c^4-3acx^5)^4$.

85. $(a^2b^3+2a^3bx^4)^5$.

86. $(\frac{2}{3}ab^2-\frac{3}{4}a^2y)^7$.

87. $(\sqrt{a}+x)^7$.

88. $(\sqrt{2b}-m)^8$.

89. $(\sqrt{3c}+2a)^7$.

90. $(\sqrt{\frac{1}{2}a}-3y)^8$.

91. $(2\sqrt{e}+a)^6$.

92. $(2b-\sqrt{x})^9$.

93. $(\frac{2}{3}a+\sqrt{2x})^6$.

94. $(\frac{3}{4}a-\sqrt{\frac{1}{2}x})^5$.

95. $(2a-3\sqrt{y})^6$.

96. $(a^2+\frac{1}{2}\sqrt{z})^7$.

97. $(\sqrt{a}+\sqrt{x})^9$.

98. $(\sqrt{b}-\sqrt{y})^8$.

99. $(\sqrt{2c}+\sqrt{3x})^6$.

100. $(\sqrt{\frac{1}{2}a}-\sqrt{2x})^7$.

101. $(\sqrt{\frac{2}{3}a}+\sqrt{\frac{3}{4}x})^6$.

102. $(\sqrt{a^3}-\sqrt{x^5})^{10}$.

103. $(\sqrt{b^5}+\sqrt{y^4})^9$.

104. $(\sqrt{\frac{2}{3}h^3}+\sqrt{\frac{1}{2}x^4})^6$.

105. $(\sqrt[3]{2a}+\sqrt[3]{3x^2})^8$.

106. $(2ax+\sqrt[3]{3xy^2})^5$.

107. $(\sqrt{3ay}+\sqrt[3]{2x^2y})^7$.

108. $(3abx^3-\sqrt[3]{5b^2xy^2})^8$.

109. $\left(\frac{\sqrt{x}}{a}+\frac{\sqrt[3]{y}}{b}\right)^{10}$.

110. $\left(\frac{\sqrt[3]{x^2}}{a^2}+\frac{\sqrt[4]{xy^3}}{b}\right)^7$.

111. $\left(\frac{\sqrt{xz}}{y}-\frac{\sqrt[3]{ay^2}}{x^2}\right)^6$.

112. $(2a-x\sqrt{-1})^7$.

113. $(3b^2+2x\sqrt{-1})^6$.

114. $(\sqrt{a^3}+i\sqrt{x})^5$.

115. $(\sqrt[3]{a^2}+i\sqrt[4]{x^3})^6$.

116. $(\sqrt[3]{a^2}-\sqrt{-1})^9$.

117. $(i\sqrt{b}+i)^{10}$.

118. $(i\sqrt[3]{x^2}-i\sqrt{2})^9$.

119. $(i\sqrt[5]{y^4}+i\sqrt[3]{5})^7$.

120. $\left(\frac{2i\sqrt{b^3}}{y}-\frac{di\sqrt[3]{y^2}}{b}\right)^8$.

Find, in the following developments, the term required:

121. The seventh term of $(2+a)^{16}$.

122. The eleventh term of $(a+d)^{21}$.

123. The sixth term of $(3+2x^2)^9$.

124. The twelfth term of $(a^3 - 3x^5)^{20}$.

125. The eighteenth term of $(1 + x^2)^{30}$.

126. The fourteenth term of $(y^3 - 1)^{40}$.

127. The seventh term of $(\frac{1}{2}a - \sqrt{x^3})^{17}$.

128. The fourth term of $(\sqrt{a} - \sqrt[3]{x^2})^{11}$.

129. The sixth term of $(\sqrt{a} - ix)^{13}$.

130. The thirtieth term of $(b - 2c^3)^{50}$.

131. Prove that the coefficient of the nth term from the beginning is equal to the coefficient of the nth term from the end.

132. What is the greatest coefficient in the development of $(1+x)^m$?

133. What is the greatest coefficient of the development of i. $(a+b^3)^{21}$? ii. $(b+x^2)^{16}$? iii. $(1+y^5)^{32}$?

134. Prove that, in the development of $(1+a)^m$, the sum of the coefficients of even places is equal to the sum of the coefficients of odd places.

135. Prove that, in the development of $(1+x)^m$, the sum of the coefficients of the terms in x is equal to $2m-1$.

Instead of $(a+b)^m$ we can write $a^m\left(1+\frac{b}{a}\right)^m$; or, replacing in the parenthesis, $\frac{b}{a}$ by x, we can develop a quantity of the form $(1+x)^m$. After the preceding formula (A), we have (B):

$$(1+x)^m = 1 + \frac{m}{1}x + \frac{m(m-1)}{1\times 2}x^2 + \frac{m(m-1)(m-2)}{1\times 2\times 3}x^3$$
$$+ \frac{m(m-1)(m-2)(m-3)}{1\times 2\times 3\times 4}x^4 + \ldots\ldots$$

If m is a negative integral number, or a positive or negative fraction, the number of the terms of the second member of (B) is unlimited. If, in absolute value, $x > 1$, this second member increases

indefinitely, and cannot be equal to the first member, whose value is finite.

In the development of $\frac{1}{1-x}$, we have

$$\frac{1}{1-x} = (1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots\ldots$$

When $x = 2$, the first member is negative, while the second is positive and indefinitely increasing.

If, in absolute value, $x < 1$, the second member converges towards a fixed finite value, which is the value of the first member.

Then $(a+b)^m$ can be developed after the formula (A), where m is integral and negative, or fractional. Thus, if $a > b$, (C):

$$(a+b)^m = a^m\left(1+\frac{b}{a}\right)^m = a^m + \frac{m}{1}a^{m-1}b + \frac{m(m-1)}{1\times 2}a^{m-2}b^2 + \frac{m(m-1)(m-2)}{1\times 2\times 3}a^{m-3}b^3 + \ldots\ldots$$

If $a < b$, it will be developed thus, (D):

$$(a+b)^m = b^m\left(1+\frac{a}{b}\right)^m = b^m + \frac{m}{1}b^{m-1}a + \frac{m(m-1)}{1\times 2}b^{m-2}a^2 + \ldots\ldots$$

136. Develop $(1+x)^{-1}$, and then make:
$x = \frac{1}{3}$; $x = 0.07$; $x = -0.009$.

137. Develop $(1+x)^{\frac{1}{2}}$, and then make $x = \frac{8}{9}$.

138. Develop $(1+x)^{-\frac{1}{2}}$, and then make $x = 0.003$.

139. Develop $(1+x)^{\frac{1}{3}}$, and then make:
$x = \frac{7}{12}$; $x = -0.09$; $x = \frac{1}{8}$.

140. Develop $(1+x)^{-\frac{1}{3}}$, and then make:
$x = -\frac{2}{11}$; $x = -0.002$; $x = -\frac{1}{8}$.

141. Develop $(1-x)^{\frac{2}{3}}$, and then make $x = 0.004$.

Develop the following:

142. $(1+15)^{\frac{1}{4}}$.

143. $(2+27)^{\frac{1}{3}}$.

144. $(1+9)^{-\frac{1}{2}}$.

145. $(1+64)^{\frac{1}{6}}$.

Form the first eight terms of the following examples, in which the second term of the binomial is supposed smaller than the first:

146. $(a+x)^{-5}$.
147. $(a-x)^{-6}$.
148. $(2b-y)^{-6}$.
149. $(\frac{1}{2}c+z)^{-9}$.
150. $(3-x)^{-8}$.
151. $(a+\frac{1}{2}x)^{-9}$.
152. $(1+x)^{-10}$.
153. $(1-y)^{-11}$.
154. $(z+1)^{-9}$.
155. $(1-x)^{-1}$.
156. $(a^2+x^3)^{-5}$.
157. $(b^3-x^2)^{-8}$.
158. $(c-2x^3)^{-9}$.
159. $(a^2c+ax^2)^{-6}$.
160. $(\sqrt{a}-x^2)^{-6}$.
161. $(b+h)^{\frac{1}{2}}$.
162. $(b-x)^{\frac{1}{2}}$.
163. $(2a+y)^{\frac{1}{2}}$.
164. $(2+a)^{\frac{1}{2}}$.
165. $(x^2+a)^{\frac{1}{2}}$.
166. $(a^2-1)^{\frac{1}{2}}$.
167. $(a+2b)^{\frac{1}{2}}$.
168. $(1+a)^{\frac{1}{2}}$.
169. $(1+y)^{\frac{1}{10}}$.
170. $(1+z^2)^{\frac{1}{2}}$.
171. $(1+h)^{\frac{1}{2}}$.
172. $(2-y^3)^{\frac{1}{2}}$.
173. $(3+d^4)^{\frac{1}{2}}$.
174. $(a^3-b)^{\frac{1}{2}}$.
175. $(y^2-b^3)^{\frac{1}{2}}$.
176. $(a^3+1)^{-\frac{1}{2}}$.
177. $(x^2-a)^{-\frac{1}{2}}$.
178. $(1-x^5)^{-\frac{1}{2}}$.
179. $(1+2d)^{-\frac{1}{2}}$.
180. $(32+5h)^{-\frac{1}{5}}$.
181. $(9-2x^2)^{-\frac{1}{2}}$.
182. $(a^7+3y^4)^{-\frac{1}{7}}$.
183. $\left(\frac{1}{\sqrt{a}}+\frac{\sqrt[3]{x}}{b}\right)^{-\frac{1}{2}}$.
184. $\left(\frac{\sqrt[3]{a^2}}{x}+\frac{x^2}{a}\right)^{-\frac{1}{2}}$.
185. $\left(\frac{a^{-1}}{2}-\frac{4}{x^2}\right)^{-\frac{1}{2}}$.
186. $(2+\sqrt[4]{2x^3})^{-\frac{1}{2}}$.
187. $\frac{1}{(a^2+x^3)^{\frac{1}{3}}}$.
188. $\frac{1}{(a^3-x^3)^{\frac{1}{2}}}$.
189. $\frac{1}{(a^{\frac{1}{2}}-b^{\frac{1}{2}})^{-7}}$.

In the following examples the first term of the binomial is supposed, in absolute value, smaller than the second.

190. $(p+a)^{-1}$.
191. $(r+b^2)^{-3}$.
192. $(t-H)^{-5}$.
193. $(v-Y)^{-4}$.
194. $(3-4a^2)^{-\frac{1}{2}}$.
195. $(2+x^5)^{-\frac{1}{7}}$.
196. $(2a^{\frac{1}{2}}-3x^{\frac{1}{3}})^{-\frac{1}{2}}$.
197. $(ax^{\frac{1}{2}}+a^2y)^{-\frac{1}{2}}$.
198. $(h^{\frac{1}{2}}+3z^4)^{\frac{1}{2}}$.

Apply the formula of the binomial to the extraction of the following roots, carrying out the operation to the sixth decimal:

199. $\sqrt{10}$.
200. $\sqrt{53}$.
201. $\sqrt[3]{87}$.
202. $\sqrt[3]{30}$.
203. $\sqrt[4]{68}$.
204. $\sqrt[5]{259}$.
205. $\sqrt[5]{1121}$.
206. $\sqrt[7]{279900}$.
207. $\sqrt{0.096}$.
208. $\sqrt[3]{0.87}$.
209. $\sqrt{34\frac{1}{2}}$.
210. $\sqrt{18\frac{3}{4}}$.
211. $\sqrt[3]{58\frac{1}{3}}$.
212. $\sqrt[3]{21\frac{2}{3}}$.
213. $\sqrt[3]{127\frac{5}{27}}$.
214. $\sqrt[4]{1298\frac{5}{8}}$.
215. $\sqrt[5]{33\frac{7}{9}}$.
216. $\sqrt[3]{80^2}$.
217. $\sqrt[4]{624\frac{1}{2}}$.
218. $\sqrt[7]{2189\frac{1}{3}}$.
219. $\sqrt[8]{1.256}$.
220. $\sqrt[9]{512.048}$.
221. $\sqrt[4]{13^3}$.
222. $\sqrt[5]{33.7^2}$.

Ex. 58.

Probabilities.

1. If a bag contains 6 black balls and 4 white ones, express the probability or mathematical chance that one drawn will be (i.) a white ball, (ii.) a black ball.

2. What is the meaning of the mathematical chance

$$P = \frac{m}{m+n}?$$

What must be the condition of the question (i.) when $P = 0$, (ii.) when $P = 1$, and (iii.) when $P = \frac{1}{2}$?

3. A bag contains black balls and white. If the odds are 4 to 1 against drawing a white ball, what is the least number of balls there can be in the bag, and of what colors?

4. A picture was raffled for. I took 5 shares. What was my expectation if there were 20 shares taken by other subscribers?

5. What is the chance of throwing 5 with one die? Also find the chance of throwing with two dice 3, 4, or 12; and, lastly, find the chance of throwing with two dice either 7, or else 8, or else 9.

6. There are three events, A, B, C, one of which must happen. The odds are 3 to 7 on A, and 3 to 6 on B. Find the odds on C.

7. What are the chances, for and against, that the number 7 will come at least once in 3 throws of 2 dice?

8. The odds in one case being $2 : 2\frac{1}{2}$, in another $3\frac{1}{4} : 4$; compare the corresponding chances.

9. In a bag are 3 white, 4 red, and 5 black balls. If 2 balls be drawn, find the chance of their being (i.) white and red; (ii.) both black; (iii.) one at least red.

10. Five balls are drawn from a bag of 12, of which 3 are marked ones. Find the chance that among the 5 all the marked ones will be drawn.

11. Out of an urn which contains 4 white, 5 red, and 3 blue balls I take, blindfold, 3 balls. Required the probability that these will all be of different colors.

12. There are two bags, one containing 5 white balls and 2 black; the other containing 7 white balls and 3 black. If one be drawn, what is the probability of drawing (ii.) a white ball from second? (iii.) a white black from the first? (v.) one white ball

13. The dealer at whist, after looking at his hand, desires to know the probability of his partner holding two given cards.

14. A boy who has 7 marbles plays with me at "odd or even." Show the chance of my winning "even" is to the chance of my winning "odd" as 63 to 64. What with 5 balls?

15. In a bag there are 7 white, 5 red, 9 blue, and 14 yellow balls. If I draw out two, how much greater is the probability that they will be one red and one yellow, than one white and one blue?

16. Out of a mutinous crew of 120, the captain ordered 2 men, chosen by lot, to be executed, the real conspirators being 20. What is the chance that (i.) one and only one of them, (ii.) two of them, will be taken?

17. In a bag there are 11 balls: 3 white and 8 red. If I draw 3 balls one after the other, find the chance of their being all white ones; also, the chance if I replace the ball drawn each time.

18. A bag contains 4 white and 7 black balls, and I draw one at a time. Find the chance that, after 3 trials, I shall have drawn 2 white and 1 black (i.) without replacing the ball drawn, (ii.) replacing each time the ball drawn.

19. If I toss a shilling 5 times, what are the odds against its falling "heads" exactly 3 times?

20. A bag contains 4 white and 6 red balls. If I draw 4 times, replacing, show that 2 red or 3 red are events equally possible.

21. If 5 shillings are thrown into the air, show that it is just as likely that the event will show 3 heads and 2 tails as 2 heads and 3 tails.

22. In a bag are 5 black balls and 1 white. If I draw 3 times (replacing), what is the chance that I shall have drawn (i.) 3 black balls? (ii.) 3 white balls? (iii.) 2 black and 1 white?

23. With 2 dice, what is the probability of making 9 at the first throw? or, if this does not happen, 9 with the second throw?

24. A bag contains 7 balls: 5 white and 2 black. I draw a ball, receiving a shilling if it be a white one, but paying 3 pence if it be black. What is my expectation?

25. With 3 dice, what are the chances of throwing at the first time 3, 4, 5, and all numbers up to 18? and show that the odds are 1 : 7 against the two most likely numbers, namely, 10 and 11.

26. There are in a bag 4 balls: one is known to be white, and the others are either all white or else all black. A ball is drawn and is found to be white, and replaced. If, then, another is drawn, what is the chance of its being a black ball? If two are drawn, what is the chance that one will be a black ball and one a white ball?

Ex. 59.

Continued Fractions.

Change into continued fractions:

1. $\frac{108}{21}$.
2. $\frac{37}{11}$.
3. $\frac{89}{12}$.
4. $\frac{269}{18}$.
5. $\frac{67}{167}$.
6. $\frac{103}{643}$.
7. $\frac{149}{809}$.
8. $\frac{379}{53}$.
9. $\frac{484}{457}$.
10. $\frac{368}{103}$.
11. $\frac{167}{907}$.
12. $\frac{811}{991}$.
13. 9.749.
14. 5.07.
15. 0.0241.

16. $\dfrac{15a^2+1}{15a^3+6a}$.

17. $\dfrac{63m^5+1}{315m^6+9m^3+5m}$.

18. $\dfrac{x^3+x^2+x}{x^3+2x^2+x+1}$.

19. $\dfrac{24x^3+6x}{24x^4+18x^2+1}$.

20. $\dfrac{x^{10}+x^7+x^5+x^3+1}{x^9+x^4+x^2}$.

21. $\dfrac{a^3x^9+x^4+a^5x^2}{a^6x^{10}+ax^7+a^3x^5+a^5x^3+1}$.

Change to common fractions:

22. $\cfrac{1}{2+\cfrac{1}{3+\frac{1}{7}}}$.

24. $\cfrac{1}{8+\cfrac{1}{7+\frac{1}{3}}}$.

26. $2+\cfrac{1}{3+\cfrac{1}{5+\frac{1}{7}}}$.

23. $\cfrac{1}{6+\cfrac{1}{4+\frac{1}{2}}}$.

25. $\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}}$.

27. $4+\cfrac{1}{2+\cfrac{1}{4+\frac{1}{6}}}$.

28. $3+\cfrac{1}{2+\cfrac{1}{4+\frac{1}{8}}}$.

29. $7+\cfrac{1}{6+\cfrac{1}{5+\frac{1}{4}}}$.

30. $1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{1+\frac{1}{2}}}}$.

31. $5+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{4+\frac{1}{5}}}}}$.

32. $\cfrac{1}{13+\cfrac{1}{11+\cfrac{1}{9+\cfrac{1}{7+\frac{1}{5}}}}}$.

33. $\cfrac{1}{5+\cfrac{1}{7+\cfrac{1}{9+\cfrac{1}{11+\frac{1}{13}}}}}$.

Form the convergents of the continued fractions:

34. $\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5+\frac{1}{6}}}}}$.

35. $\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\frac{1}{5}}}}}$.

36. $\cfrac{1}{2+\cfrac{1}{7+\cfrac{1}{3+\cfrac{1}{9+\frac{1}{4}}}}}$.

37. $\cfrac{1}{4+\cfrac{1}{8+\cfrac{1}{16+\cfrac{1}{2+\frac{1}{4}}}}}$.

38. $\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{7+\cfrac{1}{9+\frac{1}{11}}}}}}$.

39. $\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{3}}}}}}$.

Find the approximate values of:

40. $\frac{3845}{997}$.

41. $\frac{977}{4636}$.

42. $\frac{15600}{839}$.

43. $\frac{3791}{2951}$.

44. $\frac{13733}{18983}$.

45. $\frac{36661}{43523}$.

46. $\frac{2703}{2219}$.

47. $\frac{20467}{44341}$.

48. $\frac{3.1099}{36.391}$.

49. $\frac{48.427}{45.607}$.

50. 3.1415926.

51. 2.71828183.

52. 0.4342945.

53. $\frac{418\frac{5}{6}}{319.2}$.

Change to continued fractions:

54. $\sqrt{11}$. **56.** $\sqrt{17}$. **58.** $\sqrt{37}$. **60.** $\sqrt{39}$.

55. $\sqrt{15}$. **57.** $\sqrt{27}$. **59.** $\sqrt{51}$. **61.** $\sqrt{80}$.

Find the radicals which are equivalent to the following continued periodic fractions:

62. $\cfrac{1}{5+\cfrac{1}{5+\cfrac{1}{5}}}\cdots\cdots$

63. $\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4}}}}\cdots\cdots$

64. $\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3}}}\cdots\cdots$

65. $1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}}\cdots\cdots$

66. $\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}\cdots\cdots$

67. $\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}\cdots\cdots$

68. $\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3}}}}\cdots\cdots$

69. The English yard is equivalent to 0.914383^{m}. What are the approximate ratios of the meter to the yard?

70. The English fathom is equivalent to 1.828767^{m}. What are the approximate ratios of the fathom to the meter?

71. What simple ratios express the value of the twenty-dollar gold piece, which weighs 33.437 grams, to that of the twenty-franc piece, which weighs 6.45161 grams, the two pieces being at the same standard?

72. If the equatorial radius of the earth is 6,377,398^m, and the polar radius is 6,356,080^m, what are the simple ratios of the equatorial diameter to the polar diameter?

73. The length of the tropical year is 365.242264 mean solar days. What are the simple ratios of this year with the civil year of 365 days?

Ex. 60.

Indeterminate Coefficients.

1. Find 4 terms of the quotient $\frac{1-2x}{1+4x}$.

2. Develop $\frac{1}{3-x}$.

3. Expand $\frac{1}{3-4a}$ to 5 terms.

4. Convert $\frac{1-x}{2+3x}$ into a series.

5. Expand $\frac{1}{1-2x+x^2}$.

6. Convert $\frac{1-x}{1+x-x^2}$ into a series.

7. Expand $\frac{1+x}{1-2x-x^2}$.

8. Develop $\sqrt{1+x^2}$.

9. Find the expansion of $\frac{a}{b-cx}$.

10. Find 4 terms of $\sqrt{a^2+b^2}$.

11. Expand $\frac{1}{2x-3x^2}$.

12. Convert $(1+x)^{\frac{1}{2}}$ into a series.

13. Expand $\frac{mx}{m^2-mx+x^2}$.

14. Develop $\frac{x(x-1)}{(x+1)^2}$ in a series.

15. Resolve $\frac{2x+3}{(x+2)(x-1)}$ into partial fractions.

16. Resolve $\frac{27+34x}{(3+4x)(6+7x)}$ into partial fractions.

17. Separate $\frac{1}{(x+1)(x+2)(x+3)}$ into partial fractions.

18. Separate $\frac{306x^2-450x+162}{(8x-7)(5x-4)(2x-1)}$ into partial fractions.

19. Resolve $\frac{P+Qx}{(1+ax)(1+bx)}$ into partial fractions.

20. Separate $\frac{a-bx}{(c-dx)(e-fx)}$ into partial fractions.

21. Resolve $\frac{4x-7}{x^2-x-6}$ into partial fractions.

22. Resolve $\frac{x^2+x+1}{(x-1)(x-2)(x-3)}$ into partial fractions.

23. Separate $\frac{x^2-x+1}{x^2(x-1)^2}$ into partial fractions.

24. Separate $\frac{2x^2-1}{x^3+1}$ into partial fractions.

25. Separate $\frac{3x-2}{x^2(x^2+1)^2}$ into partial fractions.

26. Resolve $\frac{x+a}{x(x-a)^2}$ into partial fractions.

Ex. 61.

Differential Method.

1. Required the fourth order of differences in the series, 1, 16, 81, 256, 625, 1296, 2401.....
2. Find the first term of the fourth order of differences in the series, 1^3, 2^3, 3^3, 4^3.....
3. Find the first term in the third order of differences in the series, 1, 5, 15, 35, 70.....; also, in the eighth order of 1, 3, 3^2, 3^3, 3^4, 3^5.....
4. Find the thirteenth term and the nth term of the series, 1, 4, 9, 16.....
5. Required the general term and the twelfth term of the series, 2, 6, 12, 20, 30.....
6. Calculate the thirtieth term of 1, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{21}$, $\frac{1}{28}$.....
7. Sum the series, 1^2, 2^2, 3^2, 4^2, 5^2, to ten terms, and also to n terms.
8. Sum to eighteen terms the series, 5, 9, 16, 26, 39.....
9. Find the sum of the series, 1, 3, 6, 10, 15, 21, continued to twelve terms, and also to n terms.
10. Sum $1^4 + 2^4 + 3^4 + 4^4 + 5^4 + \cdots\cdots + n^4$.
11. Given $\log 510 = 2.707570$, $\log 511 = 2.708420$, $\log 513 = 2.710117$, $\log 514 = 2.710963$; to find $\log 512$.
12. Given the cube roots of 45, 46, 47, and $49 = 3.5568933$, 3.5830479, 3.6088261, and 3.6593057; to find the cube root of 48.
13. Given $\log \tan 68° 54' = 10.413561$, $68° 55' = 10.413938$, $68° 56' = 10.414314$, $68° 57' = 10.414691$; to find $\log \tan 68° 56' 20''$.

14. Write down the general term, and sum *ad infinitum* the series, $\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+\frac{1}{4\times6}+\cdots\cdots$

15. Find the general term, and sum the series

$$\frac{1}{1\times1}+\frac{1}{1\times3}+\frac{1}{2\times3}+\frac{1}{2\times5}+\frac{1}{3\times5}+\frac{1}{3\times7}+\frac{1}{4\times7}+\cdots\cdots, \textit{ ad infinitum.}$$

16. Sum *ad infinitum* the series whose general term is $\frac{1}{x(x+3)}$.

17. Find the general term, and sum *ad infinitum*

$$\frac{1}{1\times5}+\frac{1}{5\times9}+\frac{1}{9\times13}+\cdots\cdots$$

18. Sum *ad infinitum* the series

$$\frac{6}{2\times7}+\frac{6}{7\times12}+\frac{6}{12\times17}+\frac{6}{17\times22}+\cdots\cdots$$

19. Sum to n terms the series whose general term is $\frac{1}{3x(4+4x)}$.

20. Sum *ad infinitum* $1+8x+27x^2+64x^3+125x^4+\cdots\cdots$

21. Sum *ad infinitum* $\frac{1}{5\times11}+\frac{1}{8\times14}+\frac{1}{11\times17}+\cdots\cdots$

22. Required the number of balls in an incomplete square pile of 17 courses; a side of the base containing 24.

23. Required the sum of the infinite series

$$\frac{1}{1\times2\times3\times4}+\frac{1}{2\times3\times4\times5}+\frac{1}{3\times4\times5\times6}+\frac{1}{4\times5\times6\times7}+\cdots\cdots$$

24. Sum $1^2 + 3^2 + 5^2 + 7^2 + \cdots$ to n terms.

25. Find the scale of relation and sum *ad infinitum*
$1 + 3x + 7x^2 + 13x^3 + 25x^4 + 51x^5 + 103x^6 + \cdots$

26. (i.) Sum *ad infinitum*, and find the nth term of the series
$1 + 5x + 15x^2 + 31x^3 + 53x^4 + 131x^5 + \cdots$

(ii.) How many balls are contained in an incomplete rectangular pile of 12 courses, the sides of the base containing 40 and 20?

Ex. 62.

Equations of Higher Degrees.

Find the equations which have the following roots:

1. 1, 2, 3.

2. 1, 1, -2.

3. 2, 3, -5.

4. 3, -4, -7.

5. 7, $+\sqrt{5}$, $-\sqrt{5}$.

6. 2, $+\sqrt{-3}$, $-\sqrt{-3}$.

7. 3, $\frac{-3+\sqrt{5}}{2}$, $\frac{-3-\sqrt{5}}{2}$.

8. 2, 3, $\frac{1}{2}$, $\frac{1}{3}$.

9. -4, -3, $3 \pm \sqrt{5}$.

10. $1 \pm \sqrt{2}$, $\frac{1}{2}(1 \pm \sqrt{-1})$.

Equations Solved like Quadratics.

11. $x^4 - 13x^2 + 36 = 0$.

12. $x^4 - 21x^2 = 100$.

13. $10x^4 - 21 = x^2$.

14. $2x^{\frac{1}{2}} - 3x^{\frac{1}{4}} + x = 0$.

15. $x^2 + 5 = 8x + 2\sqrt{x^2 - 8x + 40}$.

16. $2x^2 + 3\sqrt{x^2 - x + 1} = 2x + 3$.

17. $2x^4 + 5x^2 = 21\frac{3}{8}$.

18. $\frac{x^6}{3} - \frac{x^3}{4} = 2\frac{1}{4}$.

19. $\frac{12x^3 - 11x^2 + 10x - 78}{8x^2 - 7x + 6} = 1\frac{1}{2}x - \frac{1}{2}$.

20. $(8x+3)^2+(8x+3)=30$.

21. $\sqrt[3]{x^2}+3\sqrt[3]{x}=1\frac{3}{4}$.

22. $\sqrt{x^5}+\sqrt{x^3}=6\sqrt{x}$.

23. $3x^{\frac{1}{2}}-2x^{\frac{1}{4}}=-20$.

24. $5x^{2n}+3x^n=6\frac{3}{4}$.

25. $2(x^2-x+1)-\sqrt{x^2-x+1}=\frac{1}{4}$.

26. $x^3-2x^{\frac{3}{2}}+2x-\sqrt{x}=6$.

27. $x^8-97x^4=-1296$.

28. $x^3-31x^{\frac{3}{2}}=-108$.

29. $(2.5-x)^4+0.5625=2.5\,(2.5-x)^2$.

30. $\sqrt{x^2-8x+31}+(x-4)^2=5$.

31. $\frac{8}{x^3}+2=17\div x^{\frac{1}{2}}$.

32. $2x^2+4x+3\sqrt{2x^2+4x+9}=31$.

33. $5x-7x^2+8\sqrt{7x^2-5x+1}=8$.

34. $x^{\frac{1}{2}}-40\div x^{\frac{1}{2}}=3x$.

35. $\frac{2x-\sqrt{2x+11}}{2x+\sqrt{2x+11}}=\frac{9}{19}$.

36. $\frac{x-\sqrt{x}}{x+\sqrt{x}}=\frac{4}{x^2-x}$.

37. $\frac{a+x+\sqrt{a^2-x^2}}{a+x-\sqrt{a^2-x^2}}=\frac{c}{x}$.

38. $9x+4+2x\sqrt{9x+4}=15x^2$.

39. $x^2-7=\sqrt{x^2-42x+89}$.

40. $\frac{x+\sqrt{x}}{x-\sqrt{x}}=\frac{3\sqrt{x}+6}{2\sqrt{x}}$.

Cubic and Reciprocal Equations.

Find the roots of:

41. $x^3 + 10x = 104$.
42. $x^3 - 9x^2 + 26x = 24$.
43. $x^3 - 5x = 12$.
44. $x^3 - 8x^2 + 5x = -14$.
45. $x^3 - 18x = -27$.
46. $x^3 - 18x^2 + 87x = 110$.
47. $x^3 - 5x = -12$.
48. $x^4 - 10x^3 + 35x^2 - 50x = -24$.
49. $x^3 + 2x = 135$.
50. $x^4 - 45x^2 - 40x = -84$.
51. $x^3 = 1$.
52. $x^3 - \frac{1}{8}(14x^3) + 7x = 3\frac{1}{8}$.
53. $x^3 + \frac{8x^2}{5} + \frac{9x}{100} = \frac{9}{100}$.
54. $x^3 - \frac{49x}{3} = \frac{524}{27}$.
55. $x^3 - \frac{13x^2}{12} + \frac{3x}{8} = \frac{1}{24}$.

Find one root of:

56. $x^3 + 6x = 2$, to five decimal places.
57. $x^3 - 6x = 12$, to three decimal places.
58. $x^3 + 9x = -6$, to four decimal places.
59. $x^3 + x^2 = 500$, to four decimal places.
60. $x^3 + 12x = 20$, to four decimal places.

Remove the second term from:

61. $x^3 - 6x^2 + 5x = 84$.
62. $x^2 + 6x = 91$.
63. $x^3 - 15x^2 + 81x - 243 = 0$.
64. $x^3 - 6x^2 + 18x - 22 = 0$.
65. $x^3 + 4x^2 - 5x - 24 = 0$.
66. $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$.

Find the three roots of:

67. $x^3 - 6x^2 + 10x - 8 = 0$.
68. $x^3 + 3x = 5$.

69. $x^3 - 6x^2 - 12x + 112 = 0.$

70. $x^3 + 12x^2 + 45x + 50 = 0.$

71. $x^3 - 21x^2 + 159x - 490 = 0.$

Solve the reciprocal equations:

72. $x^4 + 2x^3 + 2x + 1 = \frac{1}{9}(142x^2).$

73. $x^6 - 6x^5 + 14x^4 - 18x^3 + 14x^2 - 6x = -1.$

Solve by means of a table of sines:

74. $x^3 - 5x + 4 = 0.$

Solve by Horner's Method:

75. $x^3 - 9x = 9.$

Approximate by double position to a value of x in:

76. $x^3 + 10x^2 + 5x = 2600.$

Approximate, by a series of converging fractions, to a value of x in:

77. $x^3 + 7x + 3 = 0.$

Find the roots of: integral roots.

78. $x^3 - 12x - 16 = 0.$

Ex. 63.

Exponential Equations.

Find x:

1. Given $6^x = 7776.$
2. Given $100^x = 250.$
3. Given $10^x = 2.7182818.$
4. Given $2.5^x = 34.56.$
5. Given $\left(\frac{1}{x}\right)^{\frac{1}{2.7182818}} = 0.6922006.$

6. Given $(-1.23)^x = -2.8153057$.

7. Given $2^{3x} \times 5^{2x-1} = 4^{5x} \times 3^{x+1}$.

8. Given $(2\frac{1}{4})^{3\frac{3}{8}} = (3\frac{3}{8})^x$.

9. Given $0.12345^{\frac{3}{5}x} = 1{,}697{,}365$.

10. Given $3^{(5x)} = 7$.

11. Given $(1\frac{2}{3})^{4+\frac{3}{8}x} = 151.884$.

12. Given $a^{mx+n} = b^{nx+m}$.

13. Given $\sqrt[x]{3^{5x+7}} = \sqrt[7x]{5^{3x+1}}$.

Find x and y:

14. Given $a^x a^{5y} = (a^7)^4$, and $a^{7x} \div a^6 = (a^y)^3$.

15. Given $a^x b^y = p$, and $c^x d^y = q$.

16. Given $\sqrt[x]{a} \times \sqrt[y]{a} = a^{\frac{1}{2}}$; $\sqrt[x]{a^3} \div \sqrt[y]{a^4} = 1$.

17. Given $a^x + a^{-x} = b$, to determine the value of x.

18. Given $x^x = 100$, to approximate to a value of x.

19. Given $x^x = 42.8454$, to approximate to a value of x.

20. Given $x^{\frac{1}{x}} = 1.2655$, to approximate to a value of x.

21. Given $x^y = 243$, and $\sqrt[y]{1024} = (\frac{2}{3}x)^2$, to find x and y.

22. Given $\sqrt[x]{x+y} = 2$, and $(x+y) \times 3^x = 279{,}936$, to find x and y.

23. Given $3^x \times 2^y = 72$; $2^x \times 6^z = 144$; $3^y \times 4^z = 432$, to find x, y, and z.

24. Given $10^{(5-x)(6-x)} = 100$, to find x.

25. Write out a proof of the Exponential Theorem.

26. Given $7^{\frac{x+1}{x+2}} = 6.70375^{\frac{x+3}{x+4}}$, to determine x.

WENTWORTH & HILL'S

EXAMINATION MANUALS.

No. II.

ALGEBRA.

BOSTON:
PUBLISHED BY GINN & COMPANY.
1904.

Entered, according to Act of Congress, in the year 1883,
BY GEORGE A. WENTWORTH AND GEORGE A. HILL,
In the Office of the Librarian of Congress at Washington.

J. S. CUSHING & Co., PRINTERS, BOSTON.

PREFACE.

THIS Manual consists of two parts: The first part contains one hundred and fifty examination papers, the questions for which have been selected mainly from the best English, French, and German collections of problems. These papers may be divided into three groups. The first fifty papers embrace the subjects of Elementary Algebra as far as Quadratic Equations; the next fifty papers also include Quadratic Equations and Radical Expressions; the last fifty papers extend over still more ground, including several topics usually regarded as belonging to Higher Algebra.

In each of these groups the earlier papers will be found somewhat easier than the later ones. The papers are intended to be *hour* papers, but if any of them are thought to be too long for one hour, the time may be increased or the length of the paper reduced by omitting one or more of the questions.

The second part of the Manual is a collection of recent papers actually set in various American and English institutions of learning.

There are two ways in which the Manual may be used.

First: To *test* the learner's knowledge in the usual way by means of an examination. For this purpose the class will come to the recitation-room provided with the Manual and blank books, and the teacher will simply designate by number the paper to be worked.

Secondly: To *exercise* the learner from day to day in the various rules and processes, to detect his weak points, and ascer-

tain where he most needs assistance. This may be done by assigning exercises to be worked in the class-room, or by assigning to each member of the class a paper with directions to hand in the solutions, neatly worked out, at a subsequent recitation.

The Manual will be found especially useful in reviewing the subject of Algebra, and in preparing for examinations.

Answers to the problems in the first one hundred and fifty papers, bound separately in paper covers, can be had by teachers *only*, on application to the publishers.

G. A. WENTWORTH.
G. A. HILL

SPECIMEN PAPER WORKED OUT.

1. Simplify

$$a-[2b+\{3c-3a-(a+b)\}+\{2a-(b+c)\}]$$
$$=a-[2b+\{3c-3a-a-b\}+\{2a-b-c\}]$$
$$=a-[2b+3c-3a-a-b+2a-b-c]$$
$$=a-2b-3c+3a+a+b-2a+b+c$$
$$=3a-2c.$$

Ans. $3a-2c$.

2. Resolve into factors

$$x^2-y^2+z^2-a^2-2xz+2ay$$
$$=(x^2-2xz+z^2)-(a^2-2ay+y^2)$$
$$=(x-z)^2-(a-y)^2$$
$$=\{(x-z)-(a-y)\}\{(x-z)+(a-y)\}$$
$$=(x-z-a+y)(x-z+a-y).$$

3. Find the H.C.F. of

$$5x^2(12x^3+4x^2+17x-3)$$
and $10x(24x^3-52x^2+14x-1)$.

$5x^2(12x^3+4x^2+17x-3)$	$10x(24x^3-52x^2+14x-1)$	Reserve $5x$
$12x^3+4x^2+17x-3$	$24x^3-52x^2+14x-1$	2
$12x^3+4x^2-x$	$24x^3+8x^2+34x-6$	
$3)18x-3$	$-5)-60x^2-20x+5$	
$6x-1$	$12x^2+4x-1$	x
	$12x^2-2x$	$2x+1$
	$6x-1$	
	$6x-1$	

Ans. $5x(6x-1)$.

4. Simplify

$$\frac{3}{1-2x}-\frac{7}{1+2x}-\frac{4-20x}{4x^2-1}$$

$$=\frac{3}{1-2x}-\frac{7}{1+2x}-\frac{4-20x}{4x^2-1}$$

$$=\frac{3}{1-2x}-\frac{7}{1+2x}+\frac{4-20x}{1-4x^2}.$$

L.C.D. $= 1-4x^2$.

$3+6x =$ first numerator,

$-7+14x =$ second numerator,

$4-20x =$ third numerator.

$0 =$ sum of numerators. *Ans.* 0.

5. Solve

$$10x+15y-24z=\ 41 \quad (1)$$
$$15x-12y+16z=\ 10 \quad (2)$$
$$18x-14y-\ 7z=-13. \quad (3)$$

Multiply (1) by 2, $20x+30y-48z=\ 82$

Multiply (2) by 3, $45x-36y+48z=\ 30$

Add, $65x-\ 6y\ =112$ (4)

Multiply (2) by 7, $105x-\ 84y+112z=\ 70$

Multiply (3) by 16, $288x-224y-112z=-208$

Add, $393x-308y=-138$ (5)

Multiply (4) by 154, $10010x-924y=17248$

Multiply (5) by 3, $1179x-924y=-414$

Subtract, $8831x\ =17662$

$\therefore x=2.$

Substitute value of x in (4), $130-6y=112.$

$\therefore y=3.$

Substitute values of x and y in (1), $20+45-24z=41.$

$\therefore z=1.$

Ans. $x=2,\ y=3,\ z=1.$

6. A passenger train, after travelling an hour, is detained 15 minutes; after which it proceeds at three-fourths of its former rate, and arrives 24 minutes late. If the detention had taken place 5 miles farther on, the train would have been only 21 minutes late. Determine the usual rate of the train.

Let x = usual rate of train per hour,

and y = number of miles train has to run.

Then $y-x$ = number of miles train has to run after detention,

$\frac{y-x}{x}$ = number of hours usually required to run $y-x$ miles.

and $\frac{y-x}{\frac{4}{5}x}$ = number of hours actually required to run $y-x$ miles,

Since the detention was 15 minutes, and the train was 24 minutes late, the loss in running-time is 9 minutes, or $\frac{3}{20}$ of an hour.

$$\therefore \frac{y-x}{\frac{4}{5}x} - \frac{y-x}{x} = \frac{3}{20} \qquad (1)$$

If the detention had occurred 5 miles farther on, the loss in running-time would have been 6 minutes, or $\frac{1}{10}$ of an hour.

$$\therefore \frac{y-x-5}{\frac{4}{5}x} - \frac{y-x-5}{x} = \frac{1}{10} \qquad (2)$$

Simplify (1), $20y - 29x = 0$

Simplify (2), $20y - 26x = 100$

Subtract (1) from (2), $3x = 100$

$\therefore x = 33\frac{1}{3}$.

Ans. $33\frac{1}{3}$ miles.

7. Solve

$$\frac{1}{2x^2+x+1} + \frac{1}{2x^2-3x+1} = \frac{a}{2bx-b} - \frac{2bx+b}{ax^2-a},$$

$$\frac{1}{(2x-1)(x+1)} + \frac{1}{(2x-1)(x-1)} = \frac{a}{b(2x-1)} - \frac{2bx+b}{a(x-1)(x+1)}.$$

L.C.D. $= ab(x-1)(x+1)(2x-1)$.

Simplify, $abx - ab + abx + ab = a^2x^2 - a^2 - 4b^2x^2 + b^2$,

$2abx = a^2x^2 - a^2 - 4b^2x^2 + b^2$,

$4b^2x^2 - a^2x^2 + 2abx = b^2 - a^2$,

$(4b^2 - a^2)x^2 + 2abx = b^2 - a^2$.

Complete the square, multiplying by 4 times the coefficient of x^2 and adding the square of the coefficient of x,

$4(4b^2-a^2)^2x^2 + (\ ) + (2ab)^2 = 16b^4 - 16b^2a^2 + 4a^4$,

$(4b^2-a^2)2x + 2ab = \pm(4b^2 - 2a^2)$,

$(4b^2-a^2)2x = 4b^2 - 2ab - 2a^2$,

or $2a^2 - 2ab - 4b^2$.

$$x = \frac{2b^2 - ab - a^2}{4b^2 - a^2}, \text{ or } \frac{a^2 - ab - 2b^2}{4b^2 - a^2}.$$

$$\textit{Ans. } x = \frac{b-a}{2b-a}, \text{ or } -\frac{b+a}{2b+a}.$$

8. Solve

$$x + y = 4 \qquad (1)$$
$$x^4 + y^4 = 82 \qquad (2)$$

Put $u + v$ for x, and $u - v$ for y.

(1) becomes $2u = 4.$

$\therefore u = 2.$

(2) becomes $u^4 + 16u^2v^2 + v^4 = 41 \qquad (3)$

Substitute 2 for u in (3), $16 + 24v^2 + v^4 = 41,$

$v^4 + 24v^2 = 25,$

$v^4 + (\ ) + 144 = 169,$

$v^2 + 12 = \pm 13,$

$v^2 = 1 \text{ or } -25,$

$v = \pm 1 \text{ or } \pm\sqrt{-25}.$

Ans. $\begin{cases} x = 3, 1, \text{ or } 2 \pm \sqrt{-25}, \\ y = 1, 3, \text{ or } 2 \mp \sqrt{-25}. \end{cases}$

9. Show that $2\sqrt[3]{a^3b^2}$, $\sqrt[3]{ab^5}$ $\frac{1}{2}\sqrt[3]{\frac{a^6}{b}}$ are similar surds.

$$2\sqrt[3]{a^3b^2} = 2a\sqrt[3]{b^2},$$

$$\sqrt[3]{ab^5} = 2b\sqrt[3]{b^2},$$

$$\tfrac{1}{2}\sqrt[3]{\frac{a^6}{b}} = \tfrac{1}{2}\sqrt[3]{\frac{a^6b^2}{b^3}} = \frac{a^2}{2b}\sqrt[3]{b^2}.$$

Since they all have the same surd factor they are similar surds.

10. Simplify

$$(2ab)^{\frac{1}{3}} \times (3ab^2)^{\frac{1}{3}} \div (5ab^3)^{\frac{1}{3}}$$
$$= (2ab)^{\frac{1}{3}} \times (3ab^2)^{\frac{1}{3}} \div (5ab^3)^{\frac{1}{3}}$$
$$= \sqrt[6]{(2ab)^2} \times \sqrt[6]{(3ab^2)^2} \div \sqrt[6]{5ab^3}$$
$$= \sqrt[6]{2^2a^2b^2} \times \sqrt[6]{3^2a^2b^4} \div \sqrt[6]{5ab^3}$$
$$= \sqrt[6]{2^2 \times 3^2 \times a^5 \times b^7} \div \sqrt[6]{5ab^3}$$
$$= \sqrt[6]{\frac{2^2 \times 3^2 a^5 b^7}{5ab^3}}$$
$$= \sqrt[6]{\frac{2^2 \times 3^2 \times 5^5 a^4 b^4}{5^6}}$$
$$= \tfrac{1}{5}\sqrt[6]{225000\,a^4b^4}.$$

Ans. $\frac{1}{5}\sqrt[6]{225000\,a^4b^4}.$

11. Expand $\left(\frac{2x^2}{y} - \sqrt[3]{y^2}\right)^6$.

$$\left(\frac{2x^2}{y} - \sqrt[3]{y^2}\right)^6 = (2x^2y^{-1} - y^{\frac{2}{3}})^6,$$

$$\begin{aligned}(2x^2y^{-1} - y^{\frac{2}{3}})^6 &= (2x^2y^{-1})^6 - 6(2x^2y^{-1})^5(y^{\frac{2}{3}}) + 15(2x^2y^{-1})^4(y^{\frac{2}{3}})^2 \\ &\quad - 20(2x^2y^{-1})^3(y^{\frac{2}{3}})^3 + 15(2x^2y^{-1})^2(y^{\frac{2}{3}})^4 \\ &\quad - 6(2x^2y^{-1})(y^{\frac{2}{3}})^5 + (y^{\frac{2}{3}})^6 \\ &= 64x^{12}y^{-6} - 192x^{10}y^{-\frac{14}{3}} + 240x^8y^{-\frac{8}{3}} - 160x^6y^{-1} \\ &\quad + 60x^4y^{\frac{2}{3}} - 12x^2y^{\frac{4}{3}} + y^4.\end{aligned}$$

Ans. $64x^{12}y^{-6} - 192x^{10}y^{-\frac{14}{3}}$, etc.

12. Find the value of

$$\sqrt[5]{\frac{3.1416 \times 4771.21 \times 2.7183^{\frac{1}{2}}}{30.103^4 \times 0.4343^{\frac{1}{2}} \times 69.897^4}}.$$

log	3.1416 =	0.4971	= 0.4971
log	4771.21 =	3.6786	= 3.6786
$\frac{1}{2}$ log	2.7183 =	0.4343 ÷ 2	= 0.2172
4 colog	30.103 =	(8.5214 − 10) × 4	= 4.0856 − 10
$\frac{1}{2}$ colog	0.4343 =	0.3622 ÷ 2	= 0.1811
4 colog	69.897 =	(8.1555 − 10) × 4	= 2.6220 − 10
			11.2816 − 20
			30 − 30
			5) 41.2816 − 50
			8.2563 − 10

Ans. 0.01804.

EXAMINATION MANUAL.

ALGEBRA.

1.

1. If $a=1$, $b=3$, $c=5$, $d=0$,
   find the value of $a^2+2b^2+3c^2+4d^2$.
2. Add $a+b-2c$, $8a+4c+2b$, $3c-2a-6b$,
   and $2b-2c+3a$.
3. Multiply $1+x+x^4+x^5$ by $1-x+x^3-x^5$.
4. Resolve into the simplest factors $a^2x^2y-b^2xy^2$.
5. Find the L.C.M. of xy, $x-y$, and y^3-x^2y.
6. Simplify $\frac{7x-10}{5}-\frac{3x-7}{6}-\frac{27x-30}{30}$.
7. Solve $2(x-3)+6(x-2)=54$.
8. Divide a line 12 inches long into two parts, such that the greater shall be five times the less.

2.

1. If $a=1$, $b=-1$, $c=0$, $d=2$,
   find the value of $(ac-bd)(ad-bc)(ab-cd)$.
2. Divide $a^3+ab+2ac-2b^2+7bc-3c^2$ by $a-b+3c$.
3. Resolve into the simplest factors, $a^2x^2-a^2y^2$.
4. Find the H.C.F. of $9x^2-25$ and $9x^2+3x-20$.
5. Add $\frac{a-b}{ab}$, $\frac{a-c}{ac}$, and $\frac{b-c}{bc}$.
6. Solve $5-3(4-x)+4(3-2x)=0$.
7. A and B together have \$4900. A invests $\frac{1}{6}$ of his money in business, and B $\frac{1}{5}$ of his, and each has the same sum remaining. How much money had each?
8. Solve $\left.\begin{aligned}2x+3y&=22\\3x+4y&=31\end{aligned}\right\}$.

3.

1. If $a=1,\ b=0,\ c=-1$,
   evaluate $(1-ab)(1-ac)(1-bc)$.
2. Write down the product of $x+\frac{a}{2}$, $x-\frac{a}{2}$, $x^2+\frac{a^2}{4}$.
3. Simplify $3a-\frac{1}{2}[b+\{2a-(b-x)\}]$.
4. Find the H.C.F. of $6a^6-6a^3x^3$ and $8a^6-8a^5x$.
5. Subtract $\frac{x}{a^2-ax}$ from $\frac{a}{ax-x^2}$.
6. Solve $\frac{2}{7}x+\frac{1}{6}(x-1)=x-4$.
7. Nine years ago A was three times as old as B, but now he is only twice as old. Find the present ages of A and B.
8. Solve $\left.\begin{aligned}\frac{x}{3}+\frac{y}{2}&=\frac{7}{6}\\ \frac{3x}{7}-\frac{2y}{3}&=\frac{4}{21}\end{aligned}\right\}$.

4.

1. Multiply $a-b+c$ by $a+b-c$, and find the value of the product when $a=9,\ b=4,\ c=3$.
2. From $5a+3c-4b-7d-e$
   take $4a+7d+5e-5b-6c$.
3. Divide x^6-y^6 by $x-y$.
4. Solve $\frac{5x}{8}-\frac{11x}{12}=\frac{7x}{9}-(x+5)$.
5. Solve $\left.\begin{aligned}7x+4y&=17y\\ 6x-10y&=8\end{aligned}\right\}$.
6. Resolve into factors $y^2+25y-150$.
7. Simplify $\frac{a^2-4}{a^2+5a}\times\frac{a^2-25}{a^2+2a}$.
8. Find a number such that the sum of its fifth and its seventh parts shall exceed the difference of its fourth and its seventh parts by 99.

5.

1. Simplify $3(x+a)(y-b)-\{-a[b-(c-d)]\}$.
2. Find the H.C.F. of $12x^4-108x^2$ and $16x^4-48x^3$.
3. Find the L.C.M. of $4ab(a^2-b^2)^3$ and $6a^3b^2-6a^2b^3$.
4. Multiply $\frac{x^2+x-2}{x^2-2x-3}$ by $\frac{x^2-x-2}{x^2+2x-3}$.
5. Solve $\frac{1}{x}+\frac{1}{2x}-\frac{1}{3x}=\frac{7}{3}$.
6. What sum is that which is as much greater than \$20 as its half is less than \$20?
7. Two trains travelling, one at 26 and the other at 30 miles an hour, start at the same time from two places 120 miles apart, and move towards each other. How long will it be before the trains meet?

6.

1. Add $2a-3b+c$, $2b-3c+a$, $2c-3a+b$, and $a+b+c$.
2. From $5a^3-7a^2b+6ab^2-b^3+5$ take $3a^3+4a^2b-3+8ab^2-3b^3$.
3. Multiply $x^3-2x^2y+2xy^2-y^3$ by $x^2+2xy+y^2$.
4. Divide $a^3-b^3-c^3-3abc$ by $a-b-c$.
5. Solve $\frac{1}{3}(x-4)+\frac{1}{4}(x+4)=\frac{1}{12}(x+20)$.
6. Simplify $\frac{x}{1+x}-\frac{1}{1-x}-\frac{x^2}{1-x^2}$.
7. Solve $\left.\begin{aligned} x+2y+3z&=10\\ 2x+3y+4z&=16\\ 4x+4y+5z&=25 \end{aligned}\right\}$.
8. A man bought 3 horses and 5 cows, and gave the same sum for the 3 horses as for the 5 cows. If he had bought 4 horses and 10 cows, his outlay would have been \$600 more. Find what his outlay was.

7.

1. If $a=1$, $b=3$, $c=5$, $d=0$, find the value of
$$\frac{12a^3-b^2}{3a^2}+\frac{2c^3}{a+b^2}+\frac{a+b^2+c^3}{b^2-2bc+c^2}+\frac{a^2-d^2}{6a-c}$$
2. Simplify $2-3x-(4-6x)-\{7-(9-2x)\}$.
3. Multiply $1+2x+3x^2$ by $1+4x+5x^2+6x^3$.
4. Find the continued product of
$x+y$, $x-y$, and $x^2+2xy+y^2$.
5. Divide $x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$ by $x+a$.
6. Reduce to the simplest form $\frac{a^2+ab+b^2}{a^2-ab+b^2}\times\frac{a^3+b^3}{a^3-b^3}$.
7. Solve the equation $\frac{x}{4}-\frac{x-5}{3}=\frac{x-2}{6}$.
8. A is twice as old as B and four years older than C. The sum of the ages of A, B, and C is 96 years. Find B's age.

8.

1. Find the value of $\frac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}$ if $a=4$, $b=\frac{1}{2}$, $c=1$.
2. What number diminished by 42 becomes 18?
3. Simplify $\{y^2-(1-y)\}x-\{x^2+(x-y)y\}$.
4. Multiply px^2+qx-r by $mx-n$, and bracket coefficients of the powers of x in the product.
5. Divide $1+x^3-8y^3+6xy$ by $1+x-2y$.
6. Solve $\frac{3x-2}{3}+\frac{90x-9}{9}=5x+4\frac{1}{3}$.
7. Find the length and breadth of a rectangular field if the length is 140 yards more than the breadth, and if it requires 1000 yards of fence to enclose it.
8. Point out first the factors, secondly the terms, and thirdly the coefficients, in the expression $5ax-2ay+abz$.

9.

1. If $a=1$, $b=2$, find the value of
$a^6-3a^5b+6a^4b^2-7a^3b^3+6a^2b^4-3ab^5+b^6$.
2. Divide $x^{4a}+4y^{4b}$ by $x^{2a}+2x^ay^b+2y^{2b}$.
3. Resolve into the simplest factors
ax^2-a; x^4-81a^4; $8a^3+27$; $x^2+7x+10$.
4. Find the L.C.M. of $a+b$, $b-c$, $c+a$, a^2-b^2, b^2+c^2, c^2-a^2.
5. Find the H.C.F. of
$x^2-y^2+xz+yz$ and $x^2+y^2+xz-yz-2xy$.
6. Simplify $2(x-y)-2\left\{3(x-y)-\left(2x-3y-\frac{x-4}{2}\right)\right\}$.
7. Solve $2a(3a-x)-3x(2b-a)=ab$.
8. The sum of two numbers is 70, and their difference is 23; find the numbers.

10

1. If $a=25$, $b=9$, $c=4$, $d=1$, find the value of
$3\sqrt{a}+2\sqrt{4b}-\sqrt{9c}+\sqrt{16d}$.
2. From $7x^3-6x^2+2x-1$ take $4x^3-3x^2-x+2$.
3. Multiply $a^{2n}+a^nx^n+x^{2n}$ by $2a^n-2x^n$.
4. Divide $x^4-9x^2-6xy-y^2$ by x^2+y+3x.
5. Resolve into the simplest factors
$x^{16}-y^{16}$; $8x^6+\frac{1}{x^3}$; $16x^{12}-y^4$; x^2+6x-7.
6. Find the H.C.F. of
$6a^2+7ax-3x^2$ and $6a^2+11ax+3x^2$.
7. Solve $\frac{2x+a}{b}-\frac{(x-b)}{a}=\frac{3ax+(a-b)^2}{ab}$.
8. A is now twice as old as B, but 10 years ago he was three times as old. Find the present ages of A and B.

11.

1. If $a=0$, $b=2$, $c=4$, $d=6$, find the value of
$$3\sqrt[3]{2b^2-a}+2\sqrt[3]{b^2+c^2+7}-\sqrt[3]{2(b+c)^2-(b+d)^2}.$$
2. From $\frac{5}{2}x^4+\frac{1}{2}x^3+3x-1\frac{3}{4}$ take $\frac{1}{2}x^4-\frac{3}{2}x-2\frac{1}{4}$.
3. Divide $1-6x^5+5x^6$ by $1-2x+x^2$.
4. Simplify $(x+y)^2-2x(3x+2y)-(y-x)(-x+y)$.
5. Resolve into the simplest factors
$2x^2-3x-5$ and $a^2-b^2+c^2-2ac$.
6. Find the H.C.F. of
$y^4+m^2y^2+m^4$ and $y^4+my^3-m^3y-m^4$.
7. Solve $\frac{x}{2}-\frac{2x-11}{3}-\frac{x+3}{4}=0$.
8. A person bought 16 yards of cloth, and if he had bought one yard less for the same money, each yard would have cost 25 cents more. What did the cloth cost per yard?

12.

1. Find the H.C.F. of
$12a^2+13ab+3b^2$ and $6a^2+23ab+7b^2$.
2. Simplify $2(x-y)-2\left\{3(x-y)-\left(2x-3y-\frac{x-4}{2}\right)\right\}$.
3. Find the square root of
$9a^4-12a^3b+34a^2b^2-20ab^3+25b^4$.
4. A is twice as old as B; 22 years ago he was four times as old; what is A's age?
5. Multiply $x^2+\frac{3xy}{4}-y^2$ by $x^2-\frac{3xy}{4}+y^2$.
6. Solve $x+\frac{11-x}{3}=\frac{19-x}{2}$.
7. What fraction is that to the numerator of which if 7 be added, its value is $\frac{3}{4}$; but if 7 be taken from the denominator, its value is $\frac{3}{5}$?
8. Solve $x+2y=23$, $3x+4z=57$, $5y+6z=94$.

13.

1. Add $5x^3 - 3x + 2y$, $-x^3 + 2x - y$, and $7x^3 - 4x + 3y$.

2. From $2x + 11a + 10b - 5c - 23$ take $2c - 10 + 5a - 3b$.

3. Multiply $9x^3 + 3xy + y^2 - 6x + 2y + 4$ by $3x - y - 2$.

4. Divide $x^3 - 5x^2 - x + 14$ by $x^2 - 3x - 7$.

5. Solve $\frac{1}{2}(x+1) + \frac{1}{3}(x+2) - 16 + \frac{1}{4}(x+3) = 0$.

6. Resolve into factors $2xy - x^2 - y^2 + z^2$.

7. Simplify $\frac{x}{y} + \frac{y}{x+y} + \frac{x^2}{x^2+xy}$.

8. Solve $\left.\begin{aligned} 3x - 2y &= 5 \\ 4x - 3y + 2z &= 11 \\ x - 2y - 5z &= -7 \end{aligned}\right\}$.

14.

1. Find the square root of $x^4 + 2x^3 - x + \frac{1}{4}$.

2. Find the H.C.F. of $x^4 + 3x^3 - 6x - 4$ and $x^4 - 3x^3 + 6x - 4$.

3. Resolve into factors $x^2 + 20x + 91$.

4. Divide 86 into two parts such that their difference shall be 18.

5. Simplify $\frac{a-b}{b} + \frac{2a}{a-b} - \frac{a^3 + a^2 b}{a^2 b - b^3}$.

6. Divide $1 - \frac{x^2}{x^2 + a^2}$ by $1 + \frac{a^2}{x^2 - a^2}$.

7. Solve $\left.\begin{aligned} x + y + z &= 90 \\ 2x + 40 &= 3y + 20 \\ 2x + 40 &= 4z + 10 \end{aligned}\right\}$.

15.

1. Multiply $a^3-2a^2b+3ab^2+4b^3$ by $a^2-2ab-3b^2$.
2. Divide $x^4+(2b^2-a^2)x^2+b^4$ by x^2+ax+b^2.
3. Find the square root of $\frac{4a^2}{b^2}+8+\frac{4b^2}{a^2}$.
4. Solve $(3x-5)(6x-7)-2(3x-5)^2=7$.
5. Solve $\left.\begin{array}{l} 3x-y+z=17 \\ 5x+3y-2z=10 \\ 7x+4y-5z=3 \end{array}\right\}$.
6. Resolve into factors $x^2-y^2+z^2-a^2-2xz+2ay$.
7. Simplify
$$\frac{1}{(2-m)(3-m)}-\frac{2}{(m-1)(m-3)}+\frac{1}{(m-1)(m-2)}.$$

16.

1. A body moves a feet toward the north and then b feet towards the south. How many feet is the body, at the end of the motion, from the starting point? On which side of the starting point is the body (i.) if $a>b$, (ii.) if $a<b$?
2. Reduce to the simplest form
$$1-\{1-(1-4x)\}+\{2x-(3-5x)\}-\{2-(-4+5x)\}.$$
3. Divide $a^2+ab+2ac-2b^2+7bc-3c^2$ by $a-b+3c$.
4. Resolve into elementary factors $a^6-9a^4b^6$.
5. Find by inspection the H.C.F. of
$4(x^3+a^3)$ and $6(x^2-2ax-3a^2)$.
6. Simplify $\frac{x+y}{y}-\frac{2x}{x+y}+\frac{x^2y-x^3}{x^2y-y^3}$.
7. Solve $\frac{1}{12}(2x-3)-\frac{1}{8}(3x-2)=\frac{1}{8}(4x-3)-3\frac{5}{24}$.
8. A and B have together \$8, A and C have \$10, B and C have \$12. What have they each?

17.

1. Resolve into four factors $(a^2-b^2-c^2)^2-4b^2c^2$.
2. Find the H.C.F. of $4x^3-xy^2$ and $4x^3-10x^2y+4xy^2$.
3. Simplify $3a-(a-b-c)-2\{a+c-2(b-c)\}$.
4. Divide $x^8+x^4y^4+y^8$ by $x^4+x^2y^2+y^4$.
5. Solve $$\left.\begin{aligned} 2x-\frac{y+3}{4}&=7+\frac{3y-2x}{5} \\ 4y+\frac{x-2}{3}&=26\tfrac{1}{2}-\frac{2y+1}{2} \end{aligned}\right\}$$
6. Divide 91 into two such parts that the quotient of the greater part divided by the difference between the parts may be 7.
7. Extract the square root of $16x^4-32x^3+24x^2-8x+1$.

18.

1. From $ax^2+bxy+cy^2$
   take $(b-c)x^2+(c-a)xy+(a-b)y^2$.
2. Multiply $x^2-(a-b)x-ab$ by $x^2+(a-b)x-ab$.
3. Resolve into the simplest factors
   $32x^5+y^5$; $5x^2+8xy-21y^2$; $6a^2x^2-7ax-3$.
4. Simplify $3a-\{2a-(3a-b)^2\}+3a\left\{2b-3a-\frac{b^2}{3a}\right\}$.
5. Find the L.C.M. of $3(1-x^2)$, $6(1-x)^2$, $5(x+x^2)$.
6. Find the H.C.F. of $7x^3-12x+5$ and $2x^3+x^2-8x+5$.
7. Solve $3x-\frac{1}{2}(x-1\frac{1}{2})=9-\frac{1}{4}(5x-7)$.
8. If A can do a piece of work in 8 days, and A and B together can do it in 6 days, how long would B take to do it alone?

19.

1. From $ax^{m+n}+(a+b)x^m+2bx^{m-n}$
   take $bx^m+(a+b)x^{m-n}-ax^{m-2n}$.
2. Find the coefficient of x^3 in the product of
   $x^3-bx^2-3abx+a^2$ and $2x^2-3ax-b^2$.
3. Reduce to lowest terms $\dfrac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}$.
4. Simplify $\dfrac{a}{a+b}+\dfrac{b}{a-b}-\dfrac{a^2+b^2}{a^2-b^2}$.
5. Find the H.C.F. of
   $7a^2-23ab+6b^2$ and $5a^3-18a^2b+11ab^2-6b^3$.
6. Reduce to a single fraction $a-b+\dfrac{1}{1+\dfrac{b}{4a}}$.
7. Solve $\dfrac{2x}{a-2b}-3=\dfrac{x}{2a-b}$.
8. A square floor would contain 17 square yards more if each side were 1 yard longer. Find its area.

20.

1. Divide $2a^{3n}-6a^{2n}b^n+6a^nb^{2n}-2b^{3n}$ by a^n-b^n.
2. Find the H.C.F. of $20x^5+x^3-x$ and $25x^5+5x^4-x^3-x$.
3. Simplify $\dfrac{2ax+3a^2}{4x^2-3ax}\times\dfrac{4ax^3-3a^2x^2}{a^2x^2-a^4}\times\dfrac{a^2-x^2}{2x^2+3ax}$.
4. Resolve into factors $256x^8-1$ and $x^2-7x-18$.
5. Simplify $\left(\dfrac{x}{x+y}+\dfrac{y}{x-y}\right)\div\left(\dfrac{x}{x-y}-\dfrac{y}{x+y}\right)$.
6. Solve $(a+x)(b+x)-a(b+c)=\dfrac{a^2c}{b}+x^2$.
7. A laborer was engaged for 36 days on condition that for every day he worked he was to receive \$3, and for every day he was idle to forfeit \$2. At the end of the time he received \$68. How many days did he work?

21.

1. Resolve into their simplest factors
$a^3 - a^2x - ax^2 + x^3$; $x^3 + 3x^2y - 4xy^2 - 12y^3$.

2. Simplify $\frac{x+1}{2x-1} - \frac{x-1}{2x+1} - \frac{1-3x}{x(1-2x)}$.

3. Simplify $\frac{1}{a(a+b)} + \frac{2b}{a(a^2-b^2)} + \frac{1}{b(a-b)}$.

4. Solve $\frac{8x+5}{14} + \frac{7x-3}{6x+2} = \frac{16x+15}{28} + \frac{2\frac{1}{2}}{7}$.

5. Find the L.C.M. of
$8x^3 - 14x + 6$, $4x^2 + 4x - 3$, and $4x^3 + 2x - 6$.

6. Add $1 - \{2 - (3 - x)\}$, $3x - (4 - 5x)$, $4 - (-5 + 6x)$.

7. Twenty-four persons subscribed the cost of a new boat, but four of the subscribers proving defaulters, each of the others had to pay \$2 more than his due share. Find the cost of the boat.

22.

1. Resolve into their simplest factors
$a^4x^4 - \frac{81}{16}y^4$; $27bx^4 - b^4xy^3$; $20ax^2 + 18a^2x - 18a^3$.

2. Simplify $\dfrac{\left(x + \frac{1}{x}\right)^2 - 2\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2}$

3. Find the L.C.M. of $x^2 - 4y^2$ and $x^2 + xy - 6y^2$.

4. Solve $14(7x + 5) - [84x - (9x - 21)] = 210$.

5. Solve $\frac{2x+1}{3} - \frac{4x+2}{3x-9} = \frac{2x-5}{3}$.

6. A and B can do a piece of work in 8 days, working together; A working alone can do it in 12 days. In how many days can B do it, working alone?

7. State the distinction between an algebraic identity and an algebraic equation, and give an example of each.

23.

1. If $a=0$, $b=2$, $c=-3$, $d=4$, find the value of $4(ad-bc)-\{(a-b)-(c-d)\}^2$.

2. Add $\frac{1}{2}a+\frac{2}{3}b-\frac{3}{4}c+\frac{1}{6}d$ and $\frac{2}{3}a-\frac{5}{6}b+c-\frac{1}{4}d$.

3. Divide $x^6-2a^3x^3+a^6$ by $x^2-2ax+a^2$.

4. Reduce to the lowest terms $\dfrac{x^3+a^3}{x^2+2ax+a^2}$.

5. Solve $\left.\begin{array}{l} 4x+9y=12 \\ 6x-3y=\ 7 \end{array}\right\}$.

6. Extract the square root of $x^4-2x^3+\dfrac{3x^2}{2}-\dfrac{x}{2}+\dfrac{1}{16}$.

7. A person walking at the rate of $4\frac{1}{2}$ miles an hour starts $1\frac{1}{2}$ hours after another person who walks only 4 miles an hour. When and where will he overtake him?

24.

1. If $a=1$, $b=3$, $c=5$, $d=7$, find the value of $a-2b-\{3c-d-[3a-(5b-c-8d)]-2b\}$.

2. Simplify $(x-3)-(3-x)(1-x)-(x-3)(5-2x)$.

3. From $(a+b)^2-2bx$ take $\{(a+b)(a-x)-(a-b)(b-x)\}$.

4. Find the H.C.F. of $12xy(x^5y-2x^3y^3+xy^5)$, $20(x^8-2x^5y^3+x^2y^6)$, and $8y(x^7-xy^6)$.

5. Solve $\left.\begin{array}{l} \dfrac{x+y}{12}-\dfrac{x-y-2}{5}=1 \\ 9y-5x=10. \end{array}\right\}$.

6. Find a number which is as much greater than 63 as its half is less than 93.

25.

1. Resolve into the simplest factors $x^{12} - y^{12}$.
2. Simplify $a^2 - (b^2 - c^2) - \{b^2 - (c^2 - a^2)\} + \{c^2 - (b^2 - a^2)\}$.
3. Find the H.C.F. of $8a^2x^3 + 12a^3x^2$ and $12x^5 + 18ax^4$.
4. Divide $\dfrac{a^2 + ab + ac + bc}{a^2 - ac + ad - cd}$ by $\dfrac{a+b}{a+d}$.
5. Solve $2ax + b = 3cx + 4a$.
6. Solve $\left.\begin{aligned} \frac{1}{x} + \frac{2}{y} &= 7 \\ \frac{2}{y} + \frac{3}{z} &= 21 \\ \frac{3}{z} + \frac{4}{x} &= 19 \end{aligned}\right\}$.
7. The sum of the two digits of a certain number is six times their difference, and the number itself exceeds six times their sum by 3. Find the number.

26.

1. Solve $\left.\begin{aligned} \frac{x}{2} + \frac{y}{3} &= 4 \\ \frac{y}{2} + \frac{z}{8} &= 4 \\ \frac{z}{16} - \frac{x}{12} &= \frac{1}{6} \end{aligned}\right\}$.
2. A person swimming in a stream which runs $1\frac{1}{2}$ miles an hour finds that it takes him four times as long to swim a mile up the stream as it does to swim the same distance down. Find his rate of swimming in still water.
3. Find the H.C.F. of $3x^4 - 4x^3 + 1$ and $4x^4 - 5x^3 - x^2 + x + 1$.
4. Solve $\dfrac{x-7}{2} + \dfrac{x}{9} = \dfrac{x+7}{3}$.
5. Divide $a^2b + (a-b)^2x - 2ax^2 - x^3$ by $b + x$.
6. Simplify $\dfrac{x^2+y^2}{x^2-y^2} - \dfrac{y}{x-y} + \dfrac{x}{x+y}$.

27.

1. Subtract
$(a+y)x^3-(2a^2-y^2)x^2-2xy^3$ from $a^3y-(2x^2-a)y^2+ax^3$, and arrange your answer in descending powers of y.

2. Divide $a^2-4ab+4b^2$ by $\frac{a^2-2ab}{a+2b}$.

3. Solve $\frac{a^2}{x}-\frac{b^2}{x}=a-b$.

4. Simplify $\frac{x^2+xy}{x-y}\div\frac{x^4-y^4}{(x-y)^2}$.

5. Find two consecutive numbers, the difference of the squares of which is equal to 51.

6. Two trains pass a station at an interval of 4 hours, travelling respectively at the rates of $11\frac{1}{2}$ and $17\frac{1}{2}$ miles an hour. How far will the slower train have run before it is overtaken by the other?

7. Solve $\frac{1}{2}x+\frac{1}{3}y=16$; $\frac{1}{3}x-\frac{1}{2}y=2$.

28.

1. Divide x^3 by $1-x^2$ to three terms. What is the difference between the answer thus obtained and the true answer?

2. Arrange according to powers of x
$3(a^2-2x)(a-x^3)+2x(a-6x^2)$.

3. Find the H.C.F. of
$6a^3-6a^2y+2ay^2-2y^3$ and $12a^2-15ay+3y^2$.

4. How do you find the L.C.M. of two expressions when their factors cannot be determined by inspection?

5. Simplify $\frac{x^3+a^3}{2ax-x^2}\times\frac{x^3}{x^2-ax+a^2}\div\frac{4a^2x^2+2ax^3}{4a^2-x^2}$.

6. A, B, and C can reap a field in 30 hours. A can do half as much again as B, and B two-thirds as much again as C. How many hours would each require to do the work alone?

7. Solve $x-\frac{1}{7}(y-2)=5$; $4y-\frac{1}{3}(x+10)=3$.

29.

1. Divide $\frac{a+x}{a-x}+\frac{a-x}{a+x}$ by $\frac{a+x}{a-x}-\frac{a-x}{a+x}$.

2. Simplify $\dfrac{\frac{1}{1+x}+\frac{x}{1-x^2}}{\frac{1}{1-x^2}-\frac{x}{1+x}}$.

3. Solve $5x=43-7y$; $11x+9y=69$.

4. Find the H.C.F. of $3x^6-10x^4+15x^3-8x$ and $2x^5-4x^4+2x^3$.

5. Find all the expressions whose L.C.M. is x^3-4xy^2

6. A number consists of two digits. The sum of the digits is 7, and if 27 be subtracted from the number the order of the digits will be reversed. Find the number.

7. Find the value of $\frac{3}{1+x}+\frac{3}{1-x}$, when $x=\frac{1}{2}$.

30.

1. A is x years old; B is 20 years younger. Express in algebraic language the age of B; the sum of their ages; the age of A five years ago; the age of B five years hence.

2. Simplify $(2a-b)^2+2b(a+b)-3a^2-(a-b)^2+(a+b)(a-b)$.

3. Multiply $a-\frac{x^2}{a}$ by $\frac{a}{x}+\frac{x}{a}$.

4. Simplify $\dfrac{a(x^2+c^2)-2acx}{b(x^2+c^2)-2bcx}$.

5. Find the H.C.F. of
$3a^3-7a^2b-33ab-7ab^2+11b^3+3b^3$ and $3a^2-10ab+3b^2$.

6. Solve $5x+2y-z=2z-x-y=y+z-2x=6$.

7. If a father was four times as old as his son seven years ago, and will be twice as old as his son in seven years more, what is the present age of each?

31.

1. What is meant by a *term*, an *index*, a *factor*? Give examples.

2. Solve $\frac{m}{x}+\frac{n}{y}=a;\quad \frac{n}{x}+\frac{m}{y}=b.$

3. Solve $\left.\begin{array}{rrrr} 2x- & 4y+ & 9z= & 28 \\ 7x+ & 3y- & 5z= & 3 \\ 9x+ & 10y- & 11z= & 4 \end{array}\right\}.$

4. Find the H.C.F. of
$2x^2+3xy+6x+9y$ and $3x^2-2xy+9x-6y.$

5. Simplify $\frac{x^4-y^4}{x^2-2xy+y^2}\div\frac{x^2y+y^3}{x^3-y^3}.$

6. Find a fraction such that if 5 be added to the numerator the fraction equals 1, but if 3 be taken from the denominator the fraction equals $\frac{1}{2}$.

7. Arrange according to descending powers of x
$x^3-ax-c^2x^2-bx+bx^3-cx^2+a^2x^3-x^3-cx.$

32.

1. What is a common multiple of two or more quantities? What is meant by their least common multiple?

2. Find the H.C.F. of
$4x^4+22x^3-26x^2-198x-90$ and $4x^3-14x^2-92x-42.$

3. Simplify $\frac{m}{m+n}+\left(\frac{n}{m+n}\right)^2+\frac{mn}{(m+n)^2}.$

4. Solve $(a+x)(b+x)-a(b+c)=\frac{a^2c}{b}+x^2.$

5. Solve $\frac{2x}{5}+\frac{y}{6}=5;\quad \frac{5x}{2}-4y=1.$

6. Solve $x+y=16;\quad 2z-2x=8;\quad 2y+3z=51.$

7. At an election there were two candidates, and 1296 persons voted; the successful candidate had a majority of 120. How many votes were cast for each?

8. What is the rule for transposing terms in equations? What is the reason for the rule?

33.

1. Solve $\frac{a+x}{a-b}+\frac{a-x}{a+b}=x-\frac{a^2x}{a^2-b^2}$.

2. Solve $x+y=a$; $bx-cy=d$.

3. Solve $5x-3y+2z=19$; $4x+5y-3z=31$; $3x+7y-4z=31$.

4. Simplify $\frac{a+b}{a-b}-\frac{a-b}{a+b}+\frac{4ab}{b^2-a^2}$.

5. Reduce to lowest terms $\frac{21x^2+14xy}{18ax+12ay}$.

6. Divide $x^6-2a^3x^3+a^6$ by $x^2-2ax+a^2$.

7. Simplify $2\{a(a-b)-(a+b)^2\}\{a^2-2b^2\}$.

8. The difference between two numbers is one-fourth of their sum, and the half of the greater diminished by 7 is equal to one-fifth of the less increased by $2\frac{1}{2}$. Find the numbers.

34.

1. Define *factor*, *common factor*, *highest common factor*.

2. Find the H.C.F. and the L.C.M. of
$(2a^2bx+2abx^2)^2$ and $(6a^2b^3-6b^2x^2)^3$.

3. Multiply a^6+a^5-a-1 by $1-a+a^2-a^3+a^4$.

4. Simplify $\dfrac{1}{1+x+\dfrac{1}{1+x+\dfrac{1}{1+x}}}$.

5. Solve $\frac{a}{b^2x}+\frac{b}{a^2x}=a^2+b^2$.

6. Solve $x+y=5(x-y)=10$.

7. A, B, C, and D have \$290 between them. A has twice as much as C, and B has three times as much as D. Also, C and D together have \$50 less than A. Find how much each has.

8. Expand $(2x^2yz^3)^5$; $(-7a^3b^4c^5)^3$; $(-3ab^4c^2)^4$.

35.

1. Simplify $\dfrac{a+b}{(b-c)(a-c)} - \dfrac{b+c}{(a-b)(c-a)} + \dfrac{a+c}{(b-a)(b-c)}$.

2. Simplify $\dfrac{a^3+3a^2b+3ab^2+b^3}{a^3-3a^2b+3ab^2-b^3} \times \dfrac{a^2-2ab+b^2}{a^2+2ab+b^2}$.

3. Solve $\frac{1}{2}x+\frac{1}{3}y=12-\frac{1}{6}z$; $\frac{1}{2}y+\frac{1}{3}z=8+\frac{1}{6}z$; $\frac{1}{2}x+\frac{1}{3}z=10$.

4. Find the L.C.M. of $9a^2-81$, $12a^2-36a$, and $16a^2+48a$.

5. Extract the square root of $x^4-8x^3-26x^2+168x+441$.

6. Two persons start at noon from towns 60 miles apart. One walks at the rate of 4 miles an hour, but stops $2\frac{1}{2}$ hours on the way. The other walks at the rate of 3 miles an hour, without stopping. When and where will they meet?

7. How many different equations must be given when the values of two or more unknown quantities are required? In what sense must the equations differ?

36.

1. Find the value of $x-(\sqrt{x+1}+2)-\dfrac{x-\sqrt[3]{x}}{\sqrt{x-4}}$ when $x=8$.

2. Divide $a^2x^4-c^2y^4-bx^2y(2ax-by)$ by $(ax-by)x-cy^2$.

3. Find the H.C.F. of $3ax^2+2axy^2-3bx^2-2by^2$ and $3ax^2-2axy^2-3bx^2+2by^2$.

4. Find the L.C.M. of a^2+x^2, a^2-x^2, a^3+x^3, and a^3-x^3.

5. Simplify $\dfrac{a^4-x^4}{a+3} \times \dfrac{1}{a^2-2x-a(a-2)} \times \dfrac{a^2+5a+6}{(a^2+x^2)(a+x)}$.

6. A is 5 years more than twice as old as B, and is also 25 years older than B. Find their present ages.

7. What are simultaneous equations? Give an example.

8. Expand $\left(\dfrac{a}{b}+\dfrac{b}{a}\right)^3$.

37.

1. Simplify $\dfrac{bc}{(a-b)(a-c)}+\dfrac{ac}{(b-a)(b-c)}-\dfrac{ab}{(b-c)(c-a)}$.

2. Simplify $\dfrac{\left(\dfrac{a}{b}-\dfrac{b}{a}\right)\left(\dfrac{a+b}{a}-\dfrac{a+b}{b}\right)}{\left(\dfrac{a}{b^2}-\dfrac{b}{a^2}\right)\left(\dfrac{1}{a}-\dfrac{1}{b}\right)}$.

3. Solve $\dfrac{1}{x}+\dfrac{2}{y}+\dfrac{3}{z}=\dfrac{23}{20}$; $\dfrac{1}{y}+\dfrac{2}{z}+\dfrac{3}{x}=\dfrac{77}{60}$; $\dfrac{1}{z}+\dfrac{2}{x}+\dfrac{3}{y}=\dfrac{38}{30}$.

4. If a rectangular court had its length diminished by 7 feet, and its width increased by 5 feet, it would become an exact square, enclosing the same area. Find the dimensions of the court.

5. Extract the square root of $x^4-x^3+\dfrac{5x^2}{4}-\dfrac{x}{2}+\dfrac{1}{4}$.

6. Expand $\left(\dfrac{x^2}{y^2}-\dfrac{y}{x}\right)^3$.

38.

1. Find the product of $\dfrac{2x}{a}-\dfrac{3y}{b}$ and $\dfrac{4x^2}{a^2}+\dfrac{6xy}{ab}+\dfrac{9y^2}{b^2}$.

2. Find the H.C.F. of $x^3+6x^2+11x+6$ and $x^3+9x^2+27x+27$.

3. Simplify $\dfrac{3x^2+1}{x^2-1}-\dfrac{5x-2}{x+1}+\dfrac{2x+5}{x-1}$.

4. Extract the square root of $x^4+\dfrac{2x^3}{3}+\dfrac{10x^2}{9}+\dfrac{x}{3}+\dfrac{1}{4}$.

5. Solve $ax-\dfrac{b^2}{a}x=a+b$.

6. A and B have each an annual income of \$4000. A spends every year \$400 more than B. At the end of 4 years their joint savings amount to \$4000. What does each spend annually?

7. Expand $\left(2a-\dfrac{3}{x}\right)^3$.

8. Explain how to clear an equation of fractions, and the reason for the method.

39.

1. Simplify $\frac{1}{a(a-b)(a-c)}+\frac{1}{b(a-b)(c-b)}-\frac{1}{c(c-a)(b-c)}$.

2. Solve $\frac{x-3}{7}-\frac{x-6}{6}=\frac{x+12}{12}-\frac{x-6}{3}+\frac{x}{8}$.

3. Solve $\frac{x}{4}+\frac{y}{2}=8$; $\frac{x}{2}+\frac{y}{6}=6$.

4. Extract the square root of $\frac{a^2}{9b^2}-2+\frac{9b^2}{a^2}$.

5. Simplify $1-\{1-(1-4x)\}+\{2x-(3-5x)\}-\{2-(-4+5x)\}$.

6. Find the L.C.M. of a^2-1, $(ab-b)c$, and $(ac+c)b$.

7. A and B can mow 225 square rods in a day; B and C can mow 223 square rods in a day; A and C can mow 230 square rods in a day. How many square rods can each mow by himself?

40.

1. Simplify $3b\{a(a-b)(3a-b)-(a-b)^2\}$.

2. Find the H.C.F. of
$12a^2+13ab+3b^2$ and $6a^2+23ab+7b^2$.

3. Find the L.C.M. of
$4(1-x)^2$, $8(1-x)$, $8(1+x)$, $4(1+x^2)$.

4. Find the cube root of
$1-6x+21x^2-44x^3+63x^4-54x^5+27x^6$.

5. Simplify $\left\{\frac{1}{3}+\frac{2a}{3(1-a)}\right\}\times\left\{\frac{3}{4}-\frac{3a}{2(1+a)}\right\}$.

6. Solve $\frac{x}{2}-1=\frac{y}{3}+1=\frac{z}{4}$; $x+y+z=17$.

7. At what time between two and three o'clock are the hour and minute hands of a clock in a straight line and pointing in opposite directions?

41.

1. Find the square root of $1-xy-\frac{15}{4}x^2y^2+2x^3y^3+4x^4y^4$.

2. Simplify $\dfrac{\frac{2}{3}(2-\frac{1}{2}a)-a}{2-\frac{1}{2}(2+2a)}-\dfrac{\frac{3}{4}(6a+4)+\frac{1}{2}a}{1+2a}+2$.

3. Solve $x+y+z=6$; $x+y-z=4$; $x-y+z=2$.

4. Multiply $\dfrac{2x}{3}-\dfrac{1}{2}$ by $\dfrac{x^2}{2}-\dfrac{x}{3}+\dfrac{1}{4}$.

5. A boat is rowed 1 mile in $5\frac{1}{2}$ minutes; another is rowed 1 mile in $5\frac{3}{4}$ minutes. If they start at the same time from opposite ends of a 4-mile course, at what distance from each starting-point will they meet?

6. Expand $\left(\dfrac{a}{2}+\dfrac{2b}{3}\right)^3$.

7. Solve $\dfrac{2x+a}{3(x-a)}+\dfrac{3x-a}{2(x+a)}=2\frac{1}{6}$.

42.

1. Simplify $\dfrac{x}{x-3}-\dfrac{x-3}{x}+\dfrac{x}{x+3}-\dfrac{x+3}{x}$.

2. Find the H.C.F. of $3x^3-3x^2y+xy^2-y^3$ and $4x^2-xy-3y^2$.

3. Find the cube root of $a^3-a^2b+\dfrac{ab^2}{3}-\dfrac{b^3}{27}$.

4. Solve $(2+x)(8+x)-22=\frac{3}{2}+x^2$.

5. Solve the simultaneous equations
$3x=2(y+2)-\frac{1}{3}$; $2y=3(z-x)+\frac{3}{2}$; $z=x+y-\frac{2}{3}$.

6. Find the value of $2(1+\sqrt{a})-\dfrac{\sqrt{x}-\sqrt{a}}{3}+\dfrac{x-a}{\sqrt{a}+\sqrt{x}}$ when $x=16a$.

7. 36 octavo volumes exactly fill a box whose length and breadth are $1\frac{1}{2}$ feet and $11\frac{1}{3}$ inches, respectively. The dimensions of each volume are 9 inches by $5\frac{2}{3}$ inches by $1\frac{1}{2}$ inches. Find the depth of the box.

43.

1. Find the L.C.M. of $9x^3+53x^2-9x-18$ and $x^2+11x+30$.
2. Find the square root of $x^4-4x^3+x^2+6x+\frac{9}{4}$.
3. Solve $3x+4y-5z=32$; $4x-5y+3z=18$; $5x-3y-4z=2$.
4. Divide $5x^4+\frac{7ax^3}{2}-\frac{107}{12}a^2x^2+\frac{5a^3x}{6}+\frac{7a^4}{6}$ by $2x^2-ax-\frac{a^2}{2}$.
5. Simplify $\dfrac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}$.
6. Simplify $\dfrac{3xyz}{yz+xz-xy} \cdot \dfrac{\dfrac{x-1}{x}+\dfrac{y-1}{y}+\dfrac{z-1}{z}}{\dfrac{1}{x}+\dfrac{1}{y}-\dfrac{1}{z}}$
7. Reduce to lowest terms $\dfrac{3x^3-2x^2-x}{6x^2-x-1}$.

44.

1. Simplify $\dfrac{1}{(2x-1)^2}+\dfrac{1}{(2x+1)(1-2x)}$.
2. Divide $\dfrac{4a^2-4ax}{(a+x)^2}$ by $\dfrac{a^2-x^2}{4ax+4x^2}$.
3. Add together $\dfrac{a^2}{2}-\dfrac{b^2}{3}+\dfrac{c^2}{4}$, $\dfrac{b^2}{2}-\dfrac{c^2}{3}+\dfrac{a^2}{4}$, $\dfrac{c^2}{2}-\dfrac{a^2}{3}+\dfrac{b^2}{4}$.
4. Find the L.C.M. of
   $x^3+x^2y+xy^2+y^3$ and $x^3-x^2y+xy^2-y^3$.
5. Extract the square root of $a^2+4b^2+c^2+2ac+4ab+4bc$.
6. Solve $\dfrac{a}{x-b}=\dfrac{b}{x-a}$.
7. Divide the number 90 into four parts so that the first part increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, may be all equal.
8. What powers of a quantity are the same whether the quantity be positive or negative?

45.

1. Simplify $\dfrac{1}{1+3a+2a^2}+\dfrac{1}{1-a-2a^2}$.
2. Multiply $1-x-x^2$ by x^2-x-1.
3. Find the H.C.F. of
   $x^3+5x^2+10x+8$ and x^5+2x^4-x-2.
4. Find the L.C.M. of
   $1-x^2$, $1-2x+x^2$, $1-x+x^2-x^3$, $1-x-x^2+x^3$.
5. Reduce to lowest terms $\dfrac{2x^2-xy-y^2}{y^2+xy-2x^2}$.
6. Expand $\left(\dfrac{ab}{4}-\dfrac{1}{ab}\right)^3$.
7. Solve $\dfrac{1}{y}+\dfrac{1}{z}-\dfrac{1}{x}=6$; $\dfrac{1}{z}+\dfrac{1}{x}-\dfrac{1}{y}=4$; $\dfrac{1}{x}+\dfrac{1}{y}-\dfrac{1}{z}=2$.

46.

1. Reduce to a simple fraction $\dfrac{x}{a-\dfrac{x}{a-\dfrac{a}{x}}}$.
2. Reduce to its lowest terms $\dfrac{x^3-3x^2+7x-21}{2x^4+19x^2+35}$.
3. Find the square root of $\dfrac{4x^4-12x^3y+x^2y^2+12xy^3+4y^4}{16x^2y^2}$.
4. Solve $3x-a=\dfrac{a+x}{3}-\dfrac{b-x}{a}-cx$.
5. The united ages of a father and son make 75 years. The father was 25 years old when the son was born. What is the age of each?
6. Solve $\left.\begin{aligned}5x-3y-28&=0\\3x-2y-13&=0\end{aligned}\right\}$.
7. Subtract ax from $\dfrac{2ax+a^2x^2+2x}{2+ax-2x^2}$.

47.

1. Multiply $x^3 - x^2 - x - 1$ by $1 - x + x^2 - x^3 + x^4$.
2. Find the H.C.F. of
   $[illegible]$ and $[illegible]$.
3. Find the L.C.M. of
   $x^2 - 1$, $x^3 - 1$, $x^2 - 1$, $x - 1$, $x - 1$.
4. Solve
   $$x + \tfrac{1}{2}y - \tfrac{1}{2}z = [illegible]$$
   $$\tfrac{1}{2}x - y + \tfrac{1}{2}z = [illegible]$$
   $$\tfrac{1}{2}x + \tfrac{1}{2}y - z = [illegible]$$
5. What sum is that which is as much greater than $\frac{3}{2}$, as its half is less than $\frac{3}{2}$?
6. A number consisting of two digits when divided by the sum of the digits gives [illegible] for the quotient, and the number formed by interchanging the digits, when divided by 6, gives 4 for the quotient. Find the number.
7. What powers of a quantity are always positive? What powers of a negative quantity are always negative?

48.

1. Simplify $\dfrac{x - 1}{\dfrac{1}{x - 1} - \dfrac{1}{x - 1} - \dfrac{1}{1 - x^2}}$.
2. Simplify
   $$\frac{x^2 - y^2}{x^2 + 2xy + y^2} \div \frac{x^2 - \frac{1}{2}x}{x - \frac{1}{2}} \qquad \frac{x^3 - \frac{1}{2}x^2}{x - y} \div \frac{x^3 - \frac{1}{2}x^2}{x - y}.$$
3. Simplify $\dfrac{1}{2x^2 + x - 1} - \dfrac{1}{\frac{1}{2}x^2 - 4x - 1}$.
4. Reduce to lowest terms $\dfrac{\frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{2}}{\frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{2}}$.
5. Simplify $\dfrac{x^2 - \frac{1}{2}x - 4}{x^2 - \frac{1}{2}x - 12} \div \dfrac{x^2 - \frac{1}{2}x - 1}{x^2 - \frac{1}{2}x - 2}$.
6. Simplify $\dfrac{2m^2 - 2n^2 - m^2n^2 - m^4}{2m^2 - 2n^2 - m^2n^2 - n^4}$. What would the result be if $n = -m^2$?

49.

1. Solve $\frac{a+b}{2b} - \frac{1}{2}c\left(\frac{a-b}{bx}\right) = \frac{bc}{(a+b)x} + \frac{a}{a+b}$.

2. Solve $ax + by = c$; $a'x + b'y = c'$.

3. Solve $x+y+z=5$; $3x-5y+7z=75$; $9x-11z+10=0$.

4. Solve $\frac{1}{x}+\frac{1}{y}=m$; $\frac{1}{y}+\frac{1}{z}=n$; $\frac{1}{x}+\frac{1}{z}=p$.

5. Solve

$$\frac{x}{3}+\frac{y}{5}+\frac{2z}{7}=58;\quad \frac{5x}{4}+\frac{y}{6}+\frac{z}{3}=76;\quad \frac{x}{2}-\frac{y}{5}+\frac{7z}{40}=29\tfrac{3}{5}.$$

50.

1. A person walking at the rate of $4\frac{3}{4}$ miles per hour starts $1\frac{1}{2}$ hours after another person who walks only 4 miles an hour. When and where will he overtake him?

2. A number consists of three digits, the right-hand digit being 0. If the left-hand and the middle digits be interchanged, the number will be diminished by 180. If the left-hand digit be halved, and the middle and right-hand digits be interchanged, the number will be diminished by 454. Find the number.

3. A had twice as much money as B; but when B had received \$5 from A, he had three times as much as A. How much had they each at first?

4. A tank of water may be filled by three pipes. By the first pipe it can be filled in 4 hours, by the second in 10 hours, by the third in 15 hours. In what time will it be filled if all three pipes are opened?

   What will be the solution of this question if the general symbols m, n, p are used in place of the numbers 4, 10, 15, respectively?

5. What are simultaneous equations? Give an example.

51.

1. Bracket like powers of x in the expression
$$ax^3 - bx^2 - cx - bx^3 + cx^2 - dx + cx^3 - dx^2 - ex.$$

2. Divide $a^3 - b^3 + c^3 + 3abc$ by $a - b + c$.

3. Simplify $\frac{a}{b} - \frac{(a^2 - b^2)x}{b^2} + \frac{a(a^2 - b^2)x^2}{b^2(b + ax)}$.

4. Divide 150 into two parts such that if one is divided by 23, and the other by 27, the sum of the quotients is 6.

5. Solve $\frac{3x^2}{4} - \frac{15x^2 + 8}{6} = 2x^2 - 3$.

6. Solve $2x = 4 + \frac{6}{x}$.

52.

1. Subtract $(a-b)x-(b-c)y$ from $(a+b)x+(b+c)y$.

2. Find the difference in value between the arithmetical expression 57 and the algebraical expression ab, when $a = 5$ and $b = 7$.

3. Find by inspection the H.C.F. of
$9(a^2x^2 - 4)$ and $12(a^2x^2 + 4ax + 4)$.

4. Simplify $(4a^{-\frac{1}{2}})^{-\frac{1}{2}}$.

5. Solve $\frac{x+2}{x-1} - \frac{4-x}{2x} = \frac{7}{3}$.

6. There is a rectangular field whose length exceeds its breadth by 16 yards. The field contains 960 square yards. Find its dimensions.

53.

1. Find the L.C.M. of

$$4(a^3 - ab^2), \quad 12(ab^2 + b^3), \text{ and } 8(a^3 - a^2b).$$

2. Simplify $\dfrac{1}{2(a-x)} + \dfrac{1}{2(a+x)} + \dfrac{a}{a^2+x^2}$.

3. Simplify $(64c^{16})^{-\frac{3}{4}}$.

4. Divide $16x - y^2$ by $2x^{\frac{1}{2}} - y^{\frac{1}{2}}$.

5. Solve $1 - \dfrac{x+5}{2x+1} = \dfrac{x-6}{x-2}$.

6. Find two numbers in the ratio of $2\frac{1}{2}$ to 2, such that when diminished each by 5 they shall be in the ratio of $1\frac{1}{3}$ to 1.

54.

1. Solve $\left.\begin{aligned} x + 2y &= 7 \\ y + 2z &= 2 \\ 3x + 2y &= z - 1 \end{aligned}\right\}$.

2. The sum of \$330 is laid out in two investments, on one of which 15 per cent is gained, and on the other 8 per cent is lost. The total amount returned from the investments is \$345. Find each investment.

3. Simplify $\{(a^{-3}b^x)^{\frac{1}{2}}\}^{-\frac{2}{3}}$.

4. Divide $x^{-1} - y^{-1}$ by $x^{-\frac{1}{2}} - y^{-\frac{1}{2}}$.

5. Reduce to an *entire* surd the expression $\frac{1}{2}(\frac{2}{3})^{-\frac{2}{3}}$.

6. Solve $\dfrac{2}{x-1} - \dfrac{1}{x+3} = \dfrac{3}{8}$.

55.

1. Find the L.C.M. of
$6(x^2y + xy^2)$, $9(x^3 - xy^2)$, and $4(y^3 + xy^2)$.

2. Simplify $\frac{x}{1-x} - \frac{x^2}{(1-x)^2} + \frac{x^3}{(1-x)^3}$.

3. Arrange in order of magnitude $2\sqrt[3]{22}$, $3\sqrt[3]{7}$, $4\sqrt{2}$.

4. Find the cube root of
$x^6 - 3ax^5 + 5a^2x^3 - 3a^5x - a^6$.

5. Simplify $3\sqrt[3]{432}$.

6. Solve $\frac{5x}{x+4} - \frac{3x-2}{2x-3} = 2$.

56.

1. Show that $\sqrt{20}$, $\sqrt{45}$, $\sqrt{\frac{4}{5}}$ are similar surds.

2. Simplify $2\sqrt{ax} \times \sqrt[3]{3a^2b} \times \sqrt{2bx}$.

3. Simplify $\sqrt[3]{128} + \sqrt[3]{686} + \sqrt[3]{16}$.

4. Simplify $\frac{2x^2 - 8x + 6}{x^2 - 5x + 4} \times \frac{x^2 - 9x + 20}{x^2 - 10x + 21} \times \frac{x^2 - 7x}{2x^2 - 10x}$.

5. Solve $\frac{1}{8}x^2 + \frac{11}{15}x = 3\frac{1}{12}$.

6. If each side of a square plot were enlarged by 10 yards, it would contain $1\frac{1}{2}$ acres more. Find the dimensions of the plot. (1 acre = 4840 sq. yds.)

57.

1. Simplify $\{x^{-\frac{1}{2}}y(xy^{-2})^{-\frac{1}{2}}(x^{-1}y)^{-\frac{1}{2}}\}^{2}$.

2. Simplify $\sqrt{128}-2\sqrt{50}+\sqrt{72}-\sqrt{18}$.

3. By selling a cow for \$24 I lose as much per cent as the cow cost me. What was the cost of the cow?

4. Solve $\dfrac{x}{100}+\dfrac{21}{25x}=-\dfrac{1}{4}$.

5. Solve $\left.\begin{aligned} x+y+z&=5\\ x+y&=z-7\\ x-3&=y+z \end{aligned}\right\}$.

6. A person has travelled 3036 miles. He has gone 7 miles by water to 4 on foot, and 5 by water to 2 on horseback. How many miles has he travelled each way?

58.

1. Solve $\dfrac{27-x}{7}+\dfrac{3x-4}{5}-\dfrac{5x-2}{9}=2$.

2. Find by inspection the L.C.M. of
$a^{2}-1$, $a^{2}+2a-3$, and $a^{3}-7a^{2}+6a$.

3. A steamer takes two hours and forty minutes less time to travel from A to B than from B to A. The steamer travels at the rate of 14 miles per hour in still water, and the stream flows at the rate of $1\frac{1}{2}$ miles per hour. Find the distance from A to B.

4. Which is the greater $\frac{1}{2}\sqrt{2}$ or $\frac{1}{3}\sqrt[4]{27}$?

5. Solve $\dfrac{15}{x}-\dfrac{72-6x}{2x^{2}}=2$.

6. A and B enter into partnership with a joint capital of \$3400. A put in his money for 12 months; B put in his money for 16 months. In closing the business, A's share was \$2070 including his profit, and B's share \$1920. Find the sum put in by each.

59.

1. Solve $x+ay=b$; $ax-by=c$.

2. If $a=4$, $b=3$, $c=1$, $d=7$, find the value of
$$\frac{a^2+ac+b^2}{a^2-ac+b^2}-\frac{\sqrt{4ab+b^2+d}}{\sqrt{4ab-b^2-2d}}-\frac{c}{a+b+c+d}.$$

3. Find the H.C.F. of
$$x^4+3x^3-6x-4 \text{ and } x^4-3x^3+6x-4.$$

4. Simplify $\frac{1}{x-y}+\frac{1}{x+y}-\frac{2}{x}$.

5. Find the square root of $a^2-4ax+6a+4x^2-12x+9$.

6. When are the hour and minute hands of a clock together between 10 and 11 o'clock?

7. Solve $\frac{1}{3}(x-1)=\frac{1}{2}(x+1)-\frac{1}{6}(x-1)^2$.

60.

1. If $x=8$, $y=1$, find the value of
$$\sqrt[3]{x}\left\{y^3+\sqrt[4]{\frac{x^2+2xy+y^2}{y^2}}\right\}.$$

2. Simplify $\frac{x^4-y^4}{x^3-2xy+y^2}\div\frac{x^2y+y^3}{x^3-y^3}$.

3. Find the H.C.F. of
$$x^3-3x^2+7x-21 \text{ and } 2x^4+19x^2+35.$$

4. Find the square root of $\left(x^2+\frac{x^2}{y}-2x\right)\left(1+\frac{1}{y}\right)+1$.

5. Solve $\frac{1}{12}x-\frac{1}{8}(8-x)-\frac{1}{4}(5+x)+\frac{11}{4}=0$.

6. Find a fraction such that if 5 be added to the numerator, it will be equal to 1, but if 3 be taken from the denominator, it will be equal to $\frac{1}{2}$.

7. A boat's crew can row 5 miles an hour in still water. If they row $3\frac{1}{2}$ miles down a river and then back again to the starting-point in 1 hour 40 minutes, what is the velocity of the current?

61.

1. Solve $7x^2 - 11x = 6$.

2. Solve $\frac{x}{a+b} + \frac{x}{a-b} = \frac{4}{a^2-b^2}$.

3. Solve $\frac{x}{a} + \frac{y}{b} = 2$; $bx - ay = 0$.

4. Find the H.C.F. of
$48x^2 + 16x - 15$ and $24x^3 - 22x^2 - 7x + 5$.

5. Divide $a^3 + b^3 + c^3 - 3abc$ by $a + b + c$.

6. Extract the square root of $x^4 - 2x^3 + \frac{3x^2}{2} - \frac{x}{2} + \frac{1}{16}$.

7. A vessel containing 120 gallons was emptied in 10 minutes by two pipes running *successively;* the first pipe discharged 14 gallons in a minute, and the second pipe discharged 9 gallons in a minute. How many minutes did each pipe run?

62.

1. Subtract $a^2 - 13ab + 10ac - c^2$ from $7a^2 + 12ab - 3ac - b^2$, and find the value of the remainder, when $a = 1$, $b = 2$, $c = -3$.

2. Divide $x^5 - \frac{1}{x^5}$ by $x - \frac{1}{x}$.

3. Simplify $\frac{m+n}{m-n} \div \left[\frac{(m+n)^3 - (m-n)^3}{(m^2-n^2)(3m^2+n^2)}\right]$.

4. Find the H.C.F. of
$96x^4 + 8x^3 - 2x$ and $32x^3 - 24x^2 - 8x + 3$.

5. Extract the square root of
$a^2 - 4ab^2 - 6ac^3 + 4b^2(b^2 + 3c^3) + 9c^6$.

6. Solve $5x + \frac{2}{3}y = 52$; $\frac{3}{5}x + 4y = 27$.

7. Solve $5x + \frac{4x+3}{3x-2} = \frac{10x^2-7}{2x-1} + 9$.

8. When are the hour and minute hands of a watch at right angles between 3 and 4 o'clock?

43.

1. Find the L.C.M. of $9x^3+53x^2-9x-18$ and $x^2+11x+30$.
2. Find the square root of $x^4-4x^3+x^2+6x+\frac{9}{4}$.
3. Solve $3x+4y-5z=32$; $4x-5y+3z=18$; $5x-3y-4z=2$.
4. Divide $5x^4+\frac{7ax^3}{2}-\frac{107}{12}a^2x^2+\frac{5a^3x}{6}+\frac{7a^4}{6}$ by $2x^2-ax-\frac{a^2}{2}$.
5. Simplify $\dfrac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}$.
6. Simplify $\dfrac{3xyz}{yz+xz-xy} \div \dfrac{\dfrac{x-1}{x}+\dfrac{y-1}{y}+\dfrac{z-1}{z}}{\dfrac{1}{x}+\dfrac{1}{y}-\dfrac{1}{z}}$.
7. Reduce to lowest terms $\dfrac{3x^3-2x^2-x}{6x^2-x-1}$.

44.

1. Simplify $\dfrac{1}{(2x-1)^2}+\dfrac{1}{(2x+1)(1-2x)}$.
2. Divide $\dfrac{4a^2-4ax}{(a+x)^2}$ by $\dfrac{a^2-x^2}{4ax+4x^2}$.
3. Add together $\frac{a^2}{2}-\frac{b^2}{3}+\frac{c^2}{4}$, $\frac{b^2}{2}-\frac{c^2}{3}+\frac{a^2}{4}$, $\frac{c^2}{2}-\frac{a^2}{3}+\frac{b^2}{4}$.
4. Find the L.C.M. of $x^3+x^2y+xy^2+y^3$ and $x^3-x^2y+xy^2-y^3$.
5. Extract the square root of $a^2+4b^2+c^2+2ac+4ab+4bc$.
6. Solve $\dfrac{a}{x-b}=\dfrac{b}{x-a}$.
7. Divide the number 90 into four parts so that the first part increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, may be all equal.
8. What powers of a quantity are the same whether the quantity be positive or negative?

45.

1. Simplify $\frac{1}{1+3a+2a^2}+\frac{1}{1-a-2a^2}$.
2. Multiply $1-x-x^3$ by x^3-x-1.
3. Find the H.C.F. of
   $x^3+5x^2+10x+8$ and x^5+2x^4-x-2.
4. Find the L.C.M. of
   $1-x^2$, $1-2x+x^2$, $1-x+x^3-x^5$, $1-x-x^2+x^3$.
5. Reduce to lowest terms $\frac{2x^2-xy-y^2}{y^2+xy-2x^2}$.
6. Expand $\left(\frac{ab}{4}-\frac{1}{ab}\right)^3$.
7. Solve $\frac{1}{y}+\frac{1}{z}-\frac{1}{x}=6$; $\frac{1}{z}+\frac{1}{x}-\frac{1}{y}=4$; $\frac{1}{x}+\frac{1}{y}-\frac{1}{z}=2$.

46.

1. Reduce to a simple fraction $\cfrac{x}{a-\cfrac{x}{a-\cfrac{a}{x}}}$.
2. Reduce to its lowest terms $\frac{x^3-3x^2+7x-21}{2x^4+19x^2+35}$.
3. Find the square root of $\frac{4x^4-12x^3y+x^2y^2+12xy^3+4y^4}{16x^2y^2}$.
4. Solve $3x-a=\frac{a+x}{3}-\frac{b-x}{a}-cx$.
5. The united ages of a father and son make 75 years. The father was 25 years old when the son was born. What is the age of each?
6. Solve $\left.\begin{aligned}5x-3y-28&=0\\3x-2y-13&=0\end{aligned}\right\}$.
7. Subtract ax from $\frac{2ax+a^2x^2+2x}{2+ax-2x^2}$.

67.

1. If $y=\frac{1-z^2}{1+z^2}$ and $z=\frac{1-x}{1+x}$, find the value of y in terms of x, in its simplest form.

2. Add together
$1-\{\frac{1}{2}-(\frac{1}{4}-x)\}$, $\frac{1}{2}-(\frac{1}{3}-5x)$, $\frac{1}{3}-(\frac{1}{4}+4x)$.

3. Simplify $(a^{p-q})^{p+q}\times(a^q)^{q+r}\div(a^p)^{p-q}$.

4. Solve $x+a+b+c=\frac{x^2+a^2+b^2+c^2}{a+b-c+x}$

5. Solve $\left.\begin{aligned}\frac{x+y}{3}-2y&=2\\ \frac{2x-4y}{5}+y&=\frac{23}{5}\end{aligned}\right\}$.

6. A person out walking has 18 miles to go, and finds that at the rate at which he is going he will be half an hour late, but if he quickens his pace by half a mile an hour, he will arrive just at the proper time. At what rate is he going?

68.

1. Solve $ax^2+b(2a+b)x-(a^3-b^3)=0$.

2. Multiply $x^3-\frac{1}{2}x+\frac{2}{3}$ by $\frac{1}{3}x+2$.

3. Divide $x^4+10x^3+35x^2+24+50x$ by $(x+1)(x+4)$.

4. Simplify $\frac{x^2+xy}{x-y}\div\frac{x^4-y^4}{(x-y)^2}$.

5. Solve $\left.\begin{aligned}\frac{x}{3}+\frac{y}{5}-5&=0\\ 2x+\frac{y}{3}-17&=0\end{aligned}\right\}$.

6. Expand $\left(\frac{a}{3}-\frac{3b}{ac}\right)^3$.

69.

1. What values of x will make the value of the expression $(x-a)(x-b)(x-c)(x-d)$ equal to 0?
2. Resolve into the simplest factors $m^2-n^2-p^2+q^2+2(mq-pn)$.
3. Solve $2x-\frac{x+3}{3}+15=\frac{12x+26}{5}$.
4. Divide \$20,000 between A, B, and C so that A's share shall be $\frac{1}{2}$ of C's, and A's and B's together shall be equal to C's.
5. Find the H.C.F. of $6a^3-6a^2y+2ay^2-2y^3$ and $12a^2-15ay+3y^2$.
6. Solve $ax^2-a^2(x+b^2)=ab(x-ab)$.

70.

1. If $a=3$, $b=1$, $c=-2$, $d=0$, find the value of $\frac{a^3-b^3}{2ab+2bc+2cd+2ad}\div\frac{a^2+b^2+c^2+d^2}{\sqrt{(a-c)(b+c)}+6(-c-b)(a-2b)}$.
2. Divide $(a^2+2bc)^3-4bc(3a^4+4b^2c^2)$ by a^2-2bc.
3. Find the H.C.F. of $6x^3-x^2+16$ and $6x^4-3x^3+9x^2+3x+12$.
4. Simplify $\frac{x^2+x+1}{x-1+\frac{1}{x}}\times\frac{\frac{1}{x^2}-\frac{1}{x}+1}{x+1+\frac{1}{x}}$.
5. Simplify $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+x+1}$.
6. Solve $\frac{1}{y}+\frac{1}{z}=7$; $\frac{1}{x}+\frac{1}{z}=6$; $\frac{1}{x}+\frac{1}{y}=5$.
7. Solve $\frac{15}{x+\frac{1}{4}}=\frac{15}{x}-\frac{1}{4}$.

71.

1. Subtract $3x-\frac{x^3}{8}-(2x+3x^2)$ from $\frac{x^3}{2}-\frac{5x}{6}-\left(\frac{3x}{4}-x^2\right)$.

2. Simplify $\frac{1}{4(1+x)}+\frac{1}{4(1-x)}+\frac{1}{2(1+x^2)}$.

3. Find the square root of
$(a+b)^2-c^2+(a+c)^2-b^2+(b+c)^2-a^2$.

4. There are 1200 books in my library. There are twice as many English books as French books, and five times as many French books as books in other foreign languages. How many English books and how many French books do I have?

5. Solve $\frac{(4a^2-b^2)(x^2+1)}{4a^2+b^2}=2x$.

6. A man walking $\frac{1}{8}$ of a mile above his ordinary rate gains $\frac{3}{4}$ of an hour in 39 miles. At what rate does he ordinarily walk?

72.

1. Divide $\frac{a^3x+ax^3+x^4}{a^3-x^3}$ by $\frac{x}{a-x}$.

2. Extract the square root of $x^2+\frac{y}{2}$ to three terms.

3. Divide $\frac{(a^{\frac{1}{2}}b^{\frac{1}{2}})^3}{\sqrt[3]{c^2}}$ by $\frac{\sqrt[6]{c^2b^5}}{\sqrt[5]{a^7}}$.

4. Solve $x+y+z=0$; $ax+by+cz=0$; $2x+3y+z=1$.

5. Solve $\frac{x}{100}+\frac{21}{25x}=-\frac{1}{4}$.

6. Find the H.C.F. of
$x^4+5x^3-7x^2-9x-10$ and $2x^4-4x^3+4x-8$.

73.

1. Simplify $\sqrt[3]{a^2bc} \times \sqrt[4]{abc^3} \div a^{\frac{11}{12}} b^{\frac{1}{12}} c^{\frac{5}{6}}$.

2. Find the L.C.M. of $x^2 - 9x + 20$ and $x^2 + 6x - 55$.

3. Solve $\dfrac{2x^2+1}{9x^2-16} = \dfrac{x}{4+3x} - \dfrac{1}{9}$.

4. The sum of two numbers exceeds three times their difference by 8, and their difference is equal to $\frac{1}{8}$ of the smaller number. Find the numbers.

5. Solve $\dfrac{x+m-2n}{x+m+2n} = \dfrac{n+2m-2x}{n-2m+2x}$.

6. Extract the square root of
$8x^2(2x^2+6ax+a)+a^2(36x^2+12x+1)$.

7. Solve $(x+10\frac{1}{8})^2 = 4(x+9\frac{1}{8})$.

74.

1. Two men run a mile. The winner finishes in 4 minutes 44 seconds, and wins by 2 seconds. How many yards start must the loser of the race have for a tie, if each runs at a uniform rate?

2. Find two consecutive numbers, the difference of whose squares is 51.

3. Solve $\frac{1}{6}(12-x) + \dfrac{3x-\frac{1}{2}}{3} = \dfrac{4x-\frac{3}{4}}{2} + \dfrac{3\frac{1}{6}-x}{7}$.

4. Find the H.C.F. of $4x^3+4x^2-9x-9$ and x^3+2x^2-x-2.

5. Simplify $\dfrac{\dfrac{1}{a+b} + \dfrac{1}{a-b}}{\dfrac{1}{a-b} - \dfrac{1}{a+b}} \div \dfrac{a}{b}$.

6. Solve $6(x-\frac{1}{2})(x+\frac{1}{3}) = 0$.

7. Simplify $\left\{\dfrac{a^{-2}}{b^{-2}}\sqrt{\dfrac{a}{b}}\right\}^{\frac{1}{2}}$.

75.

1. Find the value of $\dfrac{a+\sqrt{a^2+b^2}}{a^3-2b(a^2-b^2)}$ when $a=-4$ and $b=-3$.

2. Bracket together the equivalent expressions in the following list:

$$\frac{1}{a^3},\ a^3,\ a^{\frac{1}{3}},\ a^{-3},\ \sqrt{a},\ \sqrt{a^4},\ a^{\frac{3}{2}},\ \sqrt[6]{a^3},\ \frac{a^4}{a^7},\ a^3\times a^{-1}.$$

3. Divide $x^3-\dfrac{1}{x^3}$ by $x-\dfrac{1}{x}$.

4. Solve $\left.\begin{array}{l}(x+5)(y+7)=(x+1)(y+9)+112\\ 2x+10=3y+1\end{array}\right\}$.

5. Simplify $b^3\sqrt[3]{8a^6b^3}+4a\sqrt[3]{a^3b^{12}}-\sqrt[3]{125a^6b^{12}}$.

6. A traveller walks a certain distance at a certain rate. If he had gone $\frac{1}{2}$ a mile an hour faster, he would have walked the distance in $\frac{4}{5}$ of the time. If he had gone $\frac{1}{2}$ a mile slower, he would have been $2\frac{1}{2}$ hours longer on the road. Find the distance, and the rate at which he travelled.

7. A man walking $\frac{1}{3}$ of a mile above his ordinary rate gains $\frac{3}{4}$ of an hour in 39 miles. Find his ordinary rate of walking.

76.

1. Multiply $x^{\frac{1}{2}}-4x^{\frac{1}{4}}+2$ by $x-x^{\frac{1}{4}}$.

2. Solve $\dfrac{ax-1}{\sqrt{ax-1}}=4+\dfrac{\sqrt{ax-1}}{2}$.

3. Show that if $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}$, then $\dfrac{a}{b}=\dfrac{xa+yc+ze}{xb+yd+zf}$.

4. Divide $x^{\frac{1}{2}}y^{-\frac{1}{2}}$ by $x^{-\frac{1}{2}}y^{\frac{1}{2}}$.

5. Simplify $(3\sqrt{8})(2\sqrt{6})$.

6. Simplify $[\{a^2(ab^{\frac{1}{2}})^{\frac{1}{2}}\}^{\frac{1}{2}}]^3$.

7. Solve $x-15\frac{3}{4}+\dfrac{5}{x-15\frac{3}{4}}=6$.

77.

1. Two ships sail at the same time from the same port; one due north at the rate of 9 miles an hour, the other due east at the rate of 12 miles an hour. After how many hours will they be 60 miles apart?

2. Solve $\frac{a(a-b)}{x-a-b}+a+b-x=\frac{(b-a)b}{a+b-x}$.

3. Solve $\frac{a}{x}=\frac{x}{a-x}$.

4. Two bodies move toward each other, starting from points 1800 feet apart. The first body starts 5 seconds after the second body, and the two bodies meet halfway between the points. If the rate of the first body is 6 feet a second more than that of the second body, find the rate of each body.

5. Solve $9x^2-9x+2=0$.

78.

1. Multiply $a^2-a+1-a^{-1}$ by $a+1+a^{-1}+a^{-2}$.

2. If $\frac{a}{b}=\frac{c}{d}$ show that $\frac{a+b}{c+d}=\frac{a-b}{c-d}$.

3. What is the value of $\{(\frac{1}{64})^{\frac{3}{2}}\}^{-\frac{1}{3}}$?

4. Simplify $(a^{\frac{3}{2}}b^{-\frac{1}{2}})^{\frac{2}{3}}+(a^{-\frac{1}{2}}b^2)^{\frac{1}{2}}$.

5. Divide \$560 between A and B, so that for every dollar A receives B shall receive \$2.50.

6. Solve $\left.\begin{aligned} x^2-xy&=10\\ xy-y^2&=6\end{aligned}\right\}$.

7. For what positive values of m and n does the expression $(-a)^{\frac{m}{n}}$ denote an imaginary quantity?

79.

1. Find the value of $\frac{x^2-xy+y^2}{x^2+xy+y^2}$ when

$$x=\frac{\sqrt{3}+1}{2} \text{ and } y=\frac{\sqrt{3}-1}{2}$$

2. Find the L.C.M. of $a-1$, $1-a^2$, $a-2$, $4-a^2$.

3. Solve $a+x+\sqrt{2ax+x^2}=b$.

4. Find two numbers, the sum of which is 15, and the sum of their squares 113.

5. Two freight trains pass a station at an interval of 4 hours, proceeding at the rates of $11\frac{1}{2}$ and $17\frac{1}{2}$ miles an hour, respectively. At what distance from the station will the faster train overtake the other?

6. Solve $ax^2+bx+c=0$. What is the relation between a and c when the product of the roots of this equation is equal to unity?

80.

1. Two passengers have together 400 pounds of baggage. One pays \$1.20, the other \$1.80, for excess above the weight allowed. If all the baggage had belonged to one person he would have had to pay \$4.50. How much baggage is allowed free?

2. Find the fourteenth term of $(2a-b)^{16}$.

3. Find the square root of

$$16m^4+\tfrac{16}{3}m^3n+8m^2+\tfrac{4}{9}n^2+\tfrac{4}{3}n+1.$$

4. Find the H.C.F. of $x^4-ax^3+a^3x-a^4$ and x^3-a^3.

5. Find the simplest value of $\sqrt[3]{64x^3y^6z^9}+\sqrt[4]{x^4y^8z^{12}}$.

6. Solve $\frac{a}{2bx-b}-\frac{2bx+b}{ax^2-a}=\frac{1}{2x^2+x-1}+\frac{1}{2x^2-3x+1}$.

7. Find two numbers whose sum is 7, and the sum of their squares exceeds their product by 19.

81.

1. Find the value of $ax+by$ when
$$x=\frac{cq-br}{aq-bp} \text{ and } y=\frac{ar-cp}{aq-bp}.$$
2. Add $x^2-3xy-\frac{2}{3}y^2$, $2y^2-\frac{2}{3}y^2+z^2$, $xy-\frac{1}{3}y^2+y^2$, $2xy-\frac{1}{3}y^2$.
3. Solve $\dfrac{6x+13}{15}-\dfrac{3x+5}{5x-25}=\dfrac{2x}{5}$
4. Solve $\sqrt{4x+9}-2\sqrt{x}=1$.
5. Solve $\left.\begin{aligned} x+1-\frac{3y+4x}{7}&=7-\frac{9y+33}{14}\\ y-3-\frac{5x-4y}{2}&=x-\frac{11y-19}{4}\end{aligned}\right\}$.
6. Find two numbers whose difference is 1, and the sum of their squares 313.
7. Find the L.C.M. of
$$x^3+5x+4,\ x^3+2x-8, \text{ and } x^3+7x+12.$$

82.

1. Solve $\dfrac{x^2}{2}-3x=\dfrac{x(a+x)}{2}-\left(\dfrac{ax}{2}+12\right)$.
2. The difference between a number and its square is 18 more than the same difference would be if the number were 1 less. Find the number.
3. Find the L.C.M. of $a-b$, $a+b$, a^2-b^2, a^3-b^3, a^3+b^3.
4. Simplify $\dfrac{\dfrac{x^2+y^2}{y}-x}{\dfrac{1}{y}-\dfrac{1}{x}}\times\dfrac{x^3-y^3}{x^3+y^3}$.
5. Solve $ax^2+bx+c=0$, and state the conditions under which the values of x will be (i.) real, (ii.) equal, (iii.) imaginary.
6. Find the product of $2y^{\frac{1}{2}}\sqrt{2x^3}$, $\sqrt{6xy}$, and $\dfrac{x^{\frac{1}{2}}}{\sqrt{3y^{\frac{1}{2}}}}$.
7. Solve $x-\dfrac{1}{y}=y-\dfrac{1}{x}=\dfrac{3}{2}$.

83.

1. Find the H.C.F. of
$x^4 - 2x^3 + x^2 - 8x + 8$ and $4x^3 - 12x^2 + 9x - 1$.

2. Divide $\frac{6\sqrt{b}}{25\sqrt[5]{a^3}}$ by $\frac{20c\sqrt[4]{b^3}}{21ab\sqrt[3]{a^2}}$.

3. A can do a piece of work in 10 days which B can do in 8 days; after A has been at work upon it 3 days, B comes to help him; in what time will they finish it?

4. Solve $\frac{x+3}{x-3} - \frac{x-3}{x+3} = a$.

5. What number is that to which if 1, 5, and 13, severally, be added, the first sum shall be to the second as the second to the third?

6. Expand $(a - 3x)^6$.

84.

1. If $s = \frac{1}{2}(a+b+c)$, show that
$s^2 + (s-a)^2 + (s-b)^2 + (s-c)^2 = a^2 + b^2 + c^2$.

2. Subtract $\frac{1-3x}{1+3x}$ from $\frac{3x}{1-3x}$.

3. A man has 6 hours for an excursion. How far can he ride out in a carriage which travels 8 miles an hour so as to return home in time, walking back at the rate of four miles an hour?

4. Resolve into factors $x^8 - \frac{1}{256}$.

5. Show that, if $\frac{a}{b} = \frac{c}{d}$, then $\frac{7a+5b}{8a-3b} = \frac{7c+5d}{8c-3d}$.

6. Solve $x - y = \frac{1}{6}$; $x^2 - y^2 = \frac{5}{36}$.

7. Solve $\frac{2b-x-2a}{bx} + \frac{4b-7a}{ax-bx} = \frac{x-4a}{ab-b^2}$.

85.

1. Show that the sum of the squares of any two different numbers is always greater than twice their product.

2. Reduce to its lowest terms $\dfrac{4+12x+9x^2}{2+13x+15x^2}$.

3. Find the value of x in

$$\left(\frac{10a^{\frac{1}{2}}}{3b^{\frac{2}{3}}}\right)^2 : x :: \sqrt{\frac{5a\sqrt[3]{a^2}}{4\sqrt[5]{a^2b^9}}} : \frac{9b^{-3}}{\sqrt{5}}.$$

4. Solve $\sqrt{x(3-x)} = \sqrt{x+1} + \sqrt{2(x-1)}$.

5. Extract the square root of $33 - 20\sqrt{2}$.

6. Expand $\left(1-\dfrac{3y}{4}\right)^5$.

86.

1. Solve $\dfrac{a-2b-x}{a^2-4b^2} - \dfrac{5b-x}{ax+2bx} + \dfrac{2a-x-19b}{2bx-ax} = 0$.

2. Simplify $1 + \dfrac{x}{1+x+\dfrac{2x^2}{1-x}}$.

3. Solve $\dfrac{3x-1}{2x-1} - \dfrac{4x-2}{3x-2} = \dfrac{1}{6}$.

4. Expand $(2x-3y)^4$.

5. A vessel which has two pipes can be filled in 2 hours less time by one than by the other, and by both together in 2 hours 55 minutes. How long will it take each pipe alone to fill it?

87.

1. If $a=2$, $b=3$, $x=6$, $y=5$, what is the value of
$$a+2x-\{b+y-[a-x-(b-2y)]\}.$$

2. Simplify $\{\sqrt{3+4\sqrt{-1}}+\sqrt{3-4\sqrt{-1}}\}^2$.

3. Reduce to the lowest terms
$$\frac{2x^2-8x+6}{3x^2-5x+4}\times\frac{x^2-9x+20}{x^2-10x+21}\div\frac{2x^2-10x}{x^2-7x}.$$

4. Solve $\dfrac{x}{x+b-a}+\dfrac{b}{x+b-c}=1$.

5. When are the hands of a clock together between the hours of 6 and 7?

6. The plate of a looking-glass is 18 inches by 12 inches, and it is to be framed with a frame of uniform width, whose area is to be equal to that of the glass. Find the width of the frame.

7. Show that $a^0=1$.

88.

1. Find the fourth term of $(2x-5y)^{12}$.

2. Divide $\dfrac{2}{x}-\dfrac{1}{a+x}+\dfrac{1}{a-x}$ by $\dfrac{a+x}{a-x}-\dfrac{a-x}{a+x}$.

3. Solve $(a-b)x=(a+b)y$; $x+y=c$.

4. A carriage, horse, and harness are worth, together, \$720. The carriage is worth $\frac{4}{5}$ of the value of the horse, and the harness $\frac{2}{5}$ of the difference between the values of the horse and carriage. Find the value of each.

5. Solve $\dfrac{x+13a+3b}{5a-3b-x}-1=\dfrac{a-2b}{x+2b}$.

89.

1. Solve $x^4 - 5x^2 + 4 = 0$.

2. Solve $\dfrac{\sqrt{2a^2 - x^2} + b\sqrt{2a - x}}{\sqrt{2a^2 - x^2} - b\sqrt{2a - x}} = \dfrac{\sqrt{a} + b}{\sqrt{a} - b}$.

3. Simplify $\dfrac{a^{-\frac{1}{2}}b^{\frac{1}{2}}}{\dfrac{1}{b^{\frac{1}{2}}c^{-\frac{1}{2}}}} \times \dfrac{\dfrac{1}{b^{\frac{1}{2}}c^{\frac{1}{2}}}}{c^{\frac{1}{2}}a^{-\frac{1}{2}}} \times \dfrac{a^{\frac{1}{2}}c^{\frac{1}{2}}}{b^{\frac{1}{2}}}$.

WESTFIELD SCHOOL PROPERTY,
Purchased.....190

4. Solve $\dfrac{x}{a} - \dfrac{y}{b} = 1$, $\dfrac{x}{b} + \dfrac{y}{a} = \dfrac{a}{b}$.

5. A hare is 50 of her leaps before a greyhound, and takes four leaps to his three. Two of the greyhound's leaps are equal to three of the hare's. How many leaps must the greyhound take to catch the hare?

90.

1. Extract the square root of $81a^{-\frac{1}{2}}b^{\frac{1}{2}}c$.

2. Express in the simplest form $\frac{1}{4}(\sqrt[3]{3} + \frac{1}{2}\sqrt[3]{192} + \sqrt[3]{81})$.

3. Simplify $\{(a^{-\frac{1}{2}})^{\frac{1}{3}}\}^{-\frac{1}{2}}$.

4. If $a : b = c : d = e : f$, show that
$a : b = a + c + e : b + d + f$.

5. For building 108 rods of stone-wall, 6 days less would have been required if 3 rods more a day had been built. How many rods a day were built?

6. Solve $\left.\begin{aligned} x + y &= 5 \\ x^2 - 3xy + 4y^2 &= 7 \end{aligned}\right\}$.

91.

1. Simplify $\left(\frac{32a^{-10}x^{-\frac{1}{2}}}{b^{\frac{1}{2}}y^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$.

2. A railway train, after travelling an hour, meets with an accident, which delays it 1 hour; after this it proceeds at $\frac{3}{5}$ of its former rate, and arrives 5 hours late. If the accident had occurred 50 miles farther on, the train would have been 3 hours 20 minutes late. Find the length of the line.

3. Prove that if $a : b = c : d$, then
$a + b : a - b = c + d : c - d$.

4. Solve $x^{-1} + ax^{-\frac{1}{2}} = 2a^2$.

5. Write the 5th term of $(3x^{\frac{1}{2}} - 4y^{\frac{1}{3}})^9$.

6. What is a surd? Give an example. What are similar surds? Give an example.

92.

1. Find the value of $\frac{x^3 - y^3}{x - y}$ when $x = 2^{\frac{1}{2}}a^{\frac{1}{2}}$ and $y = 2^{-\frac{1}{2}}a^{\frac{1}{2}}$.

2. Simplify $\frac{y}{2x^2 - 2xy} - \frac{4x + 5y}{4(x^2 - y^2)} + \frac{4x - 5y}{4(x - y)^2}$.

3. Simplify $\left(\frac{ay}{x^2}\right)^{\frac{1}{2}} \times \left(\frac{bx^2}{y^2}\right)^{\frac{1}{2}} \div \left(\frac{b^2a^3x^2}{y^3}\right)^{\frac{1}{2}}$.

4. Solve $\frac{\sqrt{x} + 2a}{\sqrt{x} + 2b} = \frac{\sqrt{x} + 4a}{\sqrt{x} + 3b}$.

5. Find x and y from the proportions
$2x + y : y :: 3y : 2y - x$;
$2x + 1 : y :: 2x + 6 : y + 2$.

6. A boat's crew row $3\frac{1}{2}$ miles down the river and back again in 1 hour and 40 minutes. If the current of the river is 2 miles per hour, determine their rate of rowing in still water.

93.

1. Add
$\sqrt{20a^3m-20acm+5c^2m}$ and $\sqrt{20c^2m-60acm+45a^2m}$.

2. Find the H.C.F. of
$4x^2y^2(x^2+xy+y^2)^2$, $6xy(x^6-y^6)$, and $18x^2(x^3-y^3)^2$.

3. Solve $abx^2+\frac{3a^2x}{c}=\frac{6a^2+ab-2b^2}{c^2}-\frac{b^2x}{c}$.

4. The circumference of the fore-wheel of a carriage is 9 feet, and that of the hind-wheel 12 feet. What is the distance gone over when the fore-wheel has made two more revolutions than the hind-wheel?

5. Which is the greater, $\sqrt{14}+\sqrt{7}$ or $\sqrt{19}+\sqrt{2}$? Prove your answer.

6. Solve $\left.\begin{matrix} x+y=6 \\ x^3+y^3=72 \end{matrix}\right\}$.

94.

1. Solve $\left.\begin{matrix} x^2+xy+2y^2=44 \\ 2x^2-xy+y^2=16 \end{matrix}\right\}$.

2. Find the two middle terms of $\left(\frac{x}{y}-\frac{y}{x}\right)^7$.

3. Solve $2x^2-21x+55=0$.

4. If x' and x'' are the roots of the equation $x^2-px+q=0$, show that $x'+x''=p$, and $x'x''=q$.

5. Form and solve the quadratic equation, the product of whose roots is 42 and their sum 13.

6. A number having two digits is to the number formed by interchanging the digits as 7 is to 4, and the sum of the two numbers is 132. Find them.

95.

1. For what value of m will the equation $2x^2+8x+m=0$ have equal roots?

2. Find the value of x^2+xy+y^2 if $x=1+\sqrt{2}$, $y=1-\sqrt{2}$.

3. A and B each distribute \$100 in charity. A relieves 5 persons more than B, and B gives to each \$1 more than A. How many did they each relieve?

4. Solve $x^6-20x^3=189$.

5. Multiply $\left(\frac{ay}{x}\right)^{\frac{1}{2}}$, $\left(\frac{bx}{y^2}\right)^{\frac{1}{2}}$, and $\left(\frac{y^2}{a^2b^2}\right)^{\frac{1}{2}}$.

6. Find the H.C.F. of
$x^4+x^3-2x^2+3x-3$ and $x^5-4x^3-2x^2+3x+2$.

96.

1. Simplify $\frac{x^3-8y^3-z^3-6xyz}{x^3-x^2z-2x^2y}$.

2. Find the H.C.F. and the L.C.M. of
$34a^2bx^3y^4$, $27x^5y^3b^3$, $51y^3a^3x$.

3. Extract the square root of $x^4+\frac{1}{9}+2x+\frac{1}{x^2}-\frac{2x^2}{3}-\frac{2}{3x}$.

4. Solve $\left.\begin{aligned} x^2-xy&=\tfrac{3}{2} \\ xy+y^2&=1 \end{aligned}\right\}$.

5. Solve $ax^2+2bx+c=0$, and show that the two roots are equal if b is a mean proportional between a and c.

6. Solve $\sqrt{x}+\sqrt{4+x}=\frac{4}{\sqrt{x}}$.

97.

1. Expand $\left(\sqrt{xy} - \frac{z}{2\sqrt{y}}\right)^5$.

2. Rationalize the denominator, and then find the approximate value of $\frac{7+2\sqrt{10}}{7-2\sqrt{10}}$.

3. Solve $15x - 3x^2 + 4(x^2 - 5x + 5)^{\frac{1}{2}} = 16$.

4. A man starts from the foot of a mountain to walk to its summit. His rate of walking during the second half of the distance is $\frac{1}{2}$ mile per hour less than his rate during the first half, and he reaches the summit in $5\frac{1}{2}$ hours. He descends in $3\frac{3}{4}$ hours, by walking 1 mile more per hour than during the first half of the ascent. Find the distance to the top, and the rates of walking.

98.

1. Divide $2\sqrt{3} + 3\sqrt{2} + \sqrt{30}$ by $3\sqrt{6}$.

2. Simplify $\frac{1}{4(1+\sqrt{x})} + \frac{1}{4(1-\sqrt{x})} + \frac{1}{2(1+x)}$.

3. Solve $\sqrt{x+3} + \sqrt{x+8} = 5\sqrt{x}$.

4. Find the mean proportional between $(a+b)^2$ and $(a-b)^2$.

5. Solve $\left.\begin{aligned} x - y &= 2 \\ 15(x^2 - y^2) &= 16xy \end{aligned}\right\}$.

6. Expand $(2\sqrt[5]{x^4} - \frac{1}{2}y^2)^4$.

99.

1. Simplify $2\sqrt[3]{40}+3\sqrt[3]{108}+\sqrt[3]{500}-\sqrt[3]{320}-2\sqrt[3]{1372}$.

2. Extract the square root of
$$1+4y^{-\frac{1}{2}}-2y^{-\frac{3}{2}}-4y^{-1}+25y^{-\frac{4}{3}}-24y^{-\frac{5}{3}}+16y^{-2}.$$

3. Expand $\left(\frac{2x}{y^2}-\frac{1}{2}y\sqrt{x}\right)^4$.

4. Solve $\sqrt{2x+1}-\sqrt{x+4}=\frac{1}{3}\sqrt{x-3}$.

5. Solve $\frac{2x+1}{b}-\frac{1}{x}\left(\frac{1}{b}-\frac{2}{a}\right)=\frac{3x+1}{a}$.

6. The volume of a sphere varies as the cube of its diameter. Compare the volume of a sphere 6 inches in diameter with the sum of the volumes of three spheres whose diameters are 3, 4, 5 inches, respectively.

100.

1. Simplify $\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$.

2. Rationalize the denominator of $\frac{8-5\sqrt{2}}{3-2\sqrt{2}}$, and find the approximate value of the fraction.

3. Form the quadratic equation whose roots are $a\pm\sqrt{-5b}$.

4. Solve $(x^{-\frac{1}{2}}+2)(x^{-\frac{1}{2}}+5)=x^{-1}+8$.

5. A vessel which has two pipes can be filled in 2 hours less time by one than by the other, and by both together in 1 hour 52 minutes 30 seconds. How long will it take each pipe alone to fill the vessel?

101.

1. Multiply and free from negative exponents $-13a^{-1}c^{-3}$ by $-4a^{-3}b^{-6}c^{2}$.

2. Solve $\dfrac{x+\sqrt{9-x}}{x-\sqrt{9-x}}=\dfrac{7}{3}$.

3. Find an equivalent expression with a rational denominator for $\dfrac{\sqrt{21}+\sqrt{12}}{\sqrt{7}-\sqrt{3}}$.

4. Solve $\left.\begin{aligned} 2x^2+3y^2-xy&=31\\ x-y&=3\end{aligned}\right\}$.

5. Find the limit of the series $1+\frac{1}{2}+\frac{1}{4}\cdots\cdots$

6. Reduce to partial fractions by means of indeterminate coefficients $\dfrac{6x^3-4x-6}{(x-1)(x-2)(x-3)}$.

7. Expand $\left(1-\dfrac{x}{2}\right)^5$.

102.

1. Simplify and free from negative exponents $(-\frac{5}{2}a^{-2}b^{3}c^{-m}d^{-1})\times(\frac{2}{3}a^{2}b^{-3}c^{-2}d^{4})$.

2. Multiply $\sqrt[3]{5}-2\sqrt[3]{6}$ by $3\sqrt[3]{4}-\sqrt[3]{36}$.

3. Solve $\left.\begin{aligned} x^3-y^3&=63\\ x-y&=3\end{aligned}\right\}$.

4. How many terms of the series 54, 51, 48, etc., amount to 513? Explain the two answers.

5. Solve $3x^{\frac{1}{2}}-x^{-\frac{1}{2}}+2=0$.

6. Find the first five terms, and the $(r+1)$th term, in the expansion of $(1-x)^{-\frac{1}{4}}$.

103.

1. Simplify $\sqrt{\frac{a+b}{a-b}}\sqrt{\frac{a+b}{a-b}} \times \sqrt[3]{\frac{a-b}{a+b}}\sqrt{\frac{a-b}{a+b}}$.

2. Divide $x^{-6}+y^{-9}$ by $x^{-2}+y^{-3}$.

3. Solve $\frac{x+\sqrt{2-x^2}}{x-\sqrt{2-x^2}}=\frac{4}{3}$.

4. A debt can be discharged in a year by paying \$1 the first week, \$3 the second week, \$5 the third week, etc. Required the amount of the debt, and the last payment.

5. Find the coefficient of x^{12} in the expansion of $(a^5-b^3x^3)^{\frac{1}{2}}$.

6. Solve $\left.\begin{aligned} x^2+xy+y^2 &= 52 \\ xy-x^2 &= 8 \end{aligned}\right\}$.

104.

1. Simplify $\sqrt{5} \times \sqrt[3]{2} \times \sqrt[4]{4}$.

2. Find an equivalent expression with a rational denominator for $\frac{4+\sqrt{2}}{4+\sqrt{3}}$.

3. Solve $x^2-2x+6\sqrt{x^2-2x+5}=11$.

4. The height of a certain triangle is 4 inches less than the base. If the base be increased by 6 inches, and the height diminished by 6 inches, the area is diminished by $\frac{1}{8}$. Required the base of the triangle.

5. Find the fifth term of $(3x-2y)^{-10}$.

6. Resolve into partial fractions $\frac{x^2}{(x^2-1)(x-2)}$.

105.

1. Simplify the expression $\sqrt{\sqrt[3]{a+b}} \times \sqrt[3]{\sqrt{a-b}}$.

2. Solve $\dfrac{x}{m^2p(x+a)} = \dfrac{x+a}{n^2px}$.

3. Solve $x^2 = 21 + \sqrt{(x^2-9)}$.

4. Solve $\left.\begin{aligned} 5xy &= 84 - x^2y^2 \\ x - y &= 6 \end{aligned}\right\}$.

5. The sum of 11 terms of an arithmetical progression is 22, and the common difference is $\frac{2}{5}$. Find the first term.

6. Find the coefficient of x^9 in $(5a^3 - 4x^2)^7$.

106.

1. Simplify and free from negative exponents $(-7a^{-1}b^4c^{-5}) \times (3a^2b^{-5}c)$.

2. Find the square root of $28 + 5\sqrt{12}$.

3. Solve $x^2 - x + 5\sqrt{2x^2 - 5x + 6} = \frac{1}{2}(3x + 33)$.

4. Prove the formula for finding the sum of n terms of an arithmetical progression.

5. There are three numbers in arithmetical progression whose sum is 15. If 1, 3, 9 are added to them, respectively, they are then in geometrical progression. Find the numbers.

6. Expand to five terms $\sqrt{1+2x}$.

7. Give an example of a series of terms in harmonical progression.

107.

1. Multiply $5\sqrt{14}+3\sqrt{5}$ by $7\sqrt{14}-2\sqrt{5}$.

2. Simplify $\sqrt[6]{(a^3 b\sqrt[5]{a^3}\times b\times c)^5}$.

3. Solve $x^{-\frac{1}{2}}+2=\dfrac{x^{-1}+8}{x^{-\frac{1}{2}}+5}$.

4. Solve $\left.\begin{aligned}2x^2-3xy+y^2&=24\\3x^2-5xy+2y^2&=33\end{aligned}\right\}$.

5. How many terms of the series $19+17+15\cdots\cdots$ amount to 91?

6. Prove the formula for finding the sum of n terms in geometrical progression.

7. Find the fifth term in the expansion of $\sqrt[3]{1+x}$.

108.

1. Divide $5-7\sqrt{3}$ by $1+\sqrt{3}$.

2. Find an arithmetical progression such that the second, third, fourth, and sixth terms may form a proportion.

3. Solve $\left.\begin{aligned}\frac{x}{y}-\frac{y}{x}&=\frac{15}{4}\\x-y&=\frac{3}{2}\end{aligned}\right\}$.

4. A person bought a number of shares in a railroad, that cost him \$3000. He reserved 10 of the shares, and sold the remainder for \$2700, gaining \$4 a share. How many shares did he buy?

5. Find the limit of the sum of $9-6+4-\cdots\cdots$

6. Resolve into partial fractions $\dfrac{1}{x^4-a^4}$.

7. Find the fourth term in the expansion of $\left(5-\dfrac{x}{6}\right)^6$.

109.

1. Three numbers are as 6 : 11 : 20. If 1 is added to each they are in geometrical progression. Find the numbers.

2. Solve $\frac{x+b}{a+b}+\frac{a}{b}=\frac{x^2+ab}{bx}$.

3. Divide 28 into two parts such that their squares shall be in the ratio 16 : 9.

4. The first term of a series in arithmetical progression is 17, the last term $-12\frac{2}{3}$, and the sum $25\frac{7}{16}$. Find the common difference.

5. Find the limit of the sum of $1-\frac{2}{5}+\frac{4}{25}-\cdots\cdots$

6. Find the first five terms, and the $(r+1)$th term, of the expansion of $(1-x^2)^{-\frac{1}{2}}$.

110.

1. Simplify $\sqrt[4]{\left(\frac{a\sqrt{b}}{\sqrt[3]{ab}}\right)^3}$.

2. Find an equivalent expression with a rational denominator for $\frac{\sqrt{20}-\sqrt{8}}{\sqrt{5}+\sqrt{2}}$.

3. Solve $x+5=\sqrt{x+5}+6$.

4. A certain capital is invested at 4 per cent. If the number of dollars in the capital is multiplied by the number of dollars in the interest for 5 months, the result is $117{,}041\frac{2}{3}$. Required the capital.

5. The first and the ninth terms of a series in arithmetical progression are 5 and 22. Find the sum of twenty-one terms.

6. Find the middle term of $(1+x)^{2n}$.

7. Resolve into partial fractions $\frac{1}{x^2-9x+14}$.

111.

1. Simplify $\frac{b^2}{a^3} \times \frac{a^{-1}b^{-2}}{ab^{-3}} \times \frac{a^2b^{-1}}{b^4} \div \left(\frac{a^{-2}b^{-1}}{a^2}\right)^2$.

2. Find the H.C.F. of
$x^4+7x^3+14x^2+5x-3$ and $2x^4+9x^3+8x^2-x+6$.

3. A merchant buys some goods for \$40, and sells them to another merchant, who, in his turn, sells them for \$48⅔. If each merchant makes a profit of the same rate per cent, determine what that rate is.

4. If $a:b=c:d=e:f$, show that
$$\frac{a+2c+3e}{b+2d+3f} = \frac{2a+3c+4e}{2b+3d+4f}.$$

5. The sum of five terms of an arithmetical series is 30, and the product of the first and last terms is 20. Form the series.

6. If α, β are the roots of the equation $x^2-x+1=0$, show that $\alpha+\beta=1$, and $\alpha^2+\beta^2=-1$.

112.

1. Solve $x^2+3y^2=28$; $xy+y^2=12$.

2. In each of 3 battles 36 officers and 10 per cent of the men engaged are killed. At the end of the second battle the percentage of officers to men is two-thirds as great as at its commencement. The number of men at the end of the third battle is equal to the square of the number of officers at its commencement. How many officers and men were engaged in the first battle?

3. Find an arithmetical progression such that the sum of n terms shall be equal to n^2.

4. Solve $x^2+\frac{5x}{2}-2\sqrt{2x^2+5x+3}=\frac{9}{2}$.

5. The terms of a ratio are 7 and 3; what number must be added to each in order that the ratio may be halved?

113.

1. Solve $x^2 + 2y^2 = 22$; $2xy + y^2 = 21$.
2. Simplify $2\sqrt{\frac{5}{8}} + \sqrt{60} - \sqrt{15} - \sqrt{\frac{8}{5}}$.
3. Solve $\sqrt{9x+40} + 2\sqrt{x+7} = \sqrt{x+44}$.
4. What is a ratio? Is it a quantity?
   If $m+n : m-n = x+y : x-y$, show that
   $x^2+m^2 : x^2-m^2 = y^2+n^2 : y^2-n^2$.
5. The thickness of a hollow cylinder varies directly as the amount of material, and inversely as the length of the cylinder and the sum of the radii of its internal and external surfaces. If the amount of material in a cylinder 50 feet long, whose radii of external and internal surfaces are 4 feet and 3 feet, respectively, be 1100 cubic feet, find the thickness of a cylinder 84 feet long, having the sum of its radii 5 feet, and containing 1650 cubic feet of iron.

114.

1. In an arithmetical progression, if s_1, s_2, s_3 denote the sums to the nth, $2n$th, $3n$th terms, respectively, prove that $s_3 = 3(s_2 - s_1)$.
2. In a geometrical progression, if l_1, l_2, l_3, denote the nth, $2n$th, $3n$th terms, respectively, prove that $l_2^2 = l_1 l_3$.
3. Reduce to its lowest terms $\dfrac{2x^3 - 11x^2y + 19xy^2 - 10y^3}{x^3 - 7x^2y + 13xy^2 - 6y^3}$.
4. Find the limit of $1 + \frac{1}{3} + \frac{1}{9} + \cdots$
5. A committee of 7 members is to be chosen out of a body of 20 protestants and 15 catholics in such a way that there shall be 3 protestants and 4 catholics. In how many different ways can such a committee be chosen.

115.

1. If A varies as B^2, B^2 as C^4, C^5 as D^8, and D^7 as E^4, show that $\frac{A}{E} \times \frac{B}{E} \times \frac{C}{E} \times \frac{D}{E}$ does not vary at all.

2. In how many ways can 2 white balls and 3 red ones be selected out of an urn containing 7 white balls and 10 red balls?

3. Solve $\dfrac{1}{\sqrt{x} - \sqrt{2-x}} - \dfrac{1}{\sqrt{x} + \sqrt{2-x}} = 1.$

4. Insert 10 arithmetical means between 6 and 61, and find the sum of the whole series.

5. An express-train, travelling at uniform speed, after being an hour in motion, was delayed half an hour by an accident; after which it proceeded at three-fourths of its original rate of speed, and arrived at the end of its journey 1 hour and 50 minutes late. Had the accident occurred after the train had travelled a distance of 60 miles, it would have been 1 hour and 40 minutes late. Find the length of the line.

116.

1. Find an arithmetical progression such that the second, fourth, and eighth terms are in geometrical progression.

2. Solve $x^2 = ax + by$; $y^2 = ay + bx$.

3. If a be the greatest of the four proportionals a, b, c, d, show that $a - b > c - d$.

4. If four numbers be in proportion, and the first three be in arithmetical progression, show that the reciprocals of the last three are in arithmetical progression also.

5. In what scale of notation will 540 be the square of 23?

117.

1. If $\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}=\sqrt{5}$, show that $\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}=\pm 1$.

2. Prove that a ratio of greater inequality is diminished if the same quantity be added to both its terms.

3. Solve $4x^2+xy=6$; $3xy+y^2=10$.

4. The sum of 10 terms of a certain geometrical series is 33 times the sum of 5 terms of the same series. What is the common ratio?

5. Sum the series 3, $2\frac{7}{10}$, $2\frac{4}{10}$, ·····, to 21 terms.

6. The volume of a sphere varies as the cube of its radius, and that of a circular plate of given thickness as the square of its radius. If the volume of a sphere of radius 1 inch be equal to that of a plate of radius 2 inches, find the radius of a plate which is equal in volume to a sphere of radius 4 inches.

118.

1. In an arithmetical series the common difference is 2, and the square roots of the first, third, and sixth terms also form an arithmetical series. Find the series.

2. Find a geometrical progression such that the sum of an infinite number of terms shall be 4, and the second term shall be $\frac{3}{4}$.

3. What is the equation whose roots are double those of the equation $x^4+x^3-6=0$?

4. A cattle dealer buys sheep, and sells them at a profit of 20 per cent. With the proceeds he again buys sheep, and sells them so as to gain 25 per cent. Once more he invests the proceeds in sheep, and this time he gains 16 per cent. If his last profit amounted to $300, how much money did he invest at first?

5. Solve $x^3-y^3=a^3$; $x-y=b$.

119.

1. Solve $(x+a)^5-(x-a)^5=242a^5$.

2. Solve $x^2-x+3\sqrt{x^2-3x+36}=2(x+26)$.

3. If the roots of the equation $3x^2-8x+5=0$ be α and β, show that the equation whose roots are $\frac{\beta}{\alpha}$ and $\frac{\alpha}{\beta}$ is $15x^2-34x+15=0$.

4. The sum of three quantities is y. The first varies inversely as x^2, the second inversely as x, and the third is constant. When $x=4,\ 2,\ 1$, then $y=3,\ 7,\ 21$, respectively. Find the equation between x and y.

5. Three numbers in geometrical progression, if multiplied by 3, 2, and 1, respectively, are in arithmetical progression. If the middle number is 18, what are the others?

120.

1. If $a:b=b:c$, then $\frac{b-c}{b}:\frac{a+b}{a}=\frac{a-b}{a}:\frac{b+c}{b}$.

2. If x and y are two numbers, A their arithmetical mean, and G their geometrical mean, then $x^2+y^2=2(2A^2-G^2)$.

3. If z vary inversely as $3x+y$, and y vary inversely as x, and if, when $x=1$ and $y=2$, $z=3$, find the value of z when $x=2$.

4. Find two numbers such that their product, their sum, and three times their difference are in the proportion $5:2:4$.

5. The cost of an entertainment was \$120, which was to have been divided equally among the party; but four of them leave without paying, and the rest have each to pay \$2.50 extra, in consequence. Of how many did the party consist?

121.

1. If the number of visitors to a fair varies as the square of the number of degrees above 42° F., and if there are 1152 visitors when the temperature is 68°, how many visitors will there be if the temperature is 55°?

2. Insert between 6 and 16 two numbers such that the first three numbers may be in arithmetical progression, and the last three in geometrical progression.

3. If $\frac{1}{3}$ of the sum of the squares of the roots of the equation $ax^2 + bx + c = 0$ is equal to their product, find the relation between a, b, and c.

4. Solve $x + y = a$; $x^2 + mxy + y^2 = b$.

5. What is the price of eggs per dozen when two more in a dollar's worth lowers the price one cent per dozen?

122.

1. Solve $x - y = a$; $bx^2 - cy^2 = d$.
   What do the values of x and y become when $b = c$?

2. The side of a square is a. By joining the middle points of its sides another square is formed; by joining the middle points of the sides of this square a third square is formed. If the operation is continued indefinitely, find the limit of the sum of the areas of the squares.

3. Separate into partial fractions $\frac{6x^2 + 34x - 76}{(x-1)(x-3)(x-5)}$.

4. Find the fourth term of $\frac{1}{(a^2 - bx)^{\frac{1}{2}}}$.

5. How many different signals can be made with ten flags, of which three are white, two red, and the rest blue, if all are hoisted together, one above another?

123.

1. What number must be added to 20, 50, and 100, respectively, that the results may be in geometrical progression?

2. Find an arithmetical progression such that the second, fourth, and eighth terms are in geometrical progression.

3. The volume of a sphere varies as the cube of its radius. If three spheres, whose radii are 9 inches, 12 inches, 15 inches, respectively, are melted into one, what will be its radius?

4. Solve $x^2 - y^2 = x^2 + y^2 - xy = 3$.

5. In how many ways can a base-ball nine be arranged if three men can play in any position, and the others in any position except those of pitcher and catcher?

124.

1. What number must be added to 20, 50, and 100, respectively, that the results may be in harmonical progression?

2. The sum of four numbers in geometrical progression is 170, and the third exceeds the first by 30. Find them.

3. Solve $x^3 - 11x^2 + 37x \quad 35 = 0$, one root being 5.

4. Solve $4x + 4\sqrt{3x^2 - 7x + 3} = 3x(x - 1) + 6$.

5. Transform 3256 from the septenary to the duodenary scale.

6. Two steamers ply between the same two ports, a distance of 420 miles. One travels half a mile an hour faster than the other, and is two hours less on the journey. Find their rates.

125.

1. In an arithmetical series the second term is 21, the seventh term 41, the sum 1625. Find the number of terms.

2. The first and the seventh terms of a geometrical series are 2 and $\frac{1}{2}$. Find the intermediate terms.

3. Prove that the difference of the roots of the equation $x^2 - px + q = 0$ is equal to the difference of the roots of the equation $x^2 - 3px + 2p^2 + q = 0$.

4. A vessel is half full of a mixture of wine and water. If filled with water, the ratio of the quantity of water to the quantity of wine will be 9 times as great as if filled with wine. Determine the original quantities of water and wine.

5. Find the tenth term of $\frac{1}{(x+y)^m}$.

126.

1. If $a : b = c : d$ prove that
$a(a+b+c+d) = (a+b)(a+c)$.

2. Solve $\frac{x+\sqrt{a^2-x^2}}{x-\sqrt{a^2-x^2}} = \frac{m}{n}$.

3. Solve $\left.\begin{aligned} x^3 + y^3 &= 225y \\ x^2 - y^2 &= 75 \end{aligned}\right\}$.

4. Insert 9 arithmetical means between 1 and -1.

5. There are four numbers, of which the first three are in geometrical progression and the last three in arithmetical progression. The sum of the first and last numbers is 16, and the sum of the second and third numbers is 12. Find the numbers.

6. Find the fortieth term of $(1-x)^{-5}$.

127.

1. Find an equivalent expression with a rational denominator for $\dfrac{5\sqrt{2}+4\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$.

2. Solve $\dfrac{x^2}{y}+\dfrac{y^2}{x}=18$; $\quad x+y=12$.

3. Two vessels, one of which sails faster than the other by 2 miles an hour, start at the same time on voyages of 1152 and 720 miles, respectively. The slower vessel reaches its destination one day before the other. What is the rate per hour of the faster vessel?

4. There are five numbers in arithmetical progression. Their sum is to the sum of their squares as 9 : 89. The sum of the first four numbers is 32. Find the numbers.

5. Find the limit of the sum of $4+3+\frac{9}{4}+\cdots\cdots$

6. Find the value of x in an infinite series in terms of y, if $y=1-2x+3x^2$.

128.

1. Reduce to the simplest form
$2\sqrt[3]{3}(\sqrt[3]{9}-2\sqrt[3]{2\tfrac{2}{3}}+4\sqrt[3]{\tfrac{1}{3}}-3\sqrt[3]{2})$.

2. Solve $3x^{\frac{1}{2}}-x^{-\frac{1}{2}}+2=0$.

3. In t seconds a body falling freely describes $16t^2$ feet. The velocity of sound is 1100 feet a second, very nearly. If a stone dropped from the top of a tower is heard to strike the ground after 4 seconds, find the height of the tower.

4. The sum of three numbers in arithmetical progression is 24, and their product is 480. Find them.

5. The third and the seventh terms of a geometrical series are 12 and 192. Find the tenth term.

6. Find the tenth term of $\dfrac{1}{(1-5x)^2}$.

7. In what scale is 4954 expressed by 20305?

129.

1. Find the arithmetical mean, the geometric mean, and the harmonic mean between $a+b$ and $a-b$.

2. A man divided \$216 equally among a certain number of persons. If there had been three more persons, each would have received \$1 less. Required the number of persons.

3. Solve $x^4+y^4=706$; $\quad x-y=2$.

4. Find the value of the last term of an arithmetical series, having given the first term, the common difference, and the sum of the series.

5. Develop $(3a^{-1}-2x)^{-4}$ to five terms.

6. Separate into partial fractions $\dfrac{x^2-3x+2}{x^3-6x^2-x+30}$.

7. How many different arrangements can be made with the letters of the word *freshman*, if the letters *mna* are never separated?

130.

1. Divide $\frac{1}{2}\sqrt{\frac{1}{2}}$ by $\sqrt{2}+3\sqrt{\frac{1}{2}}$.

2. Solve $x+4+\left(\dfrac{x+4}{x-4}\right)^{\frac{1}{2}}=\dfrac{12}{x-4}$.

3. A sculptor purchased two cubical blocks of marble for \$2880, at \$5 per cubic foot. The length of the two together was 12 feet. Find the length of each.

4. An army on the march is advancing at the rate of 12 miles a day, when a detachment 55 miles in the rear is ordered to join it. How long will it take to do so, if it can advance 25 miles the first day, 24 the next, 23 the next, and so on?

5. The third term of a geometric series is 20, the eighth term is 640, and the sum of the series 20475. How many terms are there?

6. Insert five harmonic means between -1 and $\frac{1}{2}$.

7. What is the sixth term of $(a^2-x^2)^{-2}$?

131.

1. Solve $9x - 3x^{\frac{1}{2}} + 4(x^{\frac{1}{2}} - 3x + 5)^{\frac{1}{2}} = 11$.

2. Solve $x^2 + xy + y^2 = 52$; $\quad xy - x^2 = 8$.

3. From two towns, 168 miles apart, A and B set out to meet each other. A went 3 miles the first day, 5 the second, 7 the third, and so on. B went 4 miles the first day, 6 the second, 8 the third, and so on. In how many days did they meet?

4. The sum of three numbers in harmonical progression is 37, and the sum of their squares is 469. Find the numbers.

5. Expand to six terms, in a series of ascending powers of x, the fraction $\dfrac{1+2x}{1-x-x^2}$.

6. How many numbers between 10,000 and 15,000 can be formed with the digits 0, 1, 2, 3, 4, 5, 6, no repetitions being allowed, and each number to be divisible by 5?

132.

1. Rationalize the denominator of the fraction $\dfrac{\sqrt[3]{5} - \sqrt[3]{2}}{\sqrt[3]{5} + \sqrt[3]{2}}$.

2. Solve $x^{\frac{1}{2}} + 5x^{\frac{1}{4}} - 22 = 0$.

3. Solve $\dfrac{(x+y)^2}{a^2} + \dfrac{(x-y)^2}{b^2} = 8$; $\quad x^2 + y^2 = 2(a^2 + b^2)$.

4. There are two arithmetical series which have the same common difference. The first terms are 3 and 5, respectively, and the sum of seven terms of one is to the sum of seven terms of the other as 2 : 3. Determine the series.

5. What is the least integer which added to each term of the ratio 9 : 23 will make it greater than the ratio 7 : 11?

6. Ten papers are to be set at an examination, four of them being in mathematics. In how many different orders can they be set so that the mathematical papers shall always come together?

133.

1. Solve $2x - y = 2$; $8x^3 - y^3 = 98$.

2. A and B start at the same time from the places C and D, A to travel from C to D, B to travel from D to C. When they meet, A had travelled thirty miles more than B. If they should continue at the same rates, A would finish his journey in 4 days and B in 9 days. Find the distance from C to D.

3. The first term of an arithmetical progression is $\frac{3}{2}$, and the difference between the third and the seventh terms is 3. Find the sum of n terms.

4. Find the limit of the sum of $\frac{3}{8} - \frac{1}{2} + \frac{3}{8} - \cdots\cdots$

5. Resolve into partial fractions $\dfrac{x^2}{x^3 - 7x^2 + 36}$.

6. From a society consisting of 12 men and 8 women, in how many ways can a committee of 4 men and 3 women be selected? In how many of these ways will a particular woman always be included?

134.

1. If a varies directly as $\sqrt{b}$, and inversely as c^3, and if $a = 3$ when $b = 256$ and $c = 2$, find b when $a = 24$ and $c = \frac{1}{2}$.

2. Solve $\sqrt[3]{x + 22} - \sqrt[3]{x + 3} = 1$.

3. Solve $x^2 + y^2 + x + y = 102$; $x + y + xy = 39$.

4. Find four numbers in arithmetical progression, such that the sum of the squares of the first and the second shall be 29, and the sum of the squares of the third and the fourth shall be 185.

5. Insert three geometrical means between $\frac{1}{2}$ and 128.

6. Find the sixth term of $(4a^2cx - 3c^{\frac{3}{2}}y^{\frac{1}{2}})^{\frac{1}{2}}$.

139.

1. Solve $\frac{1}{x}+\frac{1}{y}=\frac{1}{x+y}$; $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{a^2}$.

2. A man wishes to make up as many *different* parties as he can out of 20 friends, each party consisting of the same number. How many should he invite at a time, and how many parties will there be?

3. If on an average 2 ships out of 5 return safe to port, what is the chance that out of 5 ships expected at least two will return?

4. A man wishes to build a wall 31 feet long, and has stones of 4 different lengths: viz., 2.2 feet, 2.5 feet, 3 feet, 4.1 feet. How many of each kind may he use in laying a course?

5. Solve by resolution into factors $x^4-3x^3+3x-1=0$.

6. Find by continued fractions the fourth convergent value for $\sqrt{10}$.

140.

1. What relation exists between the two roots of the equation $ax^2+bx+a=0$?

2. Solve $x^2-y^2=3$; $(x+y)(x^3+y^3)=7$.

3. Find by means of the binomial theorem the value of $\sqrt[4]{16.16}$ correct to five decimal places.

4. Find by the method of finite differences the sum of the infinite series $1+5x+9x^2+13x^3+17x^4+\cdots\cdots$

5. Expand to four terms by the method of indeterminate coefficients $\frac{2+x}{1+x+x^2}$.

6. Find the least integral values of x and y which will satisfy the equation $x^2-20y^2=1$.

141.

1. Define a *logarithm*, its *characteristic*, its *mantissa*. Prove that $\log mn = \log m + \log n$.

2. If 2 is the base of a system of logarithms, find the logarithms of 8, $\frac{1}{64}$, and $\sqrt[3]{16}$.

3. Given $\log a$, $\log b$, $\log c$, and $\log d$; how is the value of $\log \frac{a\sqrt[n]{c^m}}{b\sqrt{d}}$ found?

4. Find by logarithms the value of 24.13×6.052.

5. Find by logarithms the cube root of 3852.

6. Find the value of $\frac{8352 \times 3.69}{(30.57)^3}$.

142.

1. State and prove the rule for finding the logarithm of a quotient.

2. If the base of a system of logarithms is 3, find $\log 9$, $\log \frac{1}{81}$, $\log 27^4$, $\log \sqrt[7]{\frac{1}{243}}$.

3. What is colog 10 in the common system?

4. If $\log 6492 = 3.81238$, find $\log 0.00006492$.

5. Find the mean proportional between $(0.01)^{\frac{1}{2}}$ and $(0.2)^4$.

6. Find the value of $\frac{(0.005234)^{\frac{2}{3}} \times (0.017)^{\frac{1}{2}}}{24^{\frac{1}{2}}}$.

143.

1. What is the logarithm of 1 in any system? of the base of the system? of the reciprocal of the base? If the base is greater than 1, for what numbers are the logarithms positive, and for what negative? Give reasons for your answers.

2. If the base of a system of logarithms is $\frac{2}{5}$, find $\log \frac{125}{8}$, $\log \frac{625}{16}$, and $\log \sqrt[3]{\frac{4}{25}}$.

3. If $\log a = 0.7824$, and $\log b = 0.6358$, find $\log \sqrt{a^2 + b^2}$.

4. Find the value of $\frac{\sqrt[5]{0.35^4}}{\sqrt[3]{0.47^2}}$.

5. Solve $20^x = 100$.

144.

1. If the base of a system of logarithms is 64, find $\log 4$, $\log 16$, $\log 32$, $\log \frac{1}{256}$.

2. Of what number is $-\frac{1}{2}$ the logarithm in a system whose base is 16?

3. Find the value of $\frac{35^{\frac{1}{2}} \times 12^{1.7}}{13}$.

4. Find the square root of $\frac{\sqrt[3]{0.0125} \times \sqrt{31.15}}{0.00081}$.

5. Solve $10^{\frac{1}{x}} = 2.45$.

6. A man owes $14,720.20. At the end of each year he pays $2000. In how many years will the debt be paid, the rate of interest being 6 per cent?

145.

1. If the base of a system of logarithms is $-\frac{2}{3}$, find $\log -\frac{8}{27}$, $\log \frac{4}{9}$, $\log \frac{81}{16}$.

2. Of what number is -5 the logarithm in the common system? in the system whose base is 3? in the system whose base is $\frac{1}{3}$?

3. For what base is $\log \frac{81}{10000}$ equal to 4?

4. Find the sixth root of 0.000000004096.

5. Find the value of $\dfrac{42 \times (0.0016)^{\frac{1}{4}}}{\sqrt[3]{108}}$.

6. Solve $(\frac{5}{3})^x = 17.4$.

146.

1. If the base of a system of logarithms is -6, find $\log 36$, $\log 1296$, $\log -\frac{1}{216}$.

2. Find the value of $(8.31)^{-0.27}$.

3. If $\sqrt[3]{8.37} : (0.84)^2 = \sqrt[5]{0.054321} : x$, find the value of x.

4. Find the value of $\dfrac{(15.6)^{\frac{1}{3}} \times (0.0045)^{\frac{2}{3}}}{(0.00065)^{\frac{1}{4}}}$.

5. Solve $(\frac{1}{2})^x = \frac{2}{3}$.

6. What sum will amount to \$1000 in 10 years at 5 per cent per annum compound interest?

147.

1. What is the logarithm of 512 to the base $2\sqrt{2}$?

2. What is the base of the system in which $\log 81 = -4$?

3. Find the value of $\dfrac{1.265 \times 0.01628}{2.2825 \times 64.28}$.

4. Find the mean proportional between $\sqrt{4.756}$ and $\sqrt[3]{(0.0078)^2}$.

5. Find the value of $\sqrt[4]{381 + \sqrt[3]{58}}$.

6. In how many years will a given sum of money treble itself, at 3 per cent per annum compound interest, payable semi-annually?

148.

1. What is the logarithm of $\frac{1}{144}$ to the base $2\sqrt{3}$?

2. Find the value of $\sqrt[3]{(24.4)^{-2}}$.

3. Given the Napierian base 2.71828; find the Napierian logarithms of 19, 23, 29, and 31, to five decimal places.

4. Find the value of $\left(\dfrac{5\sqrt[3]{138}}{\sqrt[5]{0.01}}\right)^{\frac{1}{2}}$.

5. A man places \$25,000 at 5 per cent compound interest, and draws out \$1000 at the end of each year. What will be due at the end of 12 years?

6. Solve $2^x = 769$.

149.

1. What is $\log 1$ in any system? Why?

2. If the base of a system of logarithms is a, what is $\log a$, $\log \frac{1}{a}$, $\log a^3$, $\log \sqrt[5]{a^7}$, $\log \frac{a^m}{a^n}$, $\log \left(\frac{1}{a^n}\right)^{\frac{1}{m}}$?

3. Find the value of $\sqrt{\sqrt[3]{4} + \sqrt[3]{5}}$.

4. Find the value of $\sqrt[9]{\frac{347 \times \sqrt[7]{0.0073}}{126\sqrt[3]{\frac{4}{9}}}}$.

5. Given the Napierian base $e = 2.71828$, $\log_e 5 = 1.60943$, $\log_e 11 = 2.39789$; find to five decimal places the common logarithms of 5 and 11.

150.

1. If the base of the system is a^m, what is $\log a^m$, $\log a^{5m}$, $\log \frac{1}{a^m}$, $\log 1$, $\log a^{mn}$, $\log \frac{a^m}{a^n}$?

2. Find the value of $\frac{\sqrt[3]{172500}}{\sqrt[5]{0.01}}$.

3. Find the value of $\frac{\sqrt[3]{69 + 3\sqrt[5]{1.19}}}{18\sqrt{0.95}}$.

4. Find the modulus for changing common logarithms to those whose base is 7.

5. Find to five decimal places $\log 54$ in a system whose base is 12.

6. A spendthrift, at the age of 30, comes into possession of a fortune of $\$200{,}000$, which pays 5 per cent. He spends each year all his income, and also borrows $\$2000$ at 5 per cent compound interest. How old will he be when his fortune is all gone?

EXAMINATION PAPERS IN ALGEBRA.

I.

BOWDOIN COLLEGE, BRUNSWICK, ME.

Examination for Admission, June, 1883.

1. Divide $x^4+x^3+5x-4x^2-3$ by x^2-2x-3.

   Divide a^6-b^6 by $a^5+2a^3b+2ab^2+b^5$.

2. Resolve $x^{16}-y^{16}$ into five factors.

3. Find the L.C.M. of $(x+2a)^3$, $(x-2a)^3$, and (x^2-4a^2).

4. From $\frac{a}{a-x}$ subtract $\frac{ax}{a^2-x^2}$.

5. Multiply together $\frac{1-x^2}{1+y}$, $\frac{1-y^2}{x+x^2}$, and $1+\frac{x}{1-x}$.

6. $(x+\frac{5}{2})(x-\frac{3}{2})+\frac{3}{4}=(x+5)(x-3)$. Find x.

7. $\frac{4x+81}{10y-17}=6$. $\frac{12x+97}{15y-17}=4$. Find x and y.

8. Find the square root of $x^4-x^3+\frac{x^2}{4}+4x-2+\frac{4}{x^2}$.

9. $\frac{1+a+\sqrt{1-a^2}}{1+a-\sqrt{1-a^2}}$. Reduce this to a fraction with rational denominator.

10. $\frac{1}{2}x^2-\frac{1}{3}x+7\frac{1}{3}=8$. Find x.

    $x^6-8x^3=513$. Find x.

II.

Dartmouth College, Hanover, N.H.

Entrance Examination, June, 1883.

1. Divide $a^2+\frac{1}{a^2}-2$ by $\frac{1}{a}-a$.

2. Resolve $a^{12}-x^{12}$ into six factors.

3. Find the L.C.M. of x^3-x, x^2-x-2, and x^3+1.

4. Reduce $\left(\frac{x-1}{x^3+y^3}-\frac{y^2x-x^2}{y^4-x^4}\right)(y^2+x^2)$ to its sinplest form.

5. Solve the equations: (i.) $2x-\left(x-\frac{x-1}{3}\right)=\frac{5x}{4}$;

(ii.) $\begin{cases} \frac{x}{2}+\frac{y}{4}=3 \\ \frac{2x}{3}-\frac{y}{6}=2 \end{cases}$ (iii.) $\begin{cases} \frac{1}{x}+\frac{1}{y}=\frac{1}{4} \\ \frac{1}{x}-\frac{1}{y}=\frac{1}{12} \end{cases}$

6. Write the values of $8^{-\frac{1}{3}}$, 8^0, $16^{\frac{3}{4}}$, and $(8a^{\frac{1}{2}}b^2)^{\frac{1}{3}}$.

7. Multiply together $\sqrt[3]{ab^2}$, $a^{\frac{1}{2}}b^{-1}c^{\frac{1}{2}}$, and $a^{-\frac{1}{2}}b^{\frac{1}{2}}c^{-1}$.

8. Divide a^3-b^3 by $a^{\frac{3}{2}}-\sqrt{b}$.

III.

Boston University, Boston, Mass.

Examination for Admission, June, 1882.

1. Multiply together $\frac{1-x^2}{1+y}$, $\frac{1-y^2}{x+x^2}$, and $\frac{1}{1-x}$; and simplify the result as much as possible.

2. Given $\frac{y-z}{2}-\frac{x+z}{4}=\frac{1}{2}$, $\frac{x-y}{5}-\frac{x-z}{6}=0$, and $\frac{y+z}{4}=\frac{x+y}{2}-4$. Find the values of x, y, and z.

3. Simplify $\frac{x}{x+1}-\frac{x}{x-1}+\frac{2x}{x^2-1}$.

4. Extract the square root of $40x+25-14x^2+9x^4-24x^3$.

5. Given $3x^2-2xy=15$; $2x+3y=12$. Find values of x and y.

6. Express $\frac{\sqrt[3]{5}\times\sqrt[4]{3}}{\sqrt{2}}$ with a single radical sign.

7. Expand $(1-2x^2)^5$ by the binomial theorem.

IV.

Brown University, Providence, R.I.

Examination for Admission, June, 1883.

1. Factor x^4-y^4; also, factor $4a^4-8a^3x+4a^2x^2$.

2. $\frac{6x+7}{15}-\frac{2x-2}{7x-6}=\frac{2x+1}{5}$. Find value of x.

3. A sum of money is divided equally among a certain number of persons; if there had been four more, each would have received a dollar less than he did; if there had been five fewer, each would have received two dollars more than he did; find the number of persons and what each received.

4. Multiply $a^{\frac{1}{2}}-a^{\frac{1}{3}}+1-a^{-\frac{1}{3}}+a^{-\frac{2}{3}}$ by $a^{\frac{1}{3}}+1-a^{-\frac{1}{3}}$.

5. $\sqrt{a+x}+\sqrt{a-x}=\sqrt{b}$. Find value of x.

6. $x+y=4$; $\frac{1}{x}+\frac{1}{y}=1$. Find values of x and y.

7. A boat's crew row $3\frac{1}{2}$ miles down a river and back again in 1 hour and 40 minutes; supposing the river to have a current of 2 miles per hour, find the rate at which the crew would row in still water.

8. Find sum of six terms of the geometrical progression of which $\frac{2}{3}$ is the first term and $\frac{2}{9}$ the second term.

V.

MASS. INSTITUTE OF TECHNOLOGY, BOSTON, MASS.

Entrance Examination, June, 1882.

1. Factor the following expressions:
$4x^2 - 12xy + 9y^2$; $x^2 + 5x + 6$; $x^3 - 8y^3$.

2. Find the G.C.D. of
$2x^3 - 4x^2 - 13x - 7$ and $6x^3 - 11x^2 - 37x - 20$.

3. Find the L.C. M. of $4(1+x)$, $4(1-x)$, and $2(1-x^2)$.

4. Simplify $\frac{2a+b}{a-b} - \frac{2a-b}{a+b} - \frac{6ab}{a^2-b^2}$.

5. Multiply $a^n + a^{\frac{m}{2}}$ and $\sqrt{a}$ together.

6. Solve the equation $\frac{ax^2}{b-cx} + a + \frac{ax}{c} = 0$.

7. Solve the simultaneous equations
$\frac{x-4}{5} - \frac{y+2}{10} = 0$ and $\frac{x}{6} + \frac{y-2}{4} = 3$.

8. Extract the square root of $x^4 - 2x^3y + 3x^2y^2 - 2xy^3 + y^4$.

9. Solve the quadratic equation $x - \frac{14x-9}{8x-3} = \frac{x^2-3}{x+1}$.

10. Solve the simultaneous quadratic equations
$\frac{1}{x} + \frac{1}{y} = 5$ and $\frac{1}{x^2} + \frac{1}{y^2} = 13$.

VI.

Entrance Examination, Sept., 1882.

1. Factor the following:
$9m^2 - 24m + 16$; $x^2 - 2xy + y^2 - z^2$.

2. Find the G.C.D. of
$12x^3 - 9x^2 + 5x + 2$ and $24x^2 + 10x + 1$.

3. Find the L.C.M. of $x^2 - 1$; $x^2 + 2x - 3$; $6x^2 - x - 2$.

4. Simplify $\frac{3}{x-a}+\frac{4a}{(x-a)^2}-\frac{5a^2}{(x-a)^3}$.

5. Show that

$$(a+b\sqrt{-1})(a-b\sqrt{-1})=(a+b+\sqrt{2ab})(a+b-\sqrt{2ab}).$$

6. Solve the equation $\frac{x^2-a}{bx}-\frac{a-x}{b}=\frac{2x}{b}-\frac{a}{x}$.

7. Solve the simultaneous equations

$$\frac{x+y}{y-x}=\frac{15}{8}\ ;\ 9x-\frac{37+44}{7}=100.$$

8. Extract the square root of

$$x^6+3x^5+6x^4+7x^3+6x^2+3x+1.$$

9. Solve the quadratic equations

$$\frac{a}{3}+\frac{5x}{4}-\frac{x^2}{3a}=0 \text{ and } 19x^4+216x^7=x.$$

10. Solve the simultaneous quadratic equations

$$\frac{x}{a}+\frac{y}{b}=1, \text{ and } \frac{a}{x}+\frac{b}{y}=4.$$

VII.

HARVARD COLLEGE, CAMBRIDGE, MASS.

Examination for Admission, June, 1883.

1. Solve the equation $\frac{1}{x}=2-\frac{4ax^2-3b(x-2)}{2a(x^2+1)+3b}$.

2. A man walks, at a regular rate of speed, on a road which passes over a certain bridge, distant 21 miles from the point which the man has reached at noon. If his rate of speed were half a mile per hour greater than it is, the time at which he crosses the bridge would be an hour earlier than it is. Find his actual rate of speed, and the time at which he crosses the bridge. Explain the *negative* answer.

3. Find the prime factors of the coefficient of the 6th term of the 19th power of $(a-b)$. What are the exponents in the same term, and what is the sign?

4. Reduce the following fraction to its lowest terms:
$$\frac{x^4+2x^3+9}{x^4-4x^3+10x^2-12x+9}.$$

5. Prove that, if $a:b=c:d$, $\frac{a+b}{c+d}=\frac{a-b}{c-d}=\frac{a}{c}=\frac{b}{d}$.

6. Solve the equations, $xy=4-y^2$; $2x^2-y^2=17$. Find all the answers, and show what values of x and y belong together.

VIII.

Yale College, New Haven, Ct.

Examination for Admission, June, 1883.

1. Reduce the following expression to its simplest form:
$$\frac{1}{x(x-a)(x-b)}+\frac{1}{a(a-x)(a-b)}+\frac{1}{b(b-x)(b-a)}.$$

2. Resolve y^9-b^9 into three factors.

3. Change $xy^{-3}-2x^{\frac{1}{2}}y^{-1}z^{-\frac{1}{2}}+z^{-1}$ to an expression which will contain no negative exponents.

4. If $\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$, prove by the principles of proportion that $\frac{a}{b}=\frac{c}{d}$.

5. Find the value of $2a\sqrt{1+x^2}$ when $x=\frac{1}{2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)$.

6. Given $(7-4\sqrt{3})x^2+(2-\sqrt{3})x=2$, to find x.

7. The sum of two numbers is 16, and the sum of their reciprocals is $\frac{1}{3}$. What are the numbers?

8. Compute the value of the continued fraction

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{5}}}}$$

9. Convert $\dfrac{1}{\sqrt{1+x^2}}$ into an infinite series by the method of indeterminate coefficients, or by the binomial theorem.

10. Insert three geometrical means between $\frac{1}{2}$ and 128.

IX.

SHEFFIELD SCIENTIFIC SCHOOL, NEW HAVEN, CT.

Entrance Examination, June, 1883.

Candidates for examination in this subject as a whole should take the whole of this paper; those for the first year's partial examination, the first part of it; those for the second year's partial examination, the second part.

State what text-book you have studied, and to what extent.

I.

1. Reduce to their simplest forms the fractions

(i.) $\dfrac{ac+bd+ad+bc}{af+2bx+2ax+bf}$; (ii.) $\dfrac{ax^m-bx^{m+1}}{a^2bx-b^3x^3}$.

2. Given $\dfrac{ace}{d}-\dfrac{(a+b)^2x}{a}-bx=ae-3bx$, to find x.

3. A sum of money, at simple interest, amounted in m years to a dollars, and in n years to b dollars. Find the sum and the rate of interest.

4. Prove that if $\dfrac{x-y}{m}<1-\dfrac{y}{x}$, and m is positive, then $x<y$.

5. (i.) Simplify $(a^2b^3)^{\frac{1}{2}} + (a^3c^6)^{\frac{1}{3}}$.

(ii.) Extract the square root of $6hm^{2n} + h^2 + 9m^{4n}$.

(iii.) Reduce $\dfrac{\sqrt{x+a}+\sqrt{x-a}}{\sqrt{x+a}-\sqrt{x-a}}$ to an equivalent fraction with a rational denominator.

II.

6. Given $15x^3 - 20x = 35$, to find x.

7. Given $\dfrac{x+\sqrt{x^2-9}}{x-\sqrt{x^2-9}} = (x-2)^2$, to find x.

8. Given $x^2 - xy = 48$ and $xy - y^2 = 12$, to find x and y.

9. The number of permutations of n things taken r together is equal to 10 times the number when taken $r-1$ together; and the number of combinations of n things taken r together is to the number when taken $r-1$ together as 5 to 3; required the value of n and r.

10. Expand $\dfrac{3+2x}{5+7x}$ into a series of ascending powers of x, by the method of indeterminate coefficients. (Four terms of the series will be sufficient.)

X.

Entrance Examination, Sept., 1883.

(State what text-book you have studied, and to what extent.)

1. Given $\dfrac{x-y+1}{x-y-1} = a$ and $\dfrac{x+y+1}{x+y-1} = b$, to find x and y.

2. Simplify (i.) $\sqrt{27} + 2\sqrt{48} + 3\sqrt{108}$.

(ii.) $(\sqrt[7]{a^2b})^3 (\sqrt[7]{a^5b^{12}})^4$.

(iii.) $\dfrac{x^{2p(q-1)} - y^{2q(p-1)}}{x^{p(q-1)} + y^{q(p-1)}}$.

3. Form an equation whose roots shall be 2 and -3. Resolve $x^2 - 3x + 4$ into two factors.

4. Given $\frac{1}{x} + \frac{1}{y} = 5$ and $\frac{1}{x^2} + \frac{1}{y^2} = 13$, to find x and y.

5. Given $\frac{3x + \sqrt{4x - x^2}}{3x - \sqrt{4x - x^2}} = 2$, to find x.

6. To deduce a formula for the sum of a geometric progression in terms of the first term, the ratio, and the number of terms.

7. Having 10 different letters, how many sets of two each can you form of them, differing by at least one letter?

8. Expand $\frac{1}{1 - 2x + x^2}$ into a series of ascending powers of x by the method of indeterminate coefficients. (Four terms of the series will suffice.)

9. Express $\log \sqrt[3]{\frac{ab^2c^4}{d^5}}$ in a form adapted to computation.

10. To deduce a formula for the amount of a given sum of money for a given time at a given rate of compound interest.

XI.

AMHERST COLLEGE, AMHERST, MASS.

Examination for Admission, June, 1883.

1. Find the value of $6a - [4b - \{4a - (6a - 4b)\}]$.

2. Divide $a^{-3n} - b^{6n}$ by $a^{-n} - b^{2n}$.

3. Show that $a^0 = 1$; also, that $a^{-m} = 1 \div a^m$.

4. Resolve $a^{4m} - b^{4m}$ into its prime factors.

5. Find the G.C.D. of $a^4 - b^4$ and $a^3 + a^2b - ab^2 - b^3$.

6. Given $3ax - 2bx - \frac{1}{3}c - \frac{1}{4}mx = \frac{2}{3}c + \frac{3}{4}mx - n - bx + 2ax$, to find x.

7. Divide the number a into two parts, such that the second part shall equal m times the first part plus n.

8. $3y - 2x = 9$; $7x + y = 26$; find x and y.

9. Multiply $\sqrt{a+c}$ by $\sqrt[3]{a+c}$.

10. $3x^2 - 4x = 15$. Find x.

11. Expand $(1+x^2)^7$ by the binomial formula.

12. Find the $(2n)$th term of the series 1, 3, 5, 7, ...

XII.

WILLIAMS COLLEGE, WILLIAMSTOWN, MASS.

Entrance Examination, June, 1883.

1. Divide $x^3 + \frac{1}{x^3} + 2$ by $x + \frac{1}{x}$.

2. Add the fractions $\frac{a}{2a-2b}$ and $\frac{b}{2b-2a}$.

3. Simplify $\cfrac{a}{b + \cfrac{c}{d + \cfrac{e}{f}}}$.

4. The sum of two numbers is 5760, and their difference is equal to one-third of the greater. Find the numbers.

5. $\frac{x}{a} + \frac{y}{b} = 1$; $\frac{x}{3a} + \frac{y}{6b} = \frac{2}{3}$. Find x and y.

6. Solve the equation $\sqrt{3x+4} + \sqrt{3x-5} = 9$.

XIII.

TUFTS COLLEGE, COLLEGE HILL, MASS.

Examination for Admission, June, 1881.

1. Divide $2a^{m+1} - 2a^{n+1} - a^{m+n} + a^{2n}$ by $2a - a^n$.
2. Find the G.C.D. of $ab + am$, $bn + mn$, and $b^2n - m^2n$.
3. Simplify $\dfrac{x^4 - \dfrac{1}{x^4}}{x + x^{-1}}$.
4. Solve $3x - \dfrac{3x-19}{2} - 8 = \dfrac{23-x}{4} + \dfrac{5x-38}{3} + 10$.
5. Solve $\sqrt{4+x} = 4 - \sqrt{x}$.
6. Solve $\dfrac{1}{x} + \dfrac{2}{y} = \dfrac{11}{15}$ and $\dfrac{3}{x} + \dfrac{4}{y} = \dfrac{9}{5}$.
7. There are three numbers whose sum is 324; the second exceeds the first as much as the third exceeds the second; the first is to the third as five to seven. What are the numbers?
8. $\dfrac{x}{a} + \dfrac{b}{x} = c$. Find the values of x.

XIV.

TRINITY COLLEGE, HARTFORD, CT.

Examination for Admission, June, 1883.

(One problem may be omitted in each of the three divisions indicated by the letters A, B, C.)

A.

1. Find the G.C.D. of $2x^3 + x - 1$, $x^3 + 5x + 4$, and $x^3 + 1$.
2. Solve the equation $\dfrac{6x+7}{15} - \dfrac{2x-2}{7x-6} = \dfrac{2x-1}{5}$.

3. Two workmen together finish some work in 20 days; but if the first had worked twice as fast and the second half as fast, they would have done it in 15 days. How long would it take each alone to do the work?

B.

4. Multiply $2\sqrt{-3}-3\sqrt{-2}$ by $4\sqrt{-3}+6\sqrt{-2}$; divide $\sqrt{-5}$ by $\sqrt{-1}$. Explain the process in each case.

5. Solve the equation $\sqrt{x-3}-\sqrt{x-14}-\sqrt{4x-155}=0$. Give and explain the rule for solving a quadratic equation.

6. Solve the equation $\frac{1}{x^2-1}+\frac{1}{3}=\frac{1}{3(x-1)}+\frac{1}{x+1}$; also, $x^4+4x^2=117$.

C.

7. Find two numbers such that their product is 96, and the difference of their cubes is to the cube of their difference as 19 to 1.

8. In an arithmetical progression, $a=3$, $l=42\frac{2}{3}$, $d=2\frac{1}{3}$; find n and s. Explain the rule for the sum of a geometrical progression.

9. Expand $(a-b)^{11}$ and $\left(\frac{x}{2}+3y\right)^5$ by the binomial theorem.

XV.

WESLEYAN UNIVERSITY, MIDDLETOWN, CT.

Examination for Admission, June, 1883.

1. When $a=1$, $b=0$, $c=2$, find the value of $(3a+2)(2a+b)-a\{2c-a[(3a^2+6)+c]\}$.
Multiply $a^{\frac{1}{2}}x^{\frac{1}{2}}+2a^{\frac{1}{4}}x^{\frac{1}{4}}$ by $2a^2x^{\frac{1}{2}}-a^{\frac{1}{4}}x^{\frac{3}{4}}$.

2. Factor y^3-64; $a^2b^2-3ab-4$.

3. Given $7\left(\frac{x}{2}-1\right)-3(2y+8)=0$; $\frac{2x+2}{9}-\frac{10y+9}{2}=0$; to find x and y.

4. Solve at least two of the following:

(i.) $6x^2-13x+6=0$;

(ii.) $3y^m\sqrt[3]{y^m}+\frac{2y^m}{\sqrt[3]{y^m}}=16$;

(iii.) $x^2+y^2=8$; $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{2}$

How do you "complete the square"?

5. From $a\sqrt{\frac{b^2z+b^2}{z-1}}$ take $b\sqrt{\frac{a^2z-a^2}{z+1}}$, and express the result in its simplest form.

6. Write the repeating decimal $0.\dot{3}$ as the sum of a geometrical progression. Find the limit of the sum.

XVI.

CORNELL UNIVERSITY, ITHACA, N.Y.

Entrance Examination, June, 1882.— Elementary Algebra.

1. Define: known and unknown quantities, positive and negative quantities, addition, a common multiple of two or more numbers, a radical, an equation, a theorem.

2. Resolve m^4-n^4 into three prime factors.

3. Reduce the fraction $\frac{\sqrt{x^2+xy+y^2}}{\sqrt{x-y}}$ to an equivalent fraction having a rational denominator.

4. Divide $x+y+z-3\sqrt[3]{xyz}$ by $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}$.

5. For $8 I can buy 2 pounds of tea, 10 pounds of coffee, and 20 pounds of sugar, or 3 pounds of tea, 5 pounds of coffee, and 30 pounds of sugar, or 5 pounds of tea, 5 pounds of coffee, and 10 pounds of sugar. What are the prices?

6. Solve the equation $\frac{ax-b}{4}+\frac{a}{3}=\frac{bx}{2}-\frac{bx-a}{3}$.

7. Solve the equation $x+5+\sqrt{x+5}=6$, giving all the roots.

8. Solve the equation $\frac{x+a}{x-2a}+\frac{x-2a}{x+a}=1$, and get the sum and the product of the two roots.

XVII.

Entrance Examination, June, 1882. — Advanced Algebra.

1. Prove the formula for the development of $(a+x)^n$; and from this formula get the development of $(1+x+x^2)^3$, and four terms of $(a^3-x^2)^{-\frac{1}{2}}$.

2. Prove the formula for the sum of a geometrical progression, the first term, the ratio, and the number of terms being given. From this formula obtain an expression for the amount of a deferred annuity at compound interest; the annual payment, rate, and time being given.

3. By the method of differences, find log 24, by continuing the series: $\log 20 = 1.3010$, $\log 21 = 1.3222$, $\log 22 = 1.3424$, $\log 23 = 1.3617$.
From the same data find $\log 21\frac{1}{2}$ by interpolation.

4. Prove that $\log_a b \times \log_b x = \log_a x$.

5. By the method of undetermined coefficients prove that, if $y = x + x^2 + x^3 + \ldots\ldots$, then also $x = y - y^2 + y^3 - \ldots\ldots$

6. By continued fractions, find five successive approximations to the value of $\sqrt{2}$.

7. Depress the equation $x^4+2x^3+x^2=35x+74$, by removing a commensurable root, and then find an incommensurable root correct to two decimal places.

8. Prove that, if the coefficients of an equation with one unknown quantity be real, any imaginary roots enter it in pairs.

XVIII.

STEVENS INSTITUTE OF TECHNOLOGY, HOBOKEN, N.J.

Specimen Entrance Examination Paper.

1. Multiply $\sqrt{2a}$ by $\sqrt[3]{3a}$.

2. Divide $1-x$ by $1+x$ to four terms.

3. Divide $\sqrt{3a}$ by $\sqrt[3]{2a^2}$.

4. Solve $1-\frac{ax-b}{by-c}=0$ and $a-\frac{2y-3}{1-x}=1$.

5. Solve $a-\frac{\frac{3x-2y}{b-1}}{\frac{3y-x}{7}}=2$; $4x-dy=2$.

6. Solve $\sqrt{a-x}-\sqrt{b-x}=\sqrt{c-x}$.

7. Solve $3xy-5y^2=2$; $5xy+3x^2=1$.

8. Solve $\frac{x}{y}-\frac{y}{x}=a$; $bx-\frac{cy-dx}{5}=ay$.

9. Establish the equation for the permutations of n things taken r at a time.

10. Prove that a proportion taken by *division* is a true proportion.

11. If x varies as $a+by$, and when $x=1$, $y=2$; and $x=2$, $y=-5$; show that $7x=9-y$.

12. In an arithmetical series, given the common difference, first term, and number of terms, to find the sum and the last term.

13. In a geometrical progression, given the first term, number of terms, and the last term, to find the sum of the terms.

XIX.

MADISON UNIVERSITY, HAMILTON, N.Y.

Entrance Examination, June, 1882.

1. Define: (i.) positive and negative quantities; (ii.) system of notation; (iii.) similar terms; (iv.) ratio; (v.) compound ratio; (vi.) proportion.

2. Multiply $a^{\frac{m}{n}}b^{-\frac{t}{s}} - a^{\frac{m}{2n}}b^{-\frac{t}{2s}} + 1$ by $a^{\frac{m}{2n}}b^{-\frac{t}{2s}} + 1$.

3. Rationalize the denominators of the following fractions:

$$\frac{1}{(2)^{\frac{1}{2}}}, \quad \frac{x^{\frac{1}{m}}}{y^{\frac{1}{n}}}, \quad \frac{3}{(3)^{\frac{1}{2}} + (5)^{\frac{1}{2}}}$$

4. Given $\frac{3x^{\frac{1}{2}} - 4}{2 + x^{\frac{1}{2}}} = \frac{15 + (9x)^{\frac{1}{2}}}{40 + x^{\frac{1}{2}}}$, to find the value of x.

5. Four given numbers are represented by a, b, c, and d; what quantity added to each will make them proportional?

6. Suppose a body to move eternally in this manner: viz., 20 miles the first minute, 19 miles the second minute, $18\frac{1}{20}$ miles the third minute, and so on in geometrical progression; what is the utmost distance it can reach?

7. A hare, 50 of her leaps before a greyhound, takes 4 leaps to the greyhound's 3; but 2 of the greyhound's leaps are equal to 3 of the hare's. How many leaps must the greyhound take to catch the hare?

XX.

VASSAR COLLEGE, POUGHKEEPSIE, N.Y.

Specimen Examination Paper for Admission.

1. Factor the following expressions:

$9x^4 - y^2$; $m^2 - 2mn + n^2$; $n^5 - n$; $m^4 - n^4$; $x^3 + a^3$.

2. What is the rule for transposing a term from one side of an equation to the other; what is the principle?

3. Solve the equation $\frac{x}{a}-\frac{a}{a+b}=\frac{x}{a-b}$.

4. Find the fifth power of $2a^2$; the fourth root of $\frac{27a^{-3}b^{20}}{16d^{12}z^4}$.

5. Square $x-\frac{1}{x}$; cube $3+\sqrt{2}$.

6. A and B engage to mow a field. A alone can mow it in b days, and B alone in c days. In what time can both together mow it?

7. Solve the equation $\sqrt{x+4}-\sqrt{x}=\sqrt{x+\frac{5}{4}}$.

8. Solve the the equations
$x^3-y^3=215$; $x^2+xy+y^2=43$.

9. Find four values of x in the equation $x^2+\frac{1}{x^2}=a^2+\frac{1}{a^2}$.

10. Form the equations whose roots are a, $-a$, and b.

11. Deduce the formula for the sum of n terms of a geometrical progression.

XXI.

COLLEGE OF NEW JERSEY, PRINCETON, N.J.

Examination for Admission, Sept., 1883.

1. What text-book have you used?

2. Simplify the expression $\frac{\left(\frac{a^2+b^2}{b}-a\right)\frac{a^3-b^3}{a^3+b^3}}{\frac{1}{b}-\frac{1}{a}}$.

3. Solve the following equations:

$$5-6x+\frac{7x+14}{3}=\frac{17-3x}{5}-\frac{4x+2}{3};$$

$$\sqrt{x+x^{\frac{1}{2}}}=\frac{3}{2}\left(\frac{\sqrt{x}}{1+\sqrt{x}}\right)^{\frac{1}{2}};\quad \frac{x+2}{x-2}-\frac{4-x}{2x}=\frac{7}{3};$$

$$x^{\frac{1}{2}}+x^{\frac{1}{4}}=6.$$

4. Solve the simultaneous equations:

$$\frac{x+3y}{2}=7\tfrac{1}{2};\quad \frac{4x+5y}{4}-8.$$

5. Divide $x+y+z-3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$ by $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}$.

6. Find the square root of

$$16x^4-16abx^3+16b^2x^2+4a^2b^2-8ab^3+4b^4.$$

XXII.

JOHN C. GREEN SCHOOL OF SCIENCE, PRINCETON, N.J.

Entrance Examination, Sept., 1883.

1. What text-book have you used, and how much of it have you studied?

2. Solve the following equations:

$$(3a-x)(a+2x)=(5a+x)(a-2x);$$

$$1-\frac{x-5}{2x+1}=\frac{x-6}{x-2};\quad (x^3-5)^2+29(x^3-5)=96.$$

3. Solve the following pairs of simultaneous equations:

$$\frac{3x}{5}-y=31;\quad x+\frac{y}{5}=33;$$

$$x^2+y^2=85;\quad xy=42.$$

4. Form the quadratic equation whose roots are $\frac{3}{2}$ and $\frac{2}{3}$.

5. Add $\sqrt[3]{\frac{1}{4}}$, $\sqrt[3]{\frac{1}{32}}$, and $\sqrt[3]{\frac{2}{4}}$.

6. Find the square root of $x^4+\frac{x^{-2}}{4}+4x^{\frac{3}{2}}-x+4x^{\frac{1}{2}}-2x^{-\frac{1}{2}}$.

XXIII.

UNIVERSITY OF PENNSYLVANIA, PHILADELPHIA, PA.

Examination for Admission, June, 1882.

1. Divide $x^6 - 9x^5 + 25x^4 - 3x^3 - 98x^2 + 156x - 72$ by $x^3 - 2x^2 - 4x + 8$ by synthetic division.

2. Simplify $\frac{a}{(a-b)(a-c)} + \frac{b}{(b-a)(b-c)} + \frac{c}{(c-a)(c-b)}$.

3. Simplify
$2\sqrt[3]{189} - \sqrt[3]{9317} - 2\sqrt[3]{2401} + 2\sqrt[3]{448} - 2\sqrt[3]{56} + 3\sqrt[3]{875}$.

4. Find the difference between $2 + 1 + \frac{1}{2} + \cdots$ to infinity, and $\frac{1}{8} + \frac{1}{8} + \frac{1}{2} + \cdots$ to 12 terms.

5. Find the square root of
$x^8 - 12x^7 + 62x^6 - 180x^5 + 321x^4 - 360x^3 + 243x^2 - 96x + 16$.

6. Find the value of x from $\sqrt{2x+6} + \sqrt{2x-5} = 11$.

7. Find x and y from
$\frac{x}{x+2} + \frac{3}{y-3} = \frac{7}{12}$ and $\frac{2}{x+2} - \frac{3}{y-3} = -\frac{1}{12}$.

8. Solve the equations $x + y - z = 4$, $2x - 3y + 4z = 20$, and $5x - 6y + 3z = 10$.

9. Find x from $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$.

10. Solve the equations $x^2 + 10xy = 11$ and $5xy - 3y^2 = 2$.

XXIV.

UNIVERSITY OF MICHIGAN, ANN ARBOR, MICH.

Examination for Admission, June, 1883.

1. Define *exponent*. Illustrate the significance of fractional and negative exponents. Write $\sqrt[3]{ax^2y^{-m}}$ without using the radical sign.

2. Define *elimination*. What method do you prefer, and why? Apply to the equations

$$\frac{a}{x}+\frac{b}{y}=m\,;\quad \frac{c}{x}+\frac{d}{y}=n.$$

3. Expand $(a^2+x^2)^{\frac{1}{2}}$ by the binomial formula.

4. Solve the equation $\dfrac{3\sqrt{x}-4}{2+\sqrt{x}}=\dfrac{15+\sqrt{9x}}{40+\sqrt{x}}$.

5. Produce the formulæ for *last term* and *sum* in arithmetical and geometrical progression.

6. Simplify $\dfrac{\left(1+\dfrac{y^3}{x^2}\right)^{\frac{1}{2}}}{x}$, $\dfrac{\left(1+\dfrac{y^4}{x^4}\right)^{\frac{1}{2}}}{x^2}$.

7. Rationalize the denominator of

$$\frac{\sqrt{x^2+x+1}+\sqrt{x^2+x-1}}{\sqrt{x^2+x+1}-\sqrt{x^2+x-1}}.$$

8. Solve the simultaneous equations

$$4(x+y)=3xy\,;\quad x+y+x^2+y^2=26.$$

9. Solve the simultaneous equations

$$x^2+xy+y^2=52\,;\quad xy-x^2=8.$$

10. Define *logarithm*, *mantissa*, *characteristic*. How can you extract roots by logarithms?

Given $\log x=\bar{2}.301030$, what is $\log x^{\frac{1}{3}}$?

XXV.

LAKE FOREST UNIVERSITY, LAKE FOREST, ILL.

Entrance Examination, June, 1882.

1. Define *coefficient*, *exponent*, *similar quantities*, *monomial*, *binomial*, *equation*.

2. Find the G.C.D. of

$$x^4+2x^3+9 \text{ and } 7x^3-11x^2+15x+9.$$

3. Find the value of x in

$$\frac{3x-1}{7}+\frac{5-x}{4}-\frac{2x-4}{12}=2-\frac{x+2}{28}.$$

4. Explain the rule for subtraction, showing why the signs of the subtrahend are changed. Illustrate by diagram or numbers.

5. A man rows a boat with the tide 8 miles in 48 minutes, and returns against a tide two-fifths as strong in 80 minutes, what is the rate of the stronger tide?

6. The product of two numbers is 702, and their sum is 60. Find the numbers.

7. Factor x^2-2x-3 and $x^3-x^2-13x+24$.

8. Solve $\frac{\sqrt{a-x}}{x}-\frac{\sqrt{a-x}}{a}=\sqrt{x}$.

9. Solve $x+5\sqrt{37-x}=43$.

10. What number added to its reciprocal makes 2.9?

XXVI.

EDUCATION DEPARTMENT, ONTARIO.

Examination of Third Class Teachers, July, 1883.

1. Divide

(i.) $(a-b)c^3+(b-c)a^3+(c-a)b^3$ by $(a-b)(b-c)(c-a)$;

(ii.) $\frac{x^2+y^2}{x^2y^2}-\frac{x^2+y^2}{x^3y^3}$ by $\frac{1}{x}-\frac{1}{y}$.

2. What must be the values of a, b, and c, that x^3+ax^2+bx+c may have $x-1$, $x-2$, and $x-3$ all as factors?

3. Find the H.C.F. of

(i.) $3x^4-4x^3+1$ and $4x^4-5x^3-x^2+x+1$;

(ii.) $8x^3-y^3+27z^3+18xyz$ and $4x^2+12xz+9z^2-y^2$.

4. Simplify

(i.) $\left(\frac{4x^2}{y^2}-1\right)\left(\frac{2x}{2x-y}-1\right)+\left(\frac{8x^3}{y^3}-1\right)\left(\frac{4x^2+2xy}{4x^2+2xy+y^2}-1\right)$;

(ii.) $\frac{x^3+(a+b)x^2+(ab+1)x+b}{bx^3+(ab+1)x^2+(a+b)x+1}$.

5. Find the value of x that will make $\frac{ac+bd+ad+bc}{x-3c+2d}$ independent of c and d.

6. (i.) If $a+b+c=0$, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\left\{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right\}^2$.

(ii.) If $x=a^2+b^2+c^2$ and $y=ab+bc+ca$, then $x^3+2y^3-3xy^2=(a^3+b^3+c^3-3abc)^2$.

(iii.) If $2a=y+z$, $2b=z+x$, $2c=x+y$, express $(a+b+c)^3-2(a+b+c)(a^2+b^2+c^2)$ in terms of x, y, and z.

7. Find a value of a which will make the quantities $\frac{(a+b)(a+c)}{a+b+c}$ and $\frac{(a+c)(a+d)}{a+c+d}$ equal to one another.

8. Solve the equations

(i.) $\sqrt{x+3}+\sqrt{x+2}=5$;

(ii.) $\frac{5-x}{3}+\frac{5-2x}{4}+\frac{x+1}{3}-\frac{2+5x}{2}=0$;

(iii.) $(x+a+b)(c+d)=(x+c+d)(a+b)$, where $c+d$ is not equal to $a+b$.

9. One side of a right-angled triangle exceeds the other by 3 feet, neither being the hypotenuse, and its area is 18 square feet. What are the sides?

10. A cistern with vertical sides is h feet deep. Water is carried away from it by one pipe $\frac{5}{6}$ as fast as it is supplied by another. Find at what point in the side the former pipe must be inserted that the cistern may fill in twice the time it would did water not flow from it at all.

XXVII.

UNIVERSITY OF TORONTO, TORONTO, ONT.

Junior Matriculation. — Annual Examination for Honors in Algebra, 1883.

1. Find the product of $(a+b)$, (a^2+ab+b^2), $(a-b)$, and (a^2-ab+b^2).

2. If a and b are positive integers, show that $x^a \times x^b = x^{a+b}$.

3. Prove the rule for finding the G.C.M. of two quantities. Find the G.C.M. of $6x^5+15x^4y-4x^3z^3-10x^2yz^2$ and $9x^3y-27x^2yz-6xyz^2+18yz^3$.

4. State the rule for extracting the square root of a compound quantity.
Extract the square root of x^2+y.

5. Solve the following equations:
(i.) $3x+z=11,\ 2y+3z=16,\ 5x+4y=35$;
(ii.) $\dfrac{x+a}{x-a}-\dfrac{x+b}{x-b}=c$;
(iii.) $\dfrac{x}{a}+\dfrac{a}{x}=2+\dfrac{c}{x}$.

6. When are quantities said to be in geometrical progression, when in harmonical progression, and when in arithmetical progression? (i.) Find two harmonical means between a and b. (ii.) The first term of a geometric series is $\frac{1}{3}$, the ratio $\frac{1}{2}$, and the number of terms is 6; find the sum of the series.

7. Show that the number of combinations of n things taken r together is

$$\frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{1\times 2\times 3\ldots\ldots r}$$

How many words of four letters can be formed out of the first 13 letters of the alphabet, having one vowel in each word?

8. Expand to five terms $(a+b)^{-\frac{1}{2}}$.

Show that $\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$

$$=1+x+\tfrac{1}{2}(x^2+x^3)+\tfrac{3}{8}(x^4+x^5)+\tfrac{5}{16}(x^6+x^7)+\ldots\ldots$$

9. A number consists of two digits: when the number is divided by their sum the quotient is 4, and when divided by their difference the quotient is 12; find the number.

10. The crew of a boat rowed six miles down a river, and half-way back again, in 2 hours. Supposing the stream to have a current $2\frac{1}{2}$ miles an hour, find at what rate they would row in still water.

XXVIII.

COLLEGE OF OTTAWA, OTTAWA, CAN.

Matriculation Examination. Session 1882–83.

1. Translate the following into common language:

$$\frac{\sqrt{5b}+3\sqrt{c}}{1+2a}=5x+\tfrac{1}{4}.$$

2. Divide (i.) $2a^2b+b^3+a^3+2ab^2$ by a^2+b^2+ab;
(ii.) $12n^{-2}y^{-4}$ by $-4xy^2$.

3. Find the prime factors of $25c^2d^4-9a^6c^4$.
Find the G.C.D. and L.C.M. of $9mx^2-6mx+m$ and $9nx^2-n$.

4. Find the sum of

$$\frac{a+b}{(b-c)(c-a)}+\frac{b+c}{(c-a)(a-b)}+\frac{c-a}{(a-b)(b-c)}.$$

Find the algebraic sum of

$$\frac{3}{1-2x}-\frac{7}{2x+1}-\frac{4-20x}{4x^2-1}.$$

5. Solve $x-\frac{3x-3}{5}+4=\frac{20-x}{2}-\frac{6x-8}{7}+\frac{4x-4}{5}$.

Find x, y, and z in the following equations:

$x+y+z=24$; $x-y+z=8$; $x+y-z=6$.

6. Raise $\frac{-bax^{-2}y^4}{5mn^{-1}}$ to the cube.

7. Give the simplest form of $\sqrt{3\sqrt[3]{3}}$.

8. Show why the square may be completed in the quadratic $3x^2-7x=20$, by the same rule as in $x^2+2x=24$, without introducing fractions.

9. Given $x^2-2xy=12$ and $x^2-y^2=12$, to find x and y.

10. A boy being asked how many sheep his father had, replied that $\frac{3}{4}$ of $\frac{1}{2}$ the flock would be 25 less than the whole flock. How many sheep had his father?

XXIX.

COLLEGE OF OTTAWA, OTTAWA, CAN.

Matriculation Examination. Session 1883–84.

1. Clear away the parentheses, and reduce the following expression:

$$a+b-(2a-3b)-4(5a+7b)-(-13a+2b)+3\{a-6(b-a)\}.$$

2. Give the three formulas for the expansion of $(a+b)^2$, $(a-b)^2$, and $(a+b)(a-b)$, and give an example for each formula.

3. Divide $5x-3-4x^2+x^4+x^3$ by $-3+x^2-2x$.

4. Find the G.C.D. and the L.C.M. of the three following expressions:

$(2x-4)(3x-6)$; $(x-3)(4x-8)$; $(2x-6)(5x-10)$.

5. Simplify $\dfrac{\dfrac{m^2+n^2}{n}-2m}{\dfrac{1}{n}+\dfrac{1}{m}} \times \dfrac{m^2-n^2}{m-n}$.

6. Solve the equations

$2x+4y-3z=22$; $4x-2y+5z=18$; $5x+7y-z=63$.

7. Extract the square root of

$15a^4b^2+a^6-6a^5b-20a^3b^3+b^6+15a^2b^4-6ab^5$.

8. Convert $\sqrt[3]{\frac{3}{4}}$ into such an expression, not a decimal, as shall not necessitate two extractions in finding the cube root of $\frac{3}{4}$.

9. Solve the following equation: $\frac{1}{2}x^2-\frac{1}{3}x+20\frac{1}{2}=42\frac{2}{3}$.

10. The hypotenuse of a right-angled triangle is 20 feet, and the area of the triangle is 96 square feet. Find the length of the legs.

11. Find the tenth term, and the sum of ten terms, of the series 1, 4, 10, 20, 35.

12. Develop $\dfrac{1+x}{1-x+x^2}$ into an infinite series by the method of undetermined coefficients.

13. Find the value of x in the equation $5^{\frac{2}{x}}=30$.

XXX.

McGILL UNIVERSITY, MONTREAL, CAN.

School Examination, June, 1883.

1. Multiply $1+2x-x^2-\frac{1}{2}x^3$ by itself, and find the value of the result if $1-2x=3$.

2. Find the remainder when $a^5-4a^3b^2-8a^2b^3-17ab^4-15b^5$ is divided by $a^2-2ab-3b^2$.

3. Simplify $\frac{1}{2}x(x+1)\{x+2-\frac{1}{2}(2x+1)\}$; $\frac{2(x^2-\frac{1}{4})}{2x+1}+\frac{1}{2}$.

4. Reduce the following fractions to their lowest terms:
$$\frac{a^2x+a^3}{ax^2-a^3}; \quad \frac{(x^4-a^4)(x-a)}{(x^2+a^2-2ax)(ax+x^2)}; \quad \frac{1+x^3}{1+2x+2x^2+x^3}.$$

5. Find the square root of
$$x^4+2x^3-x+\frac{1}{4} \text{ and of } \frac{4x^2-4x+1}{9x^2+6x+1}.$$

6. Solve the equations
$$\text{(i.) } 2x-\frac{x}{2}=18; \quad \text{(ii.) } (m+n)(m-x)=m(n-x);$$
$$\text{(iii.) } 2x-\frac{y-3}{5}=4; \quad 3y+\frac{x-2}{3}=9.$$

7. If ax^2+bx+c becomes 8, 22, 42, respectively, when x becomes 2, 3, 4, what will it become when $x=-\frac{1}{2}$?

8. Find two numbers which produce the same result, 7, whether one be subtracted from the other, or the latter be divided by the former.

9. In a certain school there are 6 boys to every 5 girls; if there were 2 boys less and 2 girls more, there would be the same number of each. Find the number.

10. Any odd number may be represented by $2v+1$. Prove that the difference of the squares of any two odd numbers is exactly divisible by 8.

XXXI.

UNIVERSITY OF CAMBRIDGE, ENG.

Second General Examination for the Ordinary B.A. Degree, Nov., 1880.
Time allowed, 3 *hours.*

1. Solve the equations (i.) $x - \frac{x-2}{3} = \frac{x+23}{4} - \frac{10+x}{5}$;

(ii.) $\frac{a}{x-b} = \frac{b}{x-a}$; (iii.) $\frac{x}{9} + \frac{y}{8} = 43$, $\frac{x}{8} + \frac{y}{9} = 42$.

2. Solve the quadratic $ax^2 + bx + c = 0$, and determine the condition that its roots may be equal.

If α, β be the roots, form an equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

3. Solve the equations

(i.) $\frac{3x+2}{x-3} + \frac{3x-2}{x+3} - \frac{4x^2+12x+2}{x^2-9}$;

(ii.) $\sqrt{x} + \sqrt{a+x} = \frac{a}{\sqrt{x}}$;

(iii.) $(x+2y)(2x+y) = 20$, $4x(x+y) = 16 - y^2$.

4. The first term of an arithmetical progression of n terms is a, and the last term l. Find the sum, and also the common difference.

If n be odd, and the sum of the even terms be subtracted from the sum of the odd, show that the result is $\frac{a+l}{2}$.

5. Find the sum of n terms of a series in geometrical progression.

If the sum of a geometrical series to infinity be n times the first term, show that the ratio is $1 - \frac{1}{n}$.

6. Find the sum of the series:

(i.) $2 + 2\frac{1}{4} + 2\frac{1}{2} + \cdots$, to 12 terms;

(ii.) $\frac{1}{8} - \frac{1}{2} + \frac{3}{4} - \cdots$, to 8 terms;

(iii.) $3\frac{3}{8} + 2\frac{1}{4} + 1\frac{1}{2} + \cdots$, to infinity.

7. Show that a ratio of greater inequality is increased by taking the same quantity from both its terms.

Show that the ratio $a-x : a+x$ is greater or less than the ratio $a^2-x^2 : a^2+x^2$, according as the ratio $a : x$ is one of less or greater inequality.

8. Define proportion. When are quantities said to be in continued proportion?

If a, b, c, d be in continued proportion, show that $\left(\frac{a-b}{b-c}\right)^3=\frac{a}{d}$.

9. When is one quantity said to vary directly and when inversely as another?

The volume of a sphere varies as the cube of its radius: if three spheres of radii 9, 12, 15 inches be melted and formed into a single sphere, find its radius.

10. A and B start simultaneously from two towns to meet one another. A travels 2 miles per hour faster than B, and they meet in 7 hours; if B had travelled 1 mile per hour faster, and A at only half his previous pace, they would have met in 9 hours. Find the distance between the towns.

11. A wine-merchant buys spirit, and after mixing water with it, sells the mixture at two shillings per gallon more than he paid for the spirit, making $23\frac{3}{4}$ per cent on his outlay: if he had used double the quantity of water he would have made $37\frac{1}{2}$ per cent; what proportion of water was there in the mixture?

12. Two elevens, A and B, play a cricket match. A's first innings is the square of the difference of B's two innings, and A's second one-third the sum of B's two innings; A scored 60 more their first innings than in their second, and lost the match by one run. What were the respective scores, B having first innings?

XXXII.

UNIVERSITY OF CAMBRIDGE, ENG.

Second Previous Examination, Dec., 1880.—*Time allowed*, 2½ *hours.*

1. Define *coefficient, term.*
   Find the coefficient of x in the expression
   $$\frac{x+a}{2}-\{2a-b(c-x)\}.$$

2. Find the continued product of
   x^2+3x+2, x^2-5x+6, x^2+2x-3,
   and multiply together
   $x^2+(\sqrt{2}-1)x+1$, $x^2-(\sqrt{2}+1)x+1$.

3. Divide $x^4-(b-2)x^3-(2b-1)x^2-(b^2+2b-8)x+3b+3$ by $x^2+3x+b+1$.

4. Simplify (i.) $\dfrac{1}{x-2}+\dfrac{1}{x^2-3x+2}-\dfrac{2}{x^2-4x+3}$;

   (ii.) $$\frac{\dfrac{1+x}{1+x^2}-\dfrac{1+x^2}{1+x^3}}{\dfrac{1+x^2}{1+x^3}-\dfrac{1+x^3}{1+x^4}}.$$

5. If a measures both b and c, prove that it will measure the sum of any multiples of b and c.
   Find the G.C.M. of $1+x+x^3-x^5$ and $1-x^4-x^6+x^7$.

6. Solve the equations

   (i.) $\dfrac{3x-5}{4}-\dfrac{x+1}{7}=2$;

   (ii.) $\dfrac{1}{2(x+3)}=\dfrac{1}{3(x+2)}+\dfrac{1}{6(x+1)}$;

   (iii.) $\frac{1}{3}(x+y)=\frac{1}{5}(x-y)$, $3x+11y=4$;

   (iv.) $3x^3+1=\dfrac{28x}{5}$;

   (v.) $\dfrac{x+a}{a+b}+\dfrac{b}{a}=\dfrac{x^2+ab}{ax}$;

   (vi.) $(x+1)(y+2)=10$, $xy=3$.

7. If $a+b=1$, prove that $(a^2-b^2)^2=a^3+b^3-ab$.

8. If $\frac{a}{b}=\frac{c}{d}$, prove that each of these fractions is equal to $\frac{a+mc}{b+md}$.

   If $a+b$, $b+c$, $c+a$ are in continued proportion, prove that $b+c$, $c+a$, $c-a$, $a-b$ are proportionals.

9. When is one quantity said to vary as another?

   If $\frac{1}{x}+\frac{1}{y}$ varies inversely as $x+y$, prove that x^2+y^2 varies as xy.

XXXIII.

UNIVERSITY OF CAMBRIDGE, ENG.

General Examination for the Ordinary B.A. Degree, June, 1881.
Time allowed, 3 *hours*.

1. Prove that $(x+4)^3-(x+1)^3=9(x+1)(x+4)+27$.

2. Simplify

   (i.) $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\frac{(b+c)(c+a)(a+b)}{abc}$;

   (ii.) $\frac{a^2b^2-a^2-b^2+1}{ab-a-b+1}$.

3. Find the G.C.M. of x^3-4x^2+2x+3 and $2x^4-x^2-5x-3$.

4. Prove that, if m and n be positive integers, $(a^m)^n=(a^n)^m$.

   Prove that $(\sqrt{3})^{2\sqrt{3}}=(3\sqrt{3})^{\sqrt{3}}$.

5. Solve the equations

   (i.) $\frac{1}{3}(5x+1)+\frac{2x-3}{7}=x+\frac{8}{15}$;

   (ii.) $\frac{1}{a+x}+\frac{1}{b+x}=\frac{a+b}{ab}$;

   (iii.) $ax+by=2$, $ab(x+y)=a+b$.

6. Show that the product of the roots of the equation $ax^2 + bx + c = 0$ is $\frac{c}{a}$.

Prove that the difference of the roots of the equation $x^2 + px + q = 0$ is equal to the difference of the roots of the equation $x^2 + 3px + 2p^2 + q = 0$.

7. Solve the equations

(i.) $(x-2)^2 + (x+5)^2 = (x+7)^2$;

(ii.) $ax^2 + 2bx = a - 2b$;

(iii.) $\sqrt{x+y} = \sqrt{y} + 2, \quad x - y = 7$.

8. Find the sum of n terms of a G.P. of which the first and second terms are a and b.

If each term of a G.P. be squared, prove that the new series will also form a G.P.

9. If $\frac{a}{b} = \frac{c}{d}$, prove that each of these fractions is equal to $\frac{pa + qc}{pb + qd}$.

10. What is meant by saying that A varies as B?

If the volume of a cone varies jointly as its height, and the square of the radius of its base, show that if the heights of three cones of equal volume are in continued proportion, so also are the radii of their bases.

11. Find a fraction such that the denominator exceeds the square of half the numerator by unity, and the product of the sum and difference of the numerator and denominator is 64.

12. A vessel is half full of a mixture of wine and water. If filled up with water, the quantity of water bears to that of wine a ratio nine times what it would be were the vessel filled up with wine. Determine the original quantities of wine and water.

XXXIV.

UNIVERSITY OF CAMBRIDGE, ENG.

Previous Examination, June, 1881. — Time allowed, 2½ hours.
Elementary Algebra.

1. Simplify $(x-a)^3-(x-b)^3-(a-b)(a+b-3x)$; and find the value of $3(a-b)(a+2b)+2(a-2b)(2a-b)+2(2a-b)^2$, when $a=0$ and $b=-2$.

2. Divide $7a^3-22a^2b+4ab^2-3b^3$ by $a-3b$.

3. Resolve into factors
$x^2-2x-255$; $21x^2-13xy-20y^2$; $(x+2y)^3-y^3$.

4. Simplify
(i.) $\dfrac{x^2-16x-17}{x^2-22x+85}$;
(ii.) $\left(x-\dfrac{xy-y^2}{x+y}\right)\left(x-\dfrac{xy^2-y^3}{x^2+y^2}\right)\div\left(1-\dfrac{xy-y^2}{x^2}\right)$.

5. Find the L.C.M. of
(x^3-y^3), $(2x^3-3xy+y^2)$, and $(x^3+x^2y+xy^2)$.

6. Solve the equations
(i.) $\dfrac{x+1}{2}+\dfrac{x+4}{3}=\dfrac{10-x}{5}+5$;
(ii.) $2x(7x-10)=13(x-1)$;
(iii.) $7+\dfrac{5-x}{x-3}+\dfrac{4}{(x-1)(x-3)}=0$;
(iv.) $5abx+2y=16b$, $3abx+4y=18b$;
(v.) $x^2+3xy=-8$, $y^2-xy=12$.

7. Extract the square root of
$9x^4-6x^3+43x^2-14x+49$.

8. Prove that $(a^m)^n=a^{mn}$.
Simplify $\dfrac{(a^{p-q})^{p+q}\times(a^q)^{q+r}}{(a^p)^{p-q}}$.

9. If α and β be the roots of the equation $x^2 - px + q = 0$, then will $p = \alpha + \beta$ and $q = \alpha\beta$.

   Form the equation whose roots are 27 and -13.

10. When are three quantities said to be in continued proportion?

    Show that if x, $(x+y)$, and $(x+2y+z)$ be in continued proportion, then x, y, z will also be in continued proportion.

11. Prove that if $x \propto y$ and $y \propto z$, then will $x \propto z$.

    Given that x varies inversely as $(y^2 - 1)$, and is equal to 24 when $y = 10$; find x when $y = 5$.

XXXV.

University of Cambridge, Eng.

Previous Examination, June, 1881. — *Time allowed,* 2½ *hours.*

Higher Algebra.

1. If the first two terms of an arithmetical progression are given, find the sum of the first n terms.

   The sum of n terms of an arithmetical progression, whose first two terms are 43, 45, is equal to the sum of $2n$ terms of another progression, whose first two terms are 45, 43; find the value of n.

2. Find the sum of n terms of a geometrical progression whose first term and common ratio are given.

   The sum of $2n$ terms of a geometrical progression, whose first term is a and common ratio r, is equal to the sum of n terms of a progression, whose first term is b and common ratio r^2; prove that b must be equal to the sum of the first two terms of the first series.

3. Sum the following series to 12 terms:

(i.) $1 - \frac{6}{5} - \frac{17}{5} - \ldots\ldots$

(ii.) $1 + \frac{5}{6} + \frac{25}{36} + \ldots\ldots$

(iii.) $1 - 1.2 + 1.44 - \ldots\ldots$

How many strokes are struck in a week by a clock that tells the hours?

4. If the sum of the first n terms of a series be $32n^2$, find the rth term.

5. For what values of m is $x^m + y^m$ divisible by $x + y$?

Divide $a^3 + 3a^2b + 3ab^2 + b^3 + c^3$ by $a + b + c$.

6. A clock gains 4 minutes per day; what time should it indicate at 6 o'clock in the morning in order that it may be right at 7.15 P.M. on the same day?

7. The first four nights of the boat-races both divisions rowed, and 32 bumps in all were made. The greatest number on one evening, in the first division, was reached twice, and was equal to the least number in the second division, which also occurred twice. This number is the middle one of five consecutive numbers, of which the first two represent the number of bumps the other two nights in the first division, and the last two represent the other bumps of the second division. How many bumps were made in the first division?

8. Define the logarithm of a number to a given base. Prove

$$\log_a \frac{m}{n} = \log_a m - \log_a n;$$

$$\log_a b \log_b a = 1.$$

9. Find the values of

$$\log_a a^m, \quad \log_{\sqrt{49}} \sqrt[5]{49}, \quad \log_3 0.027.$$

10. Having given $\log 2 = 0.3010300$;
$\log 3 = 0.4771213$;
$\log 4.239 = 0.627263$;
$\log 4.24 = 0.627366$;
find the value of $\frac{2^{128}3^{\frac{1}{2}}}{10^{38}}$.

XXXVI.

University of Cambridge, Eng.

Second General Examination for the Ordinary B.A. Degree, Nov., 1881. — *Time Allowed*, 3 *hours.*

1. Solve the equations

(i.) $\frac{x^3+3x^2+4x+2}{4x^2+13x+14} = \frac{x^2+x+1}{4x+5}$;

(ii.) $\frac{2(x+1)}{x^2-x+1} - \frac{2}{x+1} + \frac{1}{x+3} - \frac{1}{x-3} = 0$;

(iii.) $x - 11y = 1$, $111y - 9x = 99$.

2. Find two consecutive numbers, such that the fourth and eleventh parts of the less together exceed by 1 the fifth and ninth parts of the greater.

3. A certain number of two digits is multiplied by 4, and the product is less by 3 than the number formed by inverting its digits; if it be multiplied by 5, the tens' digit in the product is greater by 1, and the units' digit less by 2 than the units' digit in the original number: find the number.

4. Solve the equations

(i.) $3x^2 - 11x - 4 = 0$;

(ii.) $\sqrt{7x+1} = 3 + \sqrt{2x-1}$;

(iii.) $x^2 + xy - 2y^2 = -44$, $xy + 3y^2 = 80$.

5. If the greater sides of a rectangle be diminished by 3 yards, and the less by 1 yard, its area is halved. If the greater be increased by 9, and the less diminished by 2, the area is unaltered; find the sides.

6. If the number of pence which a dozen apples cost is greater by 2 than twice the number of apples which can be bought for 1 *s.*, how many can be bought for 9 *s.*

7. Define *ratio*. If a be less than b, show that $a:b$ is a less ratio than $a+1:b+1$. What is the least integer which must be added to the terms of the ratio 9 : 23, so as to make it greater than the ratio 7 : 11?

8. The first and fourth terms of a proportion are 5 and 54; the sum of the second and third terms is 51; find them.

9. If A varies directly as P, inversely as Q, and directly as R, and, if when $P=a$, $Q=b$, $R=c$, $A=abc$, find A when $P=\frac{bc}{a}$, $Q=\frac{ca}{b}$, $R=\frac{ab}{c}$.

10. Find the sum of n terms of a geometrical progression, of which the first term is a and the common ratio r.

Sum:

$64+64\frac{1}{2}+\cdots\cdots$ to 29 terms in arithmetical progression;

$64+96+\cdots\cdots$ to 7 terms in geometrical progression.

11. The common difference of an arithmetical progression is 2, and the square roots of the first, third, and sixth terms are in arithmetical progression; find the series.

12. The sum of four numbers in geometrical progression is 170, and the third exceeds the first by 30; find them.

XXXVII.

UNIVERSITY OF CAMBRIDGE, ENG.

Second Previous Examination, Dec., 1881. — *Time Allowed*, 2½ *hours.*

1. Simplify $6(a-2b)(b-2a)-(a-3b)(4b-a)-12ab$, and from the sum of $(2a-b)^2$ and $(a-2b)^2$ take the square of $2(a-b)$.

2. Define *multiplication*, *product*, and *coefficient*.
Divide $14a^4+15a^3b+33a^2b^2+36ab^3+28b^4$ by $7a^2-3ab+14b^2$.

3. Find the value of $(a-b)^2+(b-c)^2+(a-b)(b-c)+5c^2$ when $a=1$, $b=-2$, $c=\frac{1}{2}$.

4. Resolve into the simplest possible factors:
(i.) $6x^2+5xy-6y^2$;
(ii.) $x^3-13x^2y+42xy^2$;
(iii.) $(a+2b+3c)^2-4(a+b-c)^2$;
(iv.) $81x^4-625y^4$.

5. Define the highest common factor of two algebraical expressions.
Find the highest common factor of
$7x^3-10x^2-7x+10$ and $2x^3-x^2-2x+1$.

6. Reduce to simple fractions in their lowest terms:

(i.) $\dfrac{x^2-7xy+12y^2}{x^2+5xy+6y^2} \div \dfrac{x^2-5xy+4y^2}{x^2+xy-2y^2}$;

(ii.) $\dfrac{x-a}{x+a}+\dfrac{a^2+3ax}{a^2-x^2}+\dfrac{x+a}{x-a}$;

(iii.) $\dfrac{a-\dfrac{ab}{a+b}}{a^2+\dfrac{a^2b^2}{a^2-b^2}} \times \dfrac{\dfrac{1}{a^2}-\dfrac{1}{b^2}}{\dfrac{1}{a}-\dfrac{1}{b}}$

7. Solve the equations

(i.) $\frac{x+1}{5} - \frac{2x-3}{9} = \frac{3x-2}{7} - 10$;

(ii.) $\frac{x}{6} + \frac{x}{5} = 14, \quad \frac{x}{9} + \frac{y}{2} = 24$;

(iii.) $5x^2 - 17x + 14 = 0$;

(iv.) $x^2 + y^2 = 5a^2 + 5b^2 + 8ab, \quad xy = 2a^2 + 2b^2 + 5ab.$

8. Find the value of $x^m \times x^n$, when m and n are positive integers.

Simplify $a^{2p+q} \times a^{p+4q} \div a^{q-p}$.

9. Define the antecedent and consequent of a ratio.

If $7(x-y) = 3(x+y)$, what is the ratio of x to y?

10. Show that if $a : b :: c : d :: e : f$, then will $a + 3c + 2e : a - c :: b + 3d + 2f : b - f$.

11. Find two numbers such that their sum, their difference, and the sum of their squares are in the ratio 5 : 3 : 51.

12. Prove that if x varies as $\frac{1}{y} + \frac{1}{z}$, and is equal to 3 when $y = 1$ and $z = 2$, then $xyz = 2(y + z)$.

XXXVIII.

UNIVERSITY OF CAMBRIDGE, ENG.

General Examination for the Ordinary B.A. Degree, June, 1882. — *Time Allowed,* 3 *hours.*

1. Simplify $\dfrac{\left(\frac{a^2}{a^2+b^2} + \frac{b^2}{a^2-b^2}\right)(a^2+b^2)^2}{\frac{a}{a+b} + \frac{b}{a-b}}$.

2. Solve the equations

(i.) $\frac{3x+1}{4} - 2(6-x) = \frac{5x-4}{7} - \frac{x-2}{3}$;

(ii.) $27x^2 - 24x - 16 = 0$;

(iii.) $2y + \frac{3}{x} - 4 = 5y + \frac{12}{x} + 2 = y - \frac{2}{x} + 4$.

3. What is the meaning of the expressions x^{-n}, $x^{\frac{1}{p}}$?

Simplify $\frac{b^{xy} \times c^{yz+zy}}{b^{yz} \times c^{xy}} \times \left(\frac{b}{c}\right)^{y(z-x+1)}$.

4. Solve the equations

(i.) $x^2 - (a-b)x + (a-b+c)c = 2cx + ab$;

(ii.) $\sqrt{14x+9} + 2\sqrt{x+1} + \sqrt{3x+1} = 0$;

(iii.) $3x - 2\sqrt{xy} + 9 = 0$, $5\sqrt{x} - 3\sqrt{y} - 3 = 0$.

5. Show that the sum of the roots of the equation $ax^2 - bx + c = 0$ is $\frac{b}{a}$.

If α and β are the roots of the above equation, form the equation whose roots are $-\frac{1}{\alpha}$, $-\frac{1}{\beta}$.

6. When is one quantity said to vary as another?

If $ax + by + 1 = 0$, where a and b are constant, and x and y are variable, and if the values of x are 2 and -9 when the values of y are 1 and -4, respectively, what will be the value of x when y is zero?

7. Define a geometrical progression.

Find the geometrical mean of

$9x^2 - 12x + 4$ and $4x^2 - 12x + 9$.

8. The first term of an arithmetical progression is 38, and the fourth term is 86; find the sum of the first twelve terms.

The first term of a geometrical progression is 27, and the third term is 48; find the sum of the first six terms.

9. Find, to four places of decimals, the sum to infinity of the series $1+\frac{1}{\sqrt{3}}+\frac{1}{3}+\cdots\cdots$

10. The perimeter of a rectangular field is 306 yards, and the diagonal is 117 yards. What is the area?

11. The expenses of a tram-car company are fixed, and when it only sells threepenny tickets for the whole journey it loses 10 per cent. It then divides the route into two parts, selling twopenny tickets for each part, thereby gaining 4 per cent, and selling 3300 tickets every week. How many persons used the cars weekly under the old system?

12. The price of a passenger's ticket on a French railway is proportional to the distance he travels; he is allowed 25 kilograms of luggage free, but on every kilogram beyond this amount he is charged a sum proportional to the distance he goes. If a journey of 200 miles with 50 kilograms of luggage cost 25 francs, and a journey of 150 miles with 35 kilograms cost $16\frac{1}{2}$ francs, what will a journey of 100 miles with 100 kilograms of luggage cost?

XXXIX.

UNIVERSITY OF CAMBRIDGE, ENG.

Previous Examination, June, 1882.—*Time Allowed*, $2\frac{1}{2}$ *hours.*—*Elementary Algebra.*

1. Find the value of
$(3a-5b)(a-c)+c\{2a-c(3a-b)-b^2(a-c)\}$,
when $a=0$, $b=1$, $c=-\frac{1}{2}$.

2. Prove that
$(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x)$.
Multiply $9a^{\frac{1}{2}}x^{\frac{1}{2}}+6a^{\frac{1}{4}}x^{\frac{1}{4}}+4x^{\frac{1}{4}}$ by $9a-6a^{\frac{1}{4}}x^{\frac{1}{4}}+4a^{\frac{1}{4}}x$.

3. Resolve into their simplest possible factors:
$10x^2+x-2$; $x^2+4x-4y^2+4$; $a^3-3a^2b+3ab^2-b^3+c^3$.

4. What is meant by a common multiple of two quantities? Prove that the sum of two quantities is a multiple of any of their common measures.
Find the L.C.M. of
$x^3-3x^2y+3xy^2-2y^3$, $x^3-x^2y-xy^2-2y^3$, and $x^4+x^2y^2+y^4$.

5. Prove the rule for the multiplication of a fraction by an integer.
Reduce to fractions in their lowest terms:
(i.) $\dfrac{1}{2a-3b+\dfrac{1}{2a-3b}-\dfrac{1}{2a-3b}}$;
(ii.) $\left\{\dfrac{1}{x+y}-\dfrac{(x-y)^2}{x^3+y^3}\right\}+\left\{\dfrac{x}{y}+\dfrac{y}{x}-1\right\}$.

6. Prove that a quadratic equation cannot have more than two roots.
Solve the equations
(i.) $4\left(x-\dfrac{3}{8}\right)-3\frac{1}{5}\left(\dfrac{x}{2}-1\right)=5\frac{9}{10}$;
(ii.) $a(2x-y)+b(2x+y)=c(2x-y)+d(2x+y)=1$;
(iii.) $x^2-11x-42=0$;
(iv.) $\dfrac{1}{(x-1)(x-2)}+6=\dfrac{3}{x-2}+\dfrac{2}{x-1}$.

7. A certain number, consisting of two digits, becomes 110 when the number obtained by reversing the digits is added to it; also the first number exceeds unity by five times the excess of the second number over unity. What is the number?

8. Define a third proportional to two quantities. Having given a third proportional to a and b, and also to b and a, determine a and b in terms of them.

If $a:b::c:d$, and x be a third proportional to a and c, and y to b and d, prove that the third proportional to x and y is equal to that to a and d.

9. When is a quantity said to vary inversely as another? If a and b each vary inversely as c, prove that the sum of any given multiples of a and b varies inversely as any given multiple of c.

Given that $x-y$ varies inversely as $z+\frac{1}{z}$, and $x+y$ inversely as $z-\frac{1}{z}$; find the relation between x and z, provided that $x=1$ and $y=3$, when $z=\frac{1}{2}$.

XL.

University of Cambridge, Eng.

Previous Examination, June, 1882. — *Time Allowed*, 2½ *hours.*
Higher Algebra.

1. A has twice as many pennies as shillings; B, who has 8*d.* more than A, has twice as many shillings as pennies; together they have one more penny than they have shillings. How much has each?

2. A man can walk a certain distance in 4 hours; if he were to increase his rate by one-fifteenth, he could walk one mile more in that time. What is his rate?

3. Solve the equations

(i.) $\sqrt{5x+1}=2+\sqrt{x+1}$.

(ii.) $x^2-2y^2=7$, $2x+y=7$.

4. A man buys a number of articles for £1, and makes £1 1*s.* 0*d.* by selling all but two at 2*d.* apiece more than they cost. How many did he buy?

5. Find the sum of n terms of the progression
$a + ar + ar^2 + \cdots\cdots$
Find the sum of 10 terms of the progression
$64 + 96 + 144 + \cdots\cdots$

6. The fifth term of an arithmetical progression is 81, and the second term is 24; find the series.

7. Find an arithmetical progression whose first term is 3, such that its second, fourth, and eighth terms may be in geometrical progression.

8. If $a : x :: b : y :: c : z$, prove that
$$\frac{la^3 + mb^3y + ncz^3}{lax + mby^3 + nz^3} = \frac{pa + qb + rc}{px + qy + rz}.$$

9. Prove that $\log_a(pq) = \log_a p + \log_a q$.

10. Having given $\log_{10} 2 = 0.3010300$, find the logarithm to base 10 of 25, 0.03125, and $(0.025)^{\frac{1}{2}}$.

XLI.

University of Oxford, Eng.

Local Examination, Junior Candidates, May, 1880. — *Time Allowed*, $2\frac{1}{2}$ *hours.*

No credit will be given for any answer, the full working of which is not shown.

I. Algebra.

1. Find the value of $\dfrac{a^2 - c - 3ac(b - 2c)}{4c(a + b)} + \sqrt{2a - \dfrac{1+b}{c}}$, when $a = 1$, $b = 0$, and $c = -\frac{1}{2}$.

2. Multiply $x^4 - ax^3 + a^3x - a^4$ by $x^2 + ax + a^2$; also divide $p^4 - 9pq^3 + 18q^4$ by $p^2 - 3pq + 3q^2$.

3. Simplify

(i.) $\left(\frac{2x+y}{y}-\frac{y}{2x+y}\right)\left(\frac{x}{x+y}-\frac{x+y}{x}\right)$;

(ii.) $\frac{1}{x+1}-2\frac{x+2}{x^2-1}+\frac{x+3}{(x-y)^2}$.

4. Find the G.C.M. of x^3-6x-4 and $3x^3-8x+8$; also the L.C.M. of $(3a^2-3ab)^2$, $18(a^3b^2-ab^4)$, and $24(a^3b^3-b^6)$.

5. Solve the equations

(i.) $\frac{5x-3}{8}-\frac{1}{8}(x-2\frac{1}{2})=\frac{2x-1}{10}+1\frac{3}{4}$.

(ii.) $\frac{x+3}{y+4}=2\frac{x+1}{2y+7}$, $6(x+\frac{1}{2})=11(y+5)$.

II. Higher Algebra.

6. Solve the equations

(i.) $\frac{2x}{x+1}+\frac{10}{9}\frac{x+1}{x}=3$;

(ii.) $a(a-b)x^2+b(a+b)x-2b^2=0$;

(iii.) $x^2+2xy=\frac{3}{4}$, $x^2-4y^2=1\frac{1}{4}$.

7. The sum of 2 numbers is 35; and their difference exceeds one-fifth of the smaller number by 2; find the numbers.

8. After £12 have been divided equally among a certain number of men, an additional shilling apiece is given to them; and it is then found that each possesses as many shillings as there are men. Find the number of the men.

9. Prove that if b be a mean proportional between a and c, then $a^2+2b^2 : a :: b^2+2c^2 : c$.

10. Sum to 6 terms the series $\frac{1}{2}+1\frac{1}{4}+3\frac{1}{8}+\ldots\ldots$ Also insert 12 arithmetic means between $-\frac{1}{8}$ and 5.

XLII.

UNIVERSITY OF OXFORD, ENG.

Local Examination, Senior Candidates, May, 1880. — *Time Allowed*, 2½ *hours.*

No credit will be given for any answer, the full working of which is not shown.

Candidates are reminded that in order to pass in mathematics they must satisfy the Examiners in the first part of this paper.

I. ALGEBRA TO QUADRATIC EQUATIONS.

1. If $m=\frac{\sqrt{17}+1}{2}$ and $n=\frac{\sqrt{17}-1}{2}$, find the value of m^2+n^2.

2. Find the G.C.M. of $2x^3+3x^2-7x-10$, $5-9x-4x^2+4x^3$; and the L.C.M. of a^2+ab, b^2+ab, ab.

3. Simplify $\frac{1}{x-1}-\frac{2}{x}+\frac{1}{x+1}$.

4. Find the square root of $4x^4-20x^3+13x^2+30x+9$.

5. Multiply $x^{\frac{1}{2}}+2x^{\frac{1}{4}}+2$ by $x^{\frac{1}{2}}-2x^{\frac{1}{4}}+2$, and divide $a^2b^{-2}+b^2a^{-2}+1$ by $ab^{-1}+ba^{-1}-1$.

6. Solve the equations

   (i.) $\frac{1+x}{1-x}=\frac{21}{20}$;

   (ii.) $x=(2-x)(2+x)$;

   (iii.) $\frac{a}{x}+\frac{x}{a}=\frac{b}{x}+\frac{x}{b}$;

   (iv.) $x^2+y^2=169=5x+12y$.

7. Two rectangular fields each contain one acre; one of the fields is four poles shorter and two poles broader than the other. Find the length and breadth of each field.

II. HIGHER ALGEBRA.

8. A spends £a in buying a number of articles, all at the same price; B spends £b in the same way, except that he buys n more articles than A buys, and pays £c less for each. Find an equation to determine the number of articles bought by A.

9. Solve the equations

(i.) $4^x + 1 = 5 \times 2^{x-1}$;

(ii.) $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$;

(iii.) $3x + 4y = 23$ (in positive integers).

10. Prove the formulæ for finding the nth term and the sum of n terms of an arithmetic progression, the first term and the common difference being known.

A man pays his gardener 15s. a week for the first fortnight; at the end of the first and of every succeeding fortnight he raises the wages 6d. per week. What will the gardener have received in all at the end of fifty weeks?

11. Find the cost of an annuity of £A per annum, to be paid quarterly, and to continue for p years; the first payment to be made at the end of the first quarter, reckoning compound interest, at the rate of £r per cent per annum, to be due at the date of each quarterly payment.

12. Enunciate the binomial theorem.

Show that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is the sum of the coefficients of the two middle terms in the expansion of $(1+x)^{2n-1}$.

13. Prove that

(i.) $\left(1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \ldots\ldots\right)^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \ldots\ldots$;

(ii.) $\log_e \sqrt{\frac{1+x}{1-x}} = \frac{x}{1} + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots$

XLIII.

UNIVERSITY OF OXFORD, ENG.

Local Examination, Junior Candidates, June, 1881. — *Time Allowed*, 2½ *hours.*

No credit will be given for any answer, the full working of which is not shown.

I. ALGEBRA.

1. Find the value of $a^3+b^3-c^3+3abc$:
   (i.) when $a=\frac{1}{2}$, $b=\frac{1}{3}$, $c=\frac{1}{5}$;
   (ii.) when $c=a+b$.

2. Multiply together x^3-7x+6, $x^3+7x-18$, x^3-1, and express the result in simple factors.

3. Find the G.C.M. of $2x^3+7x^2+10x+5$ and x^3+3x^2+4x+2, and the L.C.M. of $6xy^2(x+y)$, $3x^3(x-y)^2$, and $4(x^3-y^3)$.

4. Simplify
   (i.) $\left(\frac{x+y}{y(x^2+y^2)}-\frac{x+y}{x(x^2+y^2)}+\frac{1}{xy}\right)\left(\frac{1}{y^2}+\frac{1}{x^2}\right)$;
   (ii.) $\left(1-\frac{y^4}{x^4}\right)\div\left(\frac{x}{y}+\frac{y}{x}\right)$.

5. Solve the equations
   (i.) $\dfrac{\frac{1}{3}-x}{2}+\dfrac{1-\frac{x}{2}}{\frac{3}{2}}=\dfrac{x}{4}+\dfrac{x-1}{3}$;
   (ii.) $9x-8y=1$, $12x-10y=1$.

6. Into a cistern one-third full of water 31 gallons are poured, and the cistern is then found to be half full; find its capacity.

II. Higher Algebra.

7. Solve the equations

(i.) $x = \frac{55}{64} + \frac{x^3}{49}$;

(ii.) $ax^2 - \frac{6c^2}{a+b} = cx - bx^2$;

(iii.) $\frac{x^2+y^2}{17} + \frac{x+y}{10} = 0,\ xy = 1.$

8. A person bought a certain number of sheep for £210. He lost 10, and to make up the deficiency sold the remainder at 10*s.* profit per head. How many did he buy?

9. Prove the rule for the summation to infinity of a geometrical progression; and sum to n terms and to infinity $8\frac{1}{3} + 5 + 3 + \ldots\ldots$

10. The seventh term of an arithmetical progression is 1; and the sum of twenty-five terms is zero. Find the progression.

11. If $a : b :: c : d$, prove that $a + b : a - b :: c + d : c - d$.

12. If $2x + 3y : 2x - 3y :: 2a^2 + 3b^2 : 2a^2 - 3b^2$, then x has to y the duplicate ratio that a has to b.

XLIV.

University of Oxford, Eng.

Local Examination, Senior Candidates, June, 1881. — *Time Allowed, 2½ hours.*

No credit will be given for any answer, the full working of which is not shown.

I. Algebra to Quadratic Equations.

1. Prove that

$(a+b)(a+x)(b+x) - a(b+x)^2 - b(a+x)^2 = (a-b)^2x$,

and divide $a^3 - b^3$ by $a^{\frac{3}{2}} - 2ab^{\frac{1}{2}} + 2a^{\frac{1}{2}}b - b^{\frac{3}{2}}$.

2. Resolve into component factors

(i.) $63x^3y - 28xy^3$; (ii.) $a^5 - a^4b - ab^4 + b^5$.

Find the remainder when $a^n + b^n$ is divided by $a - b$.

3. Find the G.C.M. of

$x^4 - 6x^3 + 13x^2 - 12x + 4$ and $x^4 - 4x^3 + 8x^2 - 16x + 16$,

and the L.C.M. of $x^3 - y^3$, $x^3 + y^3$, $x^3 - xy^2$, and $x^2y + xy^2 + y^3$.

4. Simplify the fractions

(i.) $\dfrac{x+2}{x+1} - \dfrac{x+1}{x+2} - \dfrac{2}{x+4}$;

(ii.) $\dfrac{1}{1-\frac{1}{x}} - \dfrac{1}{\frac{1}{x^2}-\frac{1}{x}} - \dfrac{1}{1-\frac{2x}{x^2+1}}$;

(iii.) $\dfrac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}-\sqrt{x-1}} - \dfrac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}$.

5. Solve the equations

(i.) $\frac{1}{8}\{(2x-32)-(x+16)\} = \frac{1}{11}\{(x-20)-(2x-11)\}$;

(ii.) $(x+5)(y+7) = (x+1)(y-9)+112$, $2x+5 = 3y-4$;

(iii.) $x - \dfrac{x^3-8}{x^2+5} = 2$.

6. A person invests £500, part of it at 5 per cent and the remainder at 3 per cent; and he thus gets $4\frac{1}{2}$ per cent on the whole. How much does he invest at each rate of interest?

7. Find the square root of $9 - 24x - 68x^2 + 112x^3 + 196x^4$.

II. Higher Algebra.

8. Prove that a ratio of greater inequality is diminished, and a ratio of less inequality is increased, by adding the same number to each of its terms.

If $a : a-b :: c : c-d$, then $a+b : b :: c+d : d$.

9. Prove that the geometrical mean between two numbers is also the geometrical mean between the arithmetical and harmonical means.

Sum the series:

(i.) $25\frac{1}{2}+24+22\frac{1}{2}+21+\ldots$ to 15 and to 21 terms;

(ii.) $4\frac{4}{15}+2\frac{2}{3}+1\frac{1}{3}+\ldots$ to n terms, and to infinity.

10. Prove that the number of combinations of n things taken r together is equal to that of n things taken $n-r$ together, and greater than that of $n-1$ things taken $r-1$ together.

How many different numbers can be made with all or any of the figures of the number 1881?

11. Employ the binomial theorem to expand $(a-x)^{11}$; also $(a+x)^{\frac{1}{2}}$ in ascending powers of x to 5 terms.

12. If α, β are the roots of the equation $ax^2+bx+c=0$, find the values of $\alpha+\beta$ and $\alpha^3+\beta^3$ in terms of a, b, c.

13. Solve the equations

(i.) $x^3-\frac{1}{x^3}+7(x^3+1)=0$;

(ii.) $3x^2+y^2=3xy+7=19$.

XLV.

University of Oxford, Eng.

Local Examination, Junior Candidates, June, 1882. — Time Allowed, 2½ hours.

No credit will be given for any answer, the full working of which is not shown.

I. Algebra.

1. Find the value of $\frac{x^2-y^2}{x^2+y^2}$,

(i.) when $x=\frac{a+b}{2}$, $y=\frac{a-b}{2}$;

(ii.) when $x=\frac{1}{3}$, $y=-\frac{1}{4}$.

2. Square $2b-3c$, and find the product of $a+2b-3c$ and $a-2b+3c$.

3. Find the G.C.M. of $2a^5b+2a^2b^4$ and $4a^5+4a^3b^2+4ab^4$; and the L.C.M. of $5x(x^2+2x+1)$, $10x^2(x^2-1)$, and $15(x+1)(x^2-2x+1)$.

4. Simplify

(i.) $\dfrac{4a^4}{x^4-a^4}+\dfrac{2a^2}{x^2+a^2}+\dfrac{a}{a+x}+\dfrac{a}{a-x}$;

(ii.) $\left[x-\dfrac{1}{1+\dfrac{2}{x-1}}\right]\div\dfrac{x+\dfrac{1}{x}}{\dfrac{1}{x}+1}$.

5. Solve the equations

(i.) $\dfrac{1}{2}-\dfrac{5-x}{x-2}=\dfrac{5-x}{x-2}-\dfrac{1}{2}$;

(ii.) $2x+\dfrac{y}{2}=4,\quad 2y+\dfrac{x}{2}=1$.

6. A and B set out at the same time from the same spot to walk to a place 6 miles distant and back again. After walking for 2 hours, A meets B coming back. Supposing B to walk twice as fast as A, and each to maintain uniform speed throughout, find their respective rates of walking.

II. Higher Algebra.

7. Solve the equations

(i.) $x^2+\left(\dfrac{1}{a}-\dfrac{1}{b}\right)x=\dfrac{1}{ab}$;

(ii.) $\dfrac{1}{2}\left[\dfrac{1}{(x+2)(2x-1)}-\dfrac{1}{(x-2)(2x+1)}\right]=\dfrac{1}{4x^2-1}$;

(iii.) $\dfrac{x+y}{x-y}=\dfrac{3}{2},\quad x^2-9y^2=16$.

8. Show that the sum of any two consecutive whole numbers is equal to the difference of their squares.

9. Find the sum to n terms of an arithmetical progression, the first term and the common difference being given.

   What is the amount of a debt which can be discharged in two years by the payment of 10*s.* the first month, £1 the second, 30*s.* the third, and so on, no interest being exacted?

10. Sum to n terms, and to infinity, the series $\frac{2}{3} - \frac{1}{2} + \frac{3}{8} - \cdots$

11. If $a_1 : b_1 :: a_2 : b_2$, prove that
$(a_1 + a_2)^2 : (b_1 + b_2)^2 :: a_1^2 + a_2^2 : b_1^2 + b_2^2$.

12. Show that if b is a mean proportional between a and c, then $(a^2 + b^2)(b^2 + c^2) = (ab + bc)^2$.

XLVI.

University of Oxford, Eng.

Local Examination, Senior Candidates, June, 1882. — *Time Allowed, 2½ hours.*

No credit will be given for any answer, the full working of which is not shown.

Candidates are reminded that in order to pass in mathematics, they must satisfy the examiners in the first part of this paper.

I. Algebra to Quadratic Equations.

1. Prove that $(a + 2b)^3 = a^3 + 2b^3 + 6b(a + b)^2$, and find the quotient $a^3 + ab^2 + b \div a + a^{\frac{1}{2}}b^{\frac{1}{2}} + b^{\frac{1}{2}}$.

2. Simplify

(i.) $\dfrac{x-y}{x+y}[(x+2y)^3 + (2x+y)^3]$;

(ii.) $\dfrac{1}{1+\dfrac{x}{y+z}} + \dfrac{1}{1+\dfrac{y}{z+x}} + \dfrac{1}{1+\dfrac{z}{x+y}}$;

(iii.) $\dfrac{(b-c)^2}{(c-a)(a-b)} + \dfrac{(c-a)^2}{(a-b)(b-c)} + \dfrac{(a-b)^2}{(b-c)(c-a)}$.

3. Find the G.C.M. of $3x^3 + 17x^2 + 22x + 8$ and $6x^3 + 25x^2 + 23x + 6$, and the L.C.M. of $(x^4 - y^4)^2$, $x^3 + y^3$, $x^3 - y^3$, and $(x^2 - y^2)^2$.

4. Extract the square root of
   (i.) $7 - 4\sqrt{3}$;
   (ii.) $x^3(x^3 + 2) + 2x^2(x^3 + 1) - x(x - 2) + 1$.

5. Solve the equations
   (i.) $11x - 5(x - 6) - 6(3x - 11) + 9(x - 7) = 0$;
   (ii.) $\frac{x}{8} + \frac{y}{6} = \frac{x}{12} + \frac{y}{4} = 20$;
   (iii.) $(3x - 6)^2 + 7x^2 - 256 = 0$.

6. My income of £240 is derived from two sums invested at three and nine per cent respectively; but, if the rates of interest were interchanged, my income would be doubled; find the sums invested.

II. Higher Algebra.

7. If $a : b :: c : d$, prove that
   (i.) $a^3 + d^3 > b^3 + c^3$;
   (ii.) $3a + 2b : 3a - 2b :: 3c + 2d : 3c - 2d$.

8. Find an expression for the sum of n terms of a geometrical series, and explain the expression "sum to infinity."
   Find the fourth term in
   (i.) $2 + 2\frac{1}{2} + 3 + \cdots$
   (ii.) $2 + 2\frac{1}{2} + 3\frac{1}{8} + \cdots$
   (iii.) $2 + 2\frac{1}{2} + 3\frac{1}{8} + \cdots$

9. Find the number of permutations ${}^nP_r$ of n things taken r together. If ${}^nP_4 : {}^{n-1}P_5 :: 3 : 22$, find n.

10. Show that the number of terms in the expansion of $(a + x)^n$ is $n + 1$, if n is a positive integer.
    Apply the binomial theorem to find $(10.001)^7$ to five places of decimals.

11. If α, β are the roots of $ax^2+bx+c=0$, find the equation whose roots are
(i.) α^2, β^2; (ii.) $\alpha(1+\beta)$, $\beta(1+\alpha)$.

12. Solve the equations
(i.) $144x^2-1+6\sqrt{9x^2-x}=16x$;
(ii.) $x^2(x^2-y^2)=25$, $y^2(x^2+y^2)=19\frac{1}{2}$;
(iii.) $\sqrt[3]{14-x}+\sqrt[3]{14+x}=4$.

XLVII.

UNIVERSITY OF OXFORD, ENG.

First Examination of Women, May, 1880. — *Time Allowed*, $2\frac{1}{2}$ *hours.*

1. Find the value of $\frac{b^2c-2a^2b}{a^3+4b^3}$ when $a=1$, $b=-\frac{1}{2}$, $c=0$.

2. Take $a+2b+3c-4d-5e$ from $3a-4b+c-d+e$.

3. Multiply $a^3-a^2b+ab^2-b^3$ by $a+b$,
and divide $a^4+a^2b^2+b^4$ by a^2+ab+b^2.

4. Find the H.C.F. of a^4+5a^2-6 and a^4+5a^3+4, and the L.C.M. of $12(a^3-b^3)$, $15(a^3+b^3)$, $20ab(a^2-b^2)$.

5. Simplify $\frac{a^2+4a+3}{a^2-a-2}$.

6. Extract the square root of $a^4-2a^3+2a^2-a+\frac{1}{4}$.

7. Solve
(i.) $7(x-1)-6(x-2)=3(x-3)$;
(ii.) $\frac{x}{9}+\frac{x}{6}+\frac{x}{3}=x-7$;
(iii.) $(x-3)(x-13)=(x-4)(x-9)$;
(iv.) $8x+3y=74$, $9x-2y=51$.

8. I have in my purse £1 13*s.* 9*d.* made up of a certain number of pence, twice the same number of farthings, and thrice the same number of fourpenny pieces. Find the number of each coin.

9. A is thrice as old as B. Seven years ago A was four times as old as B. Find their ages now.

10. A and B play at cards. A wins six shillings, and finds he has thrice as much as B. The game is continued till A finds he has lost twenty-four shillings, and then has a third of what B has. With what sum did each begin?

XLVIII.

UNIVERSITY OF OXFORD, ENG.

Second Examination of Women, May, 1880. — *Time Allowed*, 2 *hours*.

1. Divide $a^{\frac{1}{2}} - a^{-\frac{1}{2}}$ by $a^{\frac{1}{4}} - a^{-\frac{1}{4}}$.

2. Simplify

$$\frac{(6a^2-a-2)(8a^2-10a+3)(12a^2+17a+6)}{(8a^2-2a-3)(12a^2+a-6)(6a^2+a-2)};$$

$$\frac{6a^2-17a+12}{12a^2-25a+12}+\frac{27a^2+18a-24}{12a^2+7a-12}+\frac{25a^2-25a+6}{20a^2-23a+6}.$$

3. Extract the square root of

(i.) $4a^2+\frac{9}{a^2}-11-6a^{-1}+4a$;

(ii.) $41-12\sqrt{5}$.

4. Solve the equations

(i.) $2\frac{x+a}{x+b}+3\frac{x+b}{x+a}=5$;

(ii.) $x^2-7x+10=0$;

(iii.) $\sqrt{x-2}+\sqrt{x+3}=\sqrt{4x+1}$;

(iv.) $x+y=8,\ xy=15$.

5. If $a:b::c:d$, prove that $a+b:a-b::c+d:c-d$.

6. £21 18*s.* is divided equally among a certain number of persons. If each received a penny less, each would have had as many pence as there were persons. Find the number of persons.

7. A steamer takes 2 hours 24 minutes less time to travel from A to B than from B to A. The steamer travels at the rate of 16 miles an hour, and the stream flows at the rate of 6 miles an hour. Find the distance of A from B.

XLIX.

UNIVERSITY OF OXFORD, ENG.

First Examination of Women, June, 1882. — *Time Allowed*, 2½ *hours.*

1. Find the value of $x(y+z)+y[x-(y+z)]-z[y-x(z-x)]$ when $x=3,\ y=2,\ z=1$.

2. Subtract $2x^4+\frac{5}{3}x^3-x+\frac{1}{4}$ from $\frac{5}{2}x^4+x^2-\frac{1}{4}x-\frac{3}{4}$.

3. Multiply $1+2x+3y+4x^2-6xy+9y^2$ by $1-2x-3y$.

4. Divide $x^4-1\frac{3}{6}x^3+x^2+\frac{4}{3}x-2$ by $x-\frac{2}{3}$.

5. Resolve into factors
   (i.) $4x^4-36x^2y^2$;
   (ii.) $(2x-3y)^2-(x-2y)^2$.

6. Find the G.C.M. of x^3-3x+2 and x^3+4x^2-5.

7. Find the L.C.M. of
   $12(1-x^3)$, $15(1-x)^2$, and $20(x+x^2)$.

8. Simplify
   (i.) $\dfrac{8a^2b}{c}\times\dfrac{c^2d}{8a^3}\times\dfrac{4ab}{cd}\times\dfrac{bcd-cd^2}{4b^2-4bd}$;
   (ii.) $\left(\dfrac{1}{1+4x}+\dfrac{4x}{1-4x}\right)\div\left(\dfrac{1}{1-4x}-\dfrac{4x}{1+4x}\right)$.

9. Solve the equations

(i.) $\frac{x}{3}+\frac{2x}{9}-\frac{x}{27}-\frac{x+1}{11}=23$;

(ii.) $\frac{x+a}{x+b}+\frac{x+b}{x+c}=2$;

(iii.) $\frac{2x-3}{y}=1\frac{3}{4}$, $\frac{2y+3}{x}=2\frac{1}{5}$.

10. A person walks a certain distance at the rate of $3\frac{1}{2}$ miles an hour, and finds that if he had walked 4 miles an hour, he would have gone the same distance in less time by one hour; what is the distance?

11. Find two numbers such that, if half the first be added to the second, or $\frac{1}{3}$ of the second be added to the first, the sum will in either case be 30.

12. Find the square root of
$a^6-4a^5+8a^4-10a^3+8a^2-4a+1$.

L.

University of Oxford, Eng.

Second Examination of Women, June, 1882. — *Time Allowed*, 2 *hours*.

1. Find the value of $(a+b-c)^2+(b+c-a)^2+(c+a-b)^2$ when $a=2$, $b=3$, $c=-\frac{1}{2}$.

2. If $a+b+c=p$, $bc+ca+ab=q^2$, and $abc=r^3$, prove that $a^3+b^3+c^3=p^3-3pq^2+3r^3$.

3. Find the G.C.M. of
$2x^3+x^2-12x+9$ and $2x^3-7x^2+12x-9$, and simplify $\frac{1}{2x^3-7x^2+12x-9}-\frac{1}{2x^3+x^2-12x+9}$.

4. Add together $\frac{x-a}{(x+a)^2}$, $\frac{x+a}{(x-a)^2}$, and $\frac{2x(3a-x)}{(x-a)(x+a)^2}$.

5. Find the square root of
$x^6 + a^2x^4 + 2a^3x^3 + \frac{1}{4}a^4x^2 - a^5x + a^6$.

6. Solve the equations

(i.) $\frac{1}{2x-1} + \frac{2}{4x-3} = \frac{1}{x-1}$;

(ii.) $\sqrt{5x^2+x-16} = 3x-2$;

(iii.) $(x-y)(x-3y) = 24,\ x-2y = 5$.

7. There are 250 flowers in a conservatory; the number of geraniums is five times the number of roses, and is less by 30 than the number of other flowers. How many roses, geraniums, and other flowers respectively are there in the conservatory?

8. Find the value of x in each of the following proportions:
(i.) $1 : x :: 2 : 3$; (ii.) $1 : 2 :: 3 : x$; (iii.) $4 : x :: x : 9$.

9. Two persons, A and B, start at the same time by their watches, from two places 24 miles apart, and drive towards each other at rates which are as 2 : 3; but in consequence of A's watch being $\frac{1}{4}$ hour too fast, and B's $\frac{1}{4}$ hour too slow, they meet half way. At what rate does each drive?

LI.

University of Oxford, Eng.

First Examination of Women, June, 1883. — *Time Allowed*, $2\frac{1}{2}$ *hours.*

1. Evaluate $(x-y)^2 + (y-z)^2 + (z-x)^2$, when $x = 3\frac{1}{2}$, $y = 2\frac{1}{2}$, $z = 1\frac{1}{2}$.

2. From the sum of $\frac{1}{6}(2x-3y+4z)$ and $\frac{1}{12}(4x+3y-11z)$ subtract $\frac{1}{24}(8x-9y+6z)$.

3. Multiply $x^3+y^3+a(x-y)$ by $xy-a(x+y)+a^2$.

4. Divide $x^3+8y^3-125z^3+30xyz$ by $x+2y-5z$.

5. Express in factors
   (i.) $7x^2-77x-182$;
   (ii.) $20x^4-60x^3y+45x^2y^2$.

6. Find the G.C.M. of $x^3+11x+30$, $9x^3+53x^2-9x-18$.

7. Find the L.C.M. of
   $15x^2(a^2-2ax+x^2)$, $21a^2(a^2+2ax+x^2)$, $35ax(a^2-x^2)$.

8. Simplify $\dfrac{\dfrac{a}{a-b}+\dfrac{b}{a+b}-1}{\dfrac{a}{a-b}-\dfrac{b}{a-b}+1}$, and find the value of
   $\dfrac{x+2}{2-x}+\dfrac{x-2}{2+x}-\dfrac{4}{4-x^2}$ when $x=\frac{1}{2}$.

9. Solve the equations
   (i.) $\frac{2}{3}(x-1)+3\left(\dfrac{x}{2}-9\right)-(x-13)=11$;
   (ii.) $\dfrac{1}{x-a}+\dfrac{1}{x-b}-\dfrac{2}{x-a-b}=0$;
   (iii.) $\dfrac{5x+7y}{7}=\dfrac{7x+5y}{8}=8$.

10. The sum of three consecutive whole numbers exceeds the greatest of them by 19; what are the numbers?

11. Find the fraction which is such that if 3 be subtracted from the numerator, and 5 added to the denominator, the value is $\frac{1}{4}$; but if 5 be subtracted from the numerator, and 3 added to the denominator, the value is $\frac{1}{6}$.

12. Extract the square root of
    $36x^4-120ax-12a^3x+100a^2+20a^3+a^4$.

LII.

UNIVERSITY OF OXFORD, ENG.

Second Examination of Women, June, 1883. — Time Allowed, 2 hours.

1. Divide $x^5 - y^5 - xy(x^3 + 2y^3)$ by $(x + y)(x - y) - xy$, and verify the result when $x = 2y = \frac{1}{2}$.

2. Prove that $(cy - bz)^2 + (az - cx)^2 + (bx - ay)^2 = x^2 + y^2 + z^2 - (ax + by + cz)^2$ if $a^2 + b^2 + c^2 = 1$.

3. Find the G.C.M. and the L.C.M. of $7x^3 - 2x^2 - 5$ and $7x^3 + 12x^2 + 10x + 5$.

4. Simplify

(i.) $\dfrac{y(x^3 - y^3)}{x(x + y)} \times \dfrac{(x^2 - y^2)^2}{(x^2x^2 + xy + y^2)} \div \dfrac{(x - y)^3}{(x + y)^2}$;

(ii.) $\dfrac{a}{b^2\left(\frac{1}{a} + \frac{1}{b}\right)} + \dfrac{b}{a^2\left(\frac{1}{a} - \frac{1}{b}\right)} + \dfrac{2ab}{a^2 - b^2}$.

5. Find the square root of

$$(x^2 + y^2)(x^2 + z^2) + 2x(x^2 + yz)(y + z) + 4x^2yz.$$

6. Solve the equations

(i.) $\dfrac{x - 3}{2\frac{1}{2}} - \dfrac{2x - 1}{3\frac{3}{4}} + \dfrac{7x - 11}{5\frac{5}{8}} = 6$;

(ii.) $\dfrac{5(3x^2 - 1)}{1 + 5x} + \dfrac{2}{x} = 3x$;

(iii.) $x^2 + y^2 = 9, \quad x - 3y + 3 = 0$.

7. A number of men are employed at 13*s.* 6*d.* per week each, and women at 10*s.* 6*d.*; the number of women exceeds that of the men by 6, but the men's wages amount to 27*s.* per week more than the women's; how many of each sex are employed?

8. Find the mean proportional between $1\frac{5}{9}$ and $\frac{5}{8}$; and the fourth proportional to 9, 11, 27.

   If the first two terms of a proportion are 3 and 2, and the third term exceeds the fourth by 5, what are the third and fourth terms?

9. Divide £500 among three persons, so that the share of the first may be to that of the second as 8 : 17, and the share of the third to the other two together as 3 : 2.

LIII.

University of London, Eng.

Matriculation Examination, Arithmetic and Algebra, June, 1877.

1. Divide $\frac{25}{21} + 6\frac{3}{8} - 7\frac{1}{28}$ by $5\frac{2}{3} - 3\frac{1}{7} + \frac{17}{12}$, and multiply the result by $6\frac{5}{18} + 9\frac{7}{25} - 13\frac{1}{5}$.

2. Find what fraction of a guinea is equal to the difference between $\frac{2}{7}$ of a crown and $\frac{3}{11}$ of a shilling.

3. Calculate to five places of decimals the fraction

$$\frac{3.70271 \times 0.64732}{0.043679}.$$

4. Reduce the circulating decimal $1.5\dot{2}37\dot{2}$ to a vulgar fraction in its lowest terms.

5. Extract the square roots, to five places of decimals, of the numbers, 3.9726523, 0.39726523, and 0.039726523.

6. Simplify the algebraic expression,

$$\frac{b}{3ax - 5by} - \frac{1}{\dfrac{ax}{b} - y - \dfrac{2by^2}{3ax - 2by}}.$$

7. Divide $15x^5 - 17x^4 - 24x^3 + 138x^2 - 130x + 63$ by $5x^3 + 6x^2 - 9x + 7$, and verify the result.

8. Prove that if $\dfrac{x^2 - xy + y^2}{u^2 - uv + v^2} = \dfrac{x^3 + y^3}{u^3 + v^3} \times \dfrac{v}{y}$, then $\dfrac{x}{y} = \dfrac{u}{v}$.

9. Given the first term, the middle term, and the number of terms in an arithmetical progression; find the sum of the series. Has this problem a meaning, if the number of terms is even?

10. Find to n terms and to infinity the sum of the geometrical progression in which the fourth term is 1 and the ninth term is $\frac{1}{243}$.

11. Solve the equations

(i.) $\frac{1-3x}{2}+\frac{3x+1}{2}=\frac{2}{1-3x}$;

(ii.) $2x-4y=7,\ 3x+7y=19.$

12. A tourist, having remained behind his companions, wishes to rejoin them on the following day. He knows that they are 5 miles ahead, and that they will start in the morning at eight o'clock, and will walk at the rate of $3\frac{1}{4}$ miles an hour. When must he start, in order to overtake them at one o'clock, P.M., walking at the rate of 4 miles an hour, and resting once for half an hour on the road?

LIV.

SCIENCE SCHOOLS AND CLASSES, ENGLAND.

Mathematics, First Stage, May, 1880.

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Find the value, when $x=5$ and $y=3$, of

$$\frac{x^4-4x^3y+6x^2y^2-5xy^3+2y^4}{2x^4-5x^3y+6x^2y^2-4xy^3+y^4}.$$ [6.]

2. Multiply a^3-x^3 by a^2-x^2, and divide the product by $(a-x)^2$. [8.]

3. Simplify $(a-b)(b+c)(c+a)+(b-c)(c+a)(a+b)+(c-a)(a+b)(b+c)$, and find its value when $a=1$, $b=3$, and $c=-2$. [10.]

4. Solve the equations [12.]

(i.) $7\left(x+\frac{1}{3}\right)-5x\left(\frac{1}{3x}+\frac{1}{2\frac{1}{2}}\right)=4.$ [6.]

(ii.) $0.5x+0.07y=0.93,\ 0.03x-0.4y=0.46.$ [6.]

5. The rent of a shop is $\frac{2}{7}$ of the rent of the whole house of which it is a part. Being separately rated, its occupier pays £10 15*s.* 0*d.* a year less in rates than the occupier of the rest of the house. The rates are 3*s.* 7*d.* in the pound. What is the rent of the whole house? [14.]

6. Find, as a fraction in its lowest terms, the value of

$$\frac{1}{x^3-3x^2-15x+25}-\frac{1}{x^3+7x^2+5x-25}.$$ [12.]

LV.

Science Schools and Classes, England.

Mathematics, Second Stage, May, 1880. — *Arithmetic and Algebra.*

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Assuming that a franc is worth 9.504*d.*, and a hard dollar 50.49*d.*, what is the smallest sum in francs that can be exactly paid in hard dollars? [16.]

2. Show how to find the square root of a vulgar fraction, so as to make sure of obtaining it in a finite form, if it has one.

Ascertain whether the square roots of the following fractions are finite or not: $\frac{343}{1183}$, $\frac{14641}{243}$, $\frac{329}{81}$, $\frac{99}{1331}$. [18.]

3. Solve two of the following sets of equations: [20.]

(i.) $\sqrt{x^2+a^2}-\frac{b^2}{\sqrt{x^2+a^2}}=\frac{b^2-a^2}{a}.$ [10.]

(ii.) $x+y=7$, $x^3+y^3=133$. [10.]

(iii.) $xy+\sqrt{x+y}=11$, $2xy-\sqrt{x+y}=13$. [10.]

4. What is the least integral multiplier which will make $17x^5-68x^4y+102x^3y^2-68x^2y^3+17xy^4$ a complete cube? [20.]

5. A rectangular plot of ground measures 42 acres, and its diagonal is 1243 yards long. What are its sides? [22.]

6. Two boys start at the same instant from the same corner of a square, the length of one of whose sides is 200 yards, and they run round it in opposite directions: one (A) runs at the rate of 100 yards in 15 seconds, and loses 2 seconds in turning a corner; the other (B) runs at the rate of 100 yards in 16 seconds, and loses 1 second in turning a corner. Where do they meet? [26.]

LVI.

SCIENCE SCHOOLS AND CLASSES, ENG.

Mathematics, First Stage, May, 1881.

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Show that
$$\frac{x^2}{a^2}+\left(\frac{z-x}{b}\right)^2 \text{ and } \frac{z^2}{a^2+b^2}+\frac{a^2+b^2}{a^2b^2}\left(x-\frac{za^2}{a^2+b^2}\right)^2$$
are identical expressions; that is to say, that the one can be deduced from the other. Find their values, when $x=3$, $z=4$, and $a=b=5$. [8.]

2. Simplify $\left(\frac{bc}{a}\right)^{m+n}\left(\frac{a}{c}\right)^m\left(a\right)^n$, and find its value when $a=7$, $b=3$, $c=2$, $m=2$, $n=1$. [8.]

3. Find the G.C.M. and L.C.M. of $2x^4+9x^3+5x+12$ and $2x^4+4x^3+13x^2+11x+12$. [12.]

4. Solve the equations [12.]

(i.) $(2+3x)\left(4-\frac{5}{x}\right)=(3-2x)\left(6-\frac{7}{x}\right)$; [6.]

(ii.) $(x+y)^2-(x-y)^2=352$, $x(y+5)=143$. [6.]

5. Of two squares of carpet, one measures 44 feet more round than the other, and 187 square feet more in area. What are their sizes? [12.]

6. Oranges are bought for half-a-crown a hundred; some are sold at 3*s.* 6*d.* a hundred, and the rest at 2*s.* 10½*d.* a hundred. The same profit is made, as if they had all been sold at 3*s.* 1½*d.* a hundred. Of a thousand oranges sold, how many fetch 3*s.* 6*d.* a hundred? [14.]

LVII.

SCIENCE SCHOOLS AND CLASSES, ENG.

Mathematics, First Stage, May, 1882.

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Divide $x^3+8y^3-27z^3+18xyz$ by $x+2y-3z$, and test your answer by substituting $x=5$, $y=-4$, $z=3$ in the dividend, divisor, and quotient. [12.]

2. Show that the product of $1-\frac{a}{x+2a}$, $1+\frac{b}{x-2b}$, and $x+2(a-b)-\frac{4ab}{x}$ is $\frac{(x+a)(x-b)}{x}$.

Write down the value of each factor and of the product, when $x=3a=3b$. [12.]

3. Reduce to its lowest terms $\frac{x^4-13x^2+36}{x^4-x^3-7x^2+x+6}$. [12.]

4. Divide x^3+px+q by $x-a$, and find the relation that must hold good between a, p, and q, when the division can be performed without leaving a remainder. [12.]

5. Solve the equations [12.]

(i.) $\frac{1}{3}(0.75-x)+\frac{1}{5}(0.47+2x)=(3-\frac{1}{15})x$. [6.]

(ii.) $\frac{x}{15}+\frac{y}{12}=\frac{x}{3}-\frac{y}{4}=1$. [6.]

6. A man has 1000 apples for sale; at first he sells so as to gain at the rate of 50 per cent on the cost price; when he has done this for a time the sale falls off, so he sells the remainder for what he can get, and finds that by doing so he loses at the rate of 10 per cent; if his total gain is at the rate of 29 per cent, how many apples did he sell for what he could get? [14.]

LVIII.

SCIENCE SCHOOLS AND CLASSES, ENGLAND.

Mathematics, Second Stage, May, 1882. — *Arithmetic and Algebra.*

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Given $\sqrt{5}=2.236068$, express $\frac{1}{\sqrt{20}}$ and $\frac{\sqrt{5-2}}{\sqrt{5+2}}$ as decimals, true to the fifth place. [18.]

2. Solve the equations [25.]

(i.) $x^3-2x^2-3x+4=0$;

(ii.) $x^2+y=8$, $3x+2y=7$.

3. A farmer sold 7 oxen and 12 cows for £250. He sold 3 more oxen for £50 than he did cows for £30. Required the price of each. [25.]

4. What is the term which must be added to $9x^4+12x^3+20x+25$ to make it a complete square? [25.]

5. Find $(x+2y)^7$, and obtain the sixth root of 6,321,363,049. [25.]

6. It is known that the volume of a cylinder varies as the base and height jointly. If the volume of the first of two cylinders is to that of the second as 11 : 8, and the height of the first is to that of the second as 3 : 4, and if the base of the first has an area of 16.5 square feet, what is the area of the base of the second? [25.]

LIX.

SCIENCE SCHOOLS AND CLASSES, ENGLAND.

Mathematics, First Stage, May, 1883.

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Explain why the product is a^{12}, when a^5 is multiplied by a^7, and why the quotient is a^5 when a^8 is divided by a^3. [8.]

2. Obtain $(x^3+3x+15)^2-(x^3-3x+15)^2$ in its simplest form, and find its value when $2x=-5$. [12.]

3. Simplify the expressions [12.]

(i.) $\dfrac{x-a}{\frac{1}{a}-\frac{1}{b}}\times\dfrac{a-b}{1-\frac{a}{x}}$;

(ii.) $\left(\frac{1}{2}+\frac{1}{3x}\right)\div\left(9x-\frac{4}{x}\right)$.

4. Find the G.C.M. of $x^4-5x^3-6x^2+35x-7$ and $3x^3-23x^2+43x-8$, and write down these expressions in factors. [12.]

5. Solve the equations [12.]

(i.) $x - \frac{2x-0.3}{0.7} = \frac{5-x}{0.35}$;

(ii.) $\frac{3y}{4} - \frac{2x}{5} = \frac{13}{5}$, $\frac{x}{4} + \frac{y}{5} = \frac{19}{12}$.

6. A sum of £23 14*s.* is to be divided between A, B, C; if B gets 20 per cent more than A and 25 per cent more than C, how much does each get? [16.]

LX.

Civil Service of Great Britain.

Competitive Examination of Candidates for Inspectorships of National Schools, Ireland, 1878. — *Time Allowed*, 3 *hours*.

1. Find the value of $\frac{a^3+a^2-a-1}{b^3-b^2+b-a}$ when $a=2$, $b=\frac{1}{2}$, and the value of $(1+a)^{\frac{1}{2}} \times (1-b)^{-\frac{1}{2}}$ when $a=\frac{1}{3}$, $b=\frac{1}{6}$.

2. Divide $6x^3+x^2+x+2$ by $2x^2-x+1$.
Find the first four terms in the quotient obtained when $1+x$ is divided by $1-2x$.

3. Simplify the expressions
(i.) $(16a^6b^2)^{\frac{1}{2}} \times (a^{\frac{1}{2}}b^{\frac{1}{2}})^2 \times (2a^{\frac{3}{2}}b)^{\frac{1}{2}}$;
(ii.) $(1+a+b)^3+(1+a-b)^3+(1-a+b)^3+(1-a-b)^3$.

4. Multiply $a^3b^{-3}-ab^{-1}+a^{-1}b-a^{-3}b^3$ by $ab^{-1}+a^{-1}b$, and divide $(x^2y^{mn})^{\frac{1}{m}}$ by $(x^{\frac{1}{m}}y^2)^{mn}$.

5. Reduce the fraction $\frac{3x^3-x^2-x-1}{3x^3-4x^2-x+2}$ to its lowest terms.

6. Find the L.C.M. of
(i.) $7a^3bx^2$, $3ax^3$, $6ab^3$;
(ii.) $3x^2-x-2$, $2x^2+x-3$.

7. Find the value of a which will make $x^4 - x^3 - x^2 - ax$ divisible, without remainder, by $x^2 + x$.

8. Solve the equations

(i.) $1 - \frac{2}{x} - \frac{3}{2x} = \frac{4}{3x}$; (ii.) $\frac{x}{1-x} + \frac{1-2x}{2-x} = 1$.

9. Extract the square roots of

(i.) $9 - 4\sqrt{2}$; (ii.) $x^2 - 6x + 13 - 12x^{-1} + 4x^{-2}$.

10. Solve the equations

(i.) $\sqrt{x} - \sqrt{1+x} = \frac{1}{\sqrt{x}}$;

(ii.) $\frac{1-ax}{1+ax} \times \frac{a-x}{a+x} = 1$;

(iii.) $x^2 + xy = 32$, $y^2 - xy = 18$.

11. A train is timed to travel between two stations, A and C, at 45 miles an hour. It travels from A to an intermediate station, B, at the rate of 40 miles per hour, and the speed is then increased to 50 miles an hour. The train arrives punctually at the time appointed. Compare distances of B and C from A.

LXI.

Civil Service of Great Britain.

Open Competitive Examination for Clerkships of the Superior Class in the India Office, 1879.— *Time Allowed*, 3 *hours*.

Full marks may be obtained by doing less than the whole of this paper.

1. If m and n be positive integers, prove that $a^m \times a^n = a^{m+n}$; and, assuming this formula to hold good for all values of the indices, deduce the meanings of $a^{\frac{m}{n}}$, a^0, and a^{-n}.

2. Divide
$x^4+y^4+z^4-2(x^2y^2+y^2z^2+z^2x^2)$ by $x^2+2yz-y^2-z^2$,
and multiply
$2x^{\frac{3}{2}}y^{\frac{1}{2}}+7x^{\frac{1}{2}}y^{\frac{1}{2}}+12y$ by $x^{\frac{1}{2}}y^{-1}-x^{\frac{1}{2}}y^{-\frac{1}{2}}+x^{\frac{1}{2}}y^{-\frac{1}{2}}$.

3. Prove that the L.C.M. of any two algebraical expressions is equal to their product divided by their G.C.M.
Find the G.C.M. and L.C.M. of the expressions
$(x^2+b^2)c+(b^2+c^2)x$ and $(x^2-b^2)c+(b^2-c^2)x$.

4. Simplify the following expressions:

(i.) $\dfrac{1+\dfrac{4x^2}{6xy+9y^2}}{1+\dfrac{9y^2}{4x^2-6xy}} \div \left(\dfrac{16x^4}{81y^4}-\dfrac{2x}{3y}\right)$;

(ii.) $\left(\dfrac{a+y}{a^2+2ax-ay-2xy}-\dfrac{a+2x}{a^2+ay-2ax-2xy}\right) \times \dfrac{a^2+2ax+ay+2xy}{3a^2+ay+2ax-2xy}$;

(iii.) $\dfrac{2\sqrt{a-b}}{3\sqrt{a+b}-2\sqrt{a-b}}+\dfrac{3\sqrt{a+b}}{3\sqrt{a+b}+2\sqrt{a-b}}$.

5. Find the condition that the roots of the equation $ax^2+bx+c=0$ may be real, and show that the roots of the equation $(x+p)(x+q)=pqx^2$ will always be real if p and q are real.

6. Solve the equations

(i.) $\dfrac{x+2a}{(x-2a)^2}-\dfrac{3}{x}=0$;

(ii.) $6\left(x^2+\dfrac{1}{x^2}\right)+5\left(x+\dfrac{1}{x}\right)=38$;

(iii.) $x^3y-x^4=27$, $xy^3-x^3y=84$.

7. A man, who drives twice as fast as he can walk, finds that it takes him 9 hours to drive to a certain town and to walk back, and that when he has accomplished half of the return journey, he meets a man who set out to walk from the same place an hour and a half later, and is travelling a mile and a half per hour more slowly than himself. Find the distance of the town from the starting point, and the rate at which each man walks.

8. Three lamps of equal brilliancy are placed in three different corners of a square room. Compare the intensities of light at the fourth corner and at the centre of the room, assuming that the illumination from a source of light varies inversely as the square of its distance.

9. Show how to insert any number of harmonic means between two given quantities.

If $2p$ and $3q$ be the pth and qth terms, respectively, of an harmonic series, prove that the $(p+q)$th term will be $6(p-q)$.

10. Prove that the number of permutations of n things taken r together is $n(n-1)(n-2)\cdots\cdots(n-r+1)$.

In how many ways can 24 ships belonging to 4 different nations be arranged in 4 lines, each consisting of 6 ships of the same nationality?

11. Expand $(a^2-x^2)^{-\frac{1}{2}}$ to five terms, and show that the middle term in the expansion of $\left(x+\frac{1}{2x}\right)^{2n}$ is equal to $\frac{1\times3\times5\cdots\cdots(2n-1)}{1\times2\times3\cdots\cdots n}$.

12. Express the fourth root of $89-28\sqrt{10}$ as the difference of two surds, and extract the fifth root of 99,999 correct to 15 places of decimals.

LXII.

Civil Service of Great Britain.

Open Competitive Examination for Admission to the Royal Indian Engineering College, July, 1879. — Time Allowed, 3 hours.

1. Find the value of $\frac{a+b}{b-c}+\frac{b+c}{c-d}+\frac{c+d}{d-a}+\frac{d+a}{a-b}$, when $a=6,\ b=4,\ c=3,\ d=1$; and of x^3-4x^2+6x-4, when $x=1+\sqrt{-1}$.

2. Multiply $x-(2+\sqrt{3})y+(1+\sqrt{3})z$ by $x-(2-\sqrt{3})y+(1-\sqrt{3})z$; and divide $x^2-4xy+y^2$ by $x-(2+\sqrt{3})y$.

3. Explain the law of indices in the multiplication and division of algebraic quantities.

   Multiply $x^{\frac{1}{2}}+x^{\frac{1}{3}}+x^{\frac{1}{4}}$ by $x^{\frac{1}{4}}-x^{-\frac{1}{4}}$; and reduce to its simplest form $(x^{\frac{1}{2}}\times x^{-\frac{1}{3}})^{-\frac{1}{2}}+x^{-\frac{1}{4}}$.

4. Simplify the following expressions:

   (i.) $\dfrac{25x^4+5x^3-x-1}{20x^4+x^3-1}$;

   (ii.) $\dfrac{\dfrac{a+x}{a-x}+\dfrac{a-x}{a+x}}{1-\dfrac{a^2-x^2}{a^2+x^2}}\div\dfrac{\dfrac{a^2+x^2}{a^2-x^2}-\dfrac{a^2-x^2}{a^2+x^2}}{1-\dfrac{a^4+x^4}{(a^2+x^2)^2}}$.

5. Solve the equations

   (i.) $\sqrt{x}+\sqrt{4+x}=\dfrac{4}{\sqrt{x}}$;

   (ii.) $\dfrac{x}{10}-\dfrac{1}{5}\left(\dfrac{x+1}{x+2}\right)+\dfrac{3-2x}{20}=0$;

   (iii.) $\dfrac{x+1}{x-1}-\dfrac{1}{2}\left(\dfrac{x-1}{x+1}\right)=1\frac{1}{4}$;

   (iv.) $x^2+y^2=8,\ 2xy-y^2=4$.

6. A number having two digits is to the number formed by inverting the order of the digits as 8 to 3, and the sum of the two numbers is 99. Find them.

7. Prove the rule for finding the G.C.M. of two algebraical quantities.

Find the L.C.M. of

a^2-ab+b^2, a^2+ab+b^2, a^3-b^3, a^3+b^3, and $(a^2-b^2)^3$.

8. Find the square roots of

$81x^4+108x^3-24x+4$ and $327-87\sqrt{15}$.

9. If p, q, and r are the arithmetic, geometric, and harmonic means of two algebraical quantities, show that

(i.) $pr=q^2$; (ii.) $p-\frac{r}{2}=\sqrt{p^2-q^2}$.

10. Sum to n terms, and where possible to infinity,

(i.) $3,\ 2\frac{1}{3},\ 1\frac{2}{3}$ .....

(ii.) $3,\ 2\frac{1}{3},\ 1\frac{22}{27}$ .....

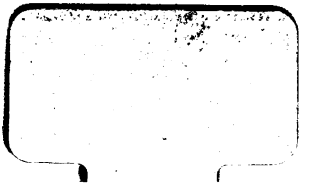
(iii.) $\frac{1}{2\times 3},\ \frac{1}{3\times 4},\ \frac{1}{4\times 5}$ .....

11. Distinguish between permutations and combinations, and find the number of the latter that can be formed from n things taken r together.

In how many ways can a guard of 8 soldiers be selected from a company of 25, and in how many of them will two particular men be on guard together?

12. Find the sixth term of $\left(3x-\frac{y}{3}\right)^{-\frac{1}{2}}$, and the two middle terms of $(a-x)^9$.

13. If $\frac{a+bx}{b+cy}=\frac{b+cx}{c+ay}=\frac{c+ax}{a+by}$, show that

$a^3+b^3+c^3-3abc=0$.

Purchased 190